ANNALES UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN-POLONIA

VOL.XXXIX,1 SECTIO A 1985

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Prestarlike Functions of Order α and Type β with Negative Coefficients

Funkcje pregwiaździste rzędu α i typu β o ujemnych współczynnikach

Презвездообразные функции порядка α и типа β с отрицательными коэффициентами

1. Introduction. Let S denote the class of functions normalized by $f(0) = 0$, $f'(0) = 1$ that are analytic and univalent in the unit disk $U = \{z : |z| < 1\}$. A function $f \in S$ is called starlike of order α $(0 \le \alpha \le 1)$, denoted $f \in S^*(\alpha)$, if $\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \alpha$, $z \in U$, and is called convex of order α, denoted $f \in K(\alpha)$, if $\operatorname{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha$, $z \in U$. Further, let T, $T^*(\alpha)$ and $C(\alpha)$ denote the subclasses of S whose elements can be written in the form

$$f(z) = z - \sum_{n=2}^{\infty} |a_n| z^n \ , \quad z \in U \ . \tag{1.1}$$

The Hadamard product (convolution) of two power series:

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

and

$$g(z) = \sum_{n=0}^{\infty} b_n z^n$$

is defined as the power series

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n .$$

Let

$$D^\alpha f(z) = f(z) * \frac{z}{(1-z)^{\alpha+1}} , \quad \alpha \geq -1 . \tag{1.2}$$

In the sequel, we let

$$C(\alpha, n) = \frac{\prod_{k=2}^{n} (k + \alpha - 1)}{(n-1)!} , \quad n = 2, 3, \ldots . \tag{1.3}$$

Thus

$$\frac{z}{(1-z)^{\alpha+1}} = z + \sum_{n=2}^{\infty} C(\alpha, n) z^n .$$

Let R_α denote the class of all analytic function $f(z)$ satisfying the relation :

$$\operatorname{Re} \frac{D^{\alpha+1} f(z)}{D^\alpha f(z)} > \frac{1}{2} , \quad \alpha \geq -1 , \quad z \in U . \tag{1.4}$$

Ruscheweyh [3] called this class *prestarlike of order* α, see also Al-Amiri [1]. Ruscheweyh obtained the basic relation $R_\alpha \subset R_\beta$, $\alpha \geq \beta \geq -1$. Since $R_0 = S^*\left(\frac{1}{2}\right) \subset S$, it follows that R_α consists of univalent functions for at least $\alpha \geq 0$.

Now we introduce the class of all analytic functions f, denoted $f \in R_\beta(\alpha)$ ($\alpha \geq 0$, $0 \leq \beta < 1$), satisfying

$$\operatorname{Re} \frac{D^{\alpha+1} f(z)}{D^\alpha f(z)} > \frac{\alpha + 2\beta}{2(\alpha+1)} = r(\alpha, \beta) , \quad z \in U . \tag{1.5}$$

R. Jenkovic, see [3,p.71], has shown that $R_0(\alpha)$ consists of univalent functions and the number $r(\alpha, 0)$ can not be replaced by any smaller number without violating the univalence property of the class. Consequently, $R_\beta(\alpha) \subset S$. Further $R_{1/2}(\alpha) = R_\alpha$ and $R_0(\beta) = S^*(\beta)$. We call $R_\beta(\alpha)$ the class of *prestarlike functions of order* α *and type* β. Let

$$T_\beta^*(\alpha) = T \cap R_\beta(\alpha) . \tag{1.6}$$

Note that $T_\beta^*(0) = T^*(\beta)$ and $T_{\beta+1/2}(0) = C(\beta)$ for $0 \leq \beta < 1$.

The purpose of this note is to investigate the class $T_\beta^*(\alpha)$, the class of prestarlike functions of order α and type β with negative coefficients. In section 2, we obtain a sufficient condition for a function f to belong to $R_\beta(\alpha)$ and show that this condition is also necessary for the subclass of $T_\beta^*(\alpha)$. In section 3, some distortion and covering theorems are obtained for $T^*_\beta(\alpha)$. Further, in section 4, we obtain the order of starlikeness for $T^*_\beta(\alpha)$. In section 5, a sequence of functions $\{f_n\}$, $f_n \in T^*_\beta(\alpha)$, $n = 2, 3, \ldots$, which characterized the class $T^*_\beta(\alpha)$ is determined. Finally, we show that if f and g are in $T^*_\beta(\alpha)$ so is their Hadamard product $f * g$.

Some special cases of our results can be found in Merkes et al.[2], Silverman [5], and Silverman and Silvia [6].

In the sequel we shall assume that the coefficients of a function in $T^*_\beta(\alpha)$ is given by (1.1) unless otherwise stated.

2. Coefficient Inequalities. We begin with a theorem that relates the order and type of $R_\beta(\alpha)$ to the modulus of the coefficients.

Theorem 1. *Let*

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n .$$

If

$$\sum_{n=2}^{\infty} (2n + \alpha - 2\beta) C(\alpha, n)|a_n| \le 2 + \alpha - 2\beta ,$$

then $f \in R_\beta(\alpha)$, $\alpha \ge 0$, $0 \le \beta < 1$, and where $C(\alpha,n)$ as given by (1.3).

Proof. It suffices to show that the values for $\dfrac{D^{\alpha+1}f}{D^\alpha f}$ lie in a circle centered at $w = 1$ whose radius is $\dfrac{2+\alpha-2\beta}{2(\alpha+1)}$.

Thus

$$\left|\frac{D^{\alpha+1}f(z)}{D^\alpha f(z)} - 1\right| = \left|\frac{D^{\alpha+1}f(z) - D^\alpha f(z)}{D^\alpha f(z)}\right| \le$$

$$\le \frac{\sum_{n=2}^{\infty} (C(\alpha+1,n) - C(\alpha,n))|a_n|\,|z|^{n-1}}{1 - \sum_{n=2}^{\infty} C(\alpha,n)|a_n|\,|z|^{n-1}} \le$$

$$\le \frac{\sum_{n=2}^{\infty} C(\alpha,n)(n-1)|a_n|/(1-\alpha)}{1 - \sum_{n=2}^{\infty} C(\alpha,n)|a_n|}$$

since from (1.3),

$$C(\alpha+1,n) = \frac{(\alpha+n)C(\alpha,n)}{1+\alpha}$$

The right hand side of the inequality is bounded above by

$$\frac{2+\alpha-2\beta}{2(1+\alpha)}$$

provided

$$2\sum_{n=2}^{\infty} C(\alpha,n)(n-1)|a_n| \le (2+\alpha-2\beta)\left(1-\sum_{n=2}^{\infty} C(\alpha,n)|a_n|\right)$$

which is equivalent to the inequality given by the hypothesis of the theorem. The proof is complete.

For functions in $T_\beta^*(\alpha)$, the converse is also true.

Theorem 2. *A function* $f(z) = z - \sum_{n=2}^{\infty} |a_n| z^n$ *is in* $T_\beta^*(\alpha)$ *if and only if*

$$\sum_{n=2}^{\infty} (2n+\alpha-2\beta)C(\alpha,n)|a_n| \le 2+\alpha-2\beta \ , \quad \alpha \ge 0 \ , \quad 0 \le \beta < 1 \ . \tag{2.1}$$

Proof. In view of Theorem 1. we need only to show the necessary part. From the identity

$$\frac{z}{(1+z)^{\alpha+2}} = \frac{z}{(1-z)^{\alpha+1}} * \left(\frac{\alpha}{\alpha+1}\cdot\frac{z}{1-z} + \frac{1}{\alpha+1}\cdot\frac{z}{(1-z)^2}\right) \ , \quad \alpha > -1$$

one can easily show

$$z(D^\alpha f(z))' = (\alpha+1)D^{\alpha+1}f(z) - \alpha D^\alpha f(z) \ . \tag{2.2}$$

Using the definition of the class $T_\beta^*(\alpha)$ and (2.2) we observe that $f \in T_\beta^*(\alpha)$ implies $D^\alpha f \in T^*(\beta - \frac{\alpha}{2})$. It is known from Merkes et al. [2] that $g \in T^*(\lambda)$ if and only if

$$\sum_{n=2}^{\infty} (n-\lambda)|b_n| \le 1-\lambda \ ,$$

where

$$g(z) = z - \sum_{n=2}^{\infty} |b_n| z^n \ .$$

Applying this inequality to

$$g(z) = D^\alpha f(z) = z - \sum_{n=2}^{\infty} C(\alpha,n)|a_n| z^n \ ,$$

we obtain (2.1) immediately. The proof of Theorem 2 is now complete.

Corollary 1. *Let* $f \in T_\beta^*(\alpha)$, $\alpha \ge 0, 0 \le \beta < 1$. *Then*

$$|a_n| \le \frac{2+\alpha-2\beta}{(2n+\alpha-2\beta)C(\alpha,n)} \ , \quad n = 2,3,\ldots \ . \tag{2.3}$$

Equality holds only for the functions

$$f_n(z) = z - \frac{2+\alpha-2\beta}{(2n+\alpha-2\beta)C(\alpha,n)}z^n \quad . \tag{2.4}$$

Remark. Theorems 1,2 and Corollary 1 have been shown by Merkes et al. [2], $\alpha = 0$, $0 \le \beta < 1$ and Silverman and Silvia [6] $\beta = \frac{1}{2}$ and $\alpha \ge 0$.

3. Distortion and covering theorems. In this section we apply inequality (2.1) to obtain some distortion and covering results for $T^*_\beta(\alpha)$.

Theorem 3. *Let* $f \in T^*_\beta(\alpha)$, $\alpha \ge 0$, $0 \le \beta < 1$. *Then*

$$r - M(\alpha,\beta,2)r^2 \le |f(z)| \le r + M(\alpha,\beta,2)r^2 \ , \ \text{where } |z| = r < 1 \ , \tag{3.1}$$

and where

$$M(\alpha,\beta,2) = \frac{2+\alpha-\beta}{(4+\alpha-2\beta)(1+\alpha)} \quad . \tag{3.2}$$

Equality only for

$$f_2(z) = z - \frac{2+\alpha-\beta}{(4+\alpha-2\beta)(1+\alpha)}z^2 \ , \quad at\ z = \pm r \ .$$

Proof. Let

$$\frac{1}{M(\alpha,\beta,n)} = A(\alpha,\beta,n) = \frac{(2n+\alpha-2\beta)C(\alpha,n)}{2+\alpha-2\beta} \ , \quad n = 2,3,\ldots \ .$$

Then

$$A(\alpha,\beta,n) = \frac{(2n+\alpha-2\beta)nC(\alpha,n+1)}{(2+\alpha-2\beta)(n+\alpha)} \le A(\alpha,\beta,n+1)$$

provided

$$n(2n+\alpha-2\beta) \le (n+\alpha)(2n+2+\alpha-2\beta) \ ,$$

which is clearly true for the specified range of α and β. Thus $A(\alpha,\beta,n)$ is an increasing function of n. Since $C(\alpha,2) = 1+\alpha$, the above result and (3.3) imply

$$A(\alpha,\beta,n) \ge A(\alpha,\beta,2) = \frac{1}{M(\alpha,\beta,2)} \ , \quad n = 2,3,\ldots \ , \tag{3.4}$$

where $M(\alpha,\beta,2)$ is as given by (3.2). Combining (2.1) and (3.4) we get

$$A(\alpha,\beta,2)\sum_{n=2}^{\infty}|a_n| \le \sum_{n=2}^{\infty}A(\alpha,\beta,n)|a_n| \le 1$$

which implies

$$\sum_{n=2}^{\infty}|a_n| \le M(\alpha,\beta,2) \tag{3.5}$$

Applying (3.5) to

$$r-\sum_{n=2}^{\infty}|a_n|r^2 \leq |f(z)| \leq r+\sum_{n=2}^{\infty}|a_n|r^2 \;, \quad |z|=r$$

we get (3.1).

Corollary 2. *The unit disk U is mapped under any function $f \in T^*_\beta(\alpha)$ onto a domain containing the disk*

$$|W| \leq \frac{\alpha^2+2(2-\beta)\alpha+2}{(\alpha+1)(4+\alpha-2\beta)} \;, \quad \alpha \geq 0 \;, \quad 0 \leq \beta < 1 \;.$$

This result is sharp for $f_2(z)$ given by Theorem 3.

Proof. Let $r \to 1^-$ in Theorem 3.

Theorem 4. *Let $f \in T^*_\beta(\alpha)$, $\alpha \geq 0$, $0 \leq \beta < 1$. Then*

$$1-2M(\alpha,\beta,2) \leq |f'(z)| \leq 1+2M(\alpha,\beta,2)r \;, \; for\; |z|=r<1 \;, \tag{3.4}$$

where $M(\alpha,\beta,2)$ is given by (3.2). Equality holds only for $f_2(z)$ of Theorem 3 at $z=\pm r$.

Proof. Let

$$B(\alpha,\beta,n)=\frac{1}{nM(\alpha,\beta,n)} \;, \text{ where } M(\alpha,\beta,n) \text{ is given by (3.3)} \;.$$

An argument similar to the one used in the proof of Theorem 3 shows that $B(\alpha,\beta,n)$ is also an increasing function of n provided $\alpha^2+\alpha+2\beta+2\alpha(n-\beta) \geq 0$, which is true for the stated range of α and β and for $n \geq 1$. Thus $B(\alpha,\beta,n) \geq B(\alpha,\beta,2)$, $n=2,3,\ldots.$ This inequality and (2.1) yield

$$B(\alpha,\beta,2)\sum_{n=2}^{\infty}n|a_n| \leq \sum_{n=2}^{\infty}B(\alpha,\beta,n)\cdot n|a_n| \leq 1 \;.$$

Consequently,

$$\sum_{n=2}^{\infty}n|a_n| \leq 2M(\alpha,\beta,2) \;. \tag{3.5}$$

Applying (3.5) to

$$1-\sum_{n=2}^{\infty}n|a_n|r^2 \leq |f'(z)| \leq 1+\sum_{n=2}^{\infty}n|a_n|r^2 \;,$$

we get (3.4).

Remark. Theorems 3,4 and Corollary 2 have been shown by Silverman [5], $\alpha=0$, $0 \leq \beta \leq 1$, and by Silverman and Silvia[6] $\alpha \geq 0$, $\beta=\frac{1}{2}$.

4. Order of starlikeness and related problems. The following theorem determines the order of starlikeness of the class $T_\beta^*(\alpha)$.

Theorem 5. *If $f \in T_\beta^*(\alpha)$, $\alpha \geq 0$, $0 \leq \beta < 1$, then $f \in T^*(\lambda)$, where*

$$\lambda = \frac{\alpha^2 + 3\alpha + 2(1-\alpha)\beta}{\alpha^2 + 4\alpha + 2 - 2\alpha\beta} .$$

Equality holds only for

$$f_2(z) = z - \frac{2 + \alpha - 2\beta}{(4 + \alpha - 2\beta)(1+\alpha)} z^2 .$$

Proof. Theorem (2.2) shows that $f \in T^*(\lambda)$ if and only if

$$\sum_{n=2}^{\infty} \frac{n-\lambda}{1-\lambda} |a_n| \leq 1 , \tag{4.1}$$

and $f \in T_\alpha^*(\beta)$ if and only if

$$\sum_{n=2}^{\infty} \frac{2n + \alpha - 2\beta}{2 + \alpha - 2\beta} \cdot C(\alpha, n) |a_n| \leq 1 . \tag{4.2}$$

Consequently, it suffices to show that (4.2) implies (4.1). However, this is the case if

$$g(\alpha, n) = \frac{2 + \alpha - 2\beta}{2n + \alpha - 2\beta} \cdot \frac{n - \lambda}{1 - \lambda} \cdot \frac{1}{C(\alpha, n)} \leq 1 .$$

Since $g(\alpha, 1) = 1$, we need only show that $g(\alpha, n)$ is a decreasing sequence of n. In view of (1.3), $g(\alpha, n+1) \leq g(\alpha, n)$ whenever

$$\frac{n(n+1-\lambda)}{(n+\alpha)(2n+2+\alpha-2\beta)} \leq \frac{n-\lambda}{2n+\alpha-2\beta} ,$$

or, equivalently, when

$$h(\alpha, n) = 2\alpha n^2 + n(\alpha^2 + \alpha - 2\alpha\beta + 2\beta - 2\alpha\lambda - 2\lambda) - \lambda(\alpha^2 + 2\alpha - 2\alpha\beta) \geq 0 . \tag{4.3}$$

Since $h(\alpha, 1) = 0$, and

$$h(\alpha, n+1) - h(\alpha, n) = 4n\alpha + \lambda(\alpha^2 + 2\alpha - 2\alpha\beta) \geq 0 ,$$

inequality (4.3) is satisfied. The proof of Theorem 5 is now completed.

Corollary 3. *If $f \in C(\alpha)$, then $f \in T^*\left(\frac{2}{3-\alpha}\right)$, $-\frac{1}{2} \leq \alpha \leq \frac{1}{2}$. This result is sharp for*

$$f(z) = z - \frac{1-\alpha}{2(2-\alpha)} z^2 .$$

Proof. Since $T^*_\beta(1) \equiv C\left(\beta - \frac{1}{2}\right)$, Theorem 5 implies that a convex function of order $\beta - \frac{1}{2}$ is a starlike function of order $\lambda = \frac{4}{7-2\beta}$. Now replacing $\beta - \frac{1}{2}$ by α, we see that $C(\alpha) \subset T^*\left(\frac{2}{3-\alpha}\right)$, for $-\frac{1}{2} \le \alpha = \beta - \frac{1}{2} \le \frac{1}{2}$.

Remark. Theorem 5 is known by Silverman and Silvia [6] for the case $\beta = 1/2$ and $0 \le \alpha \le 1$. Corollary 3 was obtained by Silverman [5] when $0 \le \alpha \le 1$.

Theorem 6. *If $f \in T^*_\beta(\alpha)$, then for $\alpha \le \lambda$,*

$$\operatorname{Re} \frac{D^{\lambda+1} f(z)}{D^\lambda f(z)} > \frac{\lambda+\beta}{2(\lambda+1)}$$

is valid in the disk of radius

$$r(\alpha,\beta,\lambda) = \min_n \left\{ \frac{2+\lambda-2\beta}{2+\alpha-2\beta} \cdot \frac{2n+\alpha-2\beta}{2n+\lambda-2\beta} \cdot \frac{C(\alpha,n)}{C(\lambda,n)} \right\}^{\frac{1}{(n-1)}} . \tag{4.4}$$

The result is sharp for

$$f_n(z) = z - \frac{2+\alpha-2\beta}{(2n+\alpha-2\beta)C(\alpha,n)} z^n \quad , \quad n = 2,3\ldots .$$

Proof. Let $f(z) = z - \sum_{n=2}^{\infty} |a_n| z^n \in T^*_\beta(\alpha)$. It suffices to show

$$\left| \frac{D^{\lambda+1} f(z)}{D^\lambda f(z)} - 1 \right| < \frac{2+\lambda-2\beta}{2(\lambda+1)}$$

is valid for $|z| \le r(\alpha,\beta,\lambda)$ where $r(\alpha,\beta,\lambda)$ is given by (4.4). This will show if

$$\frac{\sum_{n=2}^{\infty} C(\lambda,n)(n-1)|a_n|\,|z|^{n-1}/(\lambda+1)}{1 - \sum_{n=2}^{\infty} C(\lambda,n)|a_n|\,|z|^{n-1}} < \frac{2+\lambda-2\beta}{2(\lambda+1)} ,$$

or

$$\sum_{n=2}^{\infty} C(\lambda,n)(2n+\lambda-2\beta)|a_n|\,|z|^{n-1} < 2+\lambda-2\beta \tag{4.5}$$

$$\text{for } |z| = r(\alpha,\beta,\lambda) .$$

Since $f \in T^*_\beta(\alpha)$, (2.1) implies (4.5) if

$$\frac{C(\lambda,n)(2n+\lambda-2\beta)}{2+\lambda-2\beta} |z|^{n-1} \le \frac{C(\alpha,n)(2n+\alpha-2\beta)}{2+\alpha-2\beta}$$

is satisfied for $|z| \le r(\alpha, \beta, \lambda)$. This is obviously the case.The proof is complete.

Remark. For $\beta = 1/2$, $\lambda = 1$, Theorem 6 shows that the radius of univalence and convexity for functions in R_α with negative coefficients is

$$r_0 = \min_n \left[\frac{(2n+\alpha-1)C(\alpha,n)}{(1+\alpha)n^2}\right]^{\frac{1}{n-1}} , \quad 0 \le \alpha \le 1$$

This results is also known, [6].

5. Extreme points for T_β^*. Theorem 7. *Set* $f_1(z) = z$ *and*

$$f_n(z) = z - \frac{2+\alpha-2\beta}{(2n+\alpha-2\beta)C(\alpha,n)} z^n \ . \quad n = 2, 3 \dots \ . \tag{4.6}$$

Then $f \in T_\beta^*(\alpha)$, $\alpha \ge 0$, $0 \le \beta < 1$ *if and only if it can be expressed in the form*

$$f(z) = \sum_{n=1}^{\infty} \lambda_n f_n(z) \ ,$$

where $\lambda_n \ge 0$, *and* $\sum_{n=1}^{\infty} \lambda_n = 1$.

Proof. Suppose $f(z) = \sum_{n=1}^{\infty} \lambda_n f_n(z)$, where λ_n, and $f_n(z)$ are as given by the theorem. Then

$$\sum_{n=2}^{\infty} \frac{(2n+\alpha-2\beta)(C(\alpha,n)}{2+\alpha-2\beta} \cdot \frac{\lambda_n(2+\alpha-2\beta)}{(2n+\alpha-2\beta)C(\alpha,n)} = \sum_{n=2}^{\infty} \lambda_n = 1 - \lambda_1 \le 1 \ .$$

According to Theorem 2 f belongs to $T_\beta^*(\alpha)$.

Conversly, suppose

$$f(z) = z - \sum_{n=2}^{\infty} |a_n| z^n \in T_\beta^*(\alpha) \ .$$

Then by Corollary 1

$$|a_n| \le \frac{2+\alpha-2\beta}{(2n+\alpha-2\beta)C(\alpha,n)} , \quad n = 2, 3, \dots \ .$$

If we set

$$\lambda_n = \frac{2n+\alpha-2\beta}{2+\alpha-2\beta} \cdot C(\alpha,n)|a_n| \,, n = 2, 3, \dots \ , \text{ and } \lambda_1 = 1 - \sum_{n=2}^{\infty} \lambda_n \ ,$$

we conclude by virtue of Theorem 2 that $\sum_{n=2}^{\infty} \lambda_n \leq 1$ and consequently $\lambda_1 \geq 0$. Thus $f(z)$ has the desirable representation.

Remark. For $\beta = \frac{1}{2}$, $0 \leq \alpha \leq 1$, Theorem 6 is shown in [6].

Theorem 8. *If* $f \in T^*_\beta(\alpha)$, $g \in T^*_\beta(\lambda)$, *then*

$$f * g \in T^*_\beta(\alpha) \cap T^*_\beta(\lambda) .$$

Proof. Suppose

$$f(z) = z - \sum_{n=2}^{\infty} |a_n| z^n \in T^*_\beta(\alpha)$$

and

$$g(z) = z - \sum_{n=2}^{\infty} |b_n| z^n \in T^*_\beta(\lambda) .$$

We observe that $|a_n| \leq 1$, $|b_n| \leq 1$ for $n = 2, 3, \ldots$. Thus using (2.1) for f and $|b_n| \leq 1$ for g we get

$$\sum_{n=2}^{\infty} \frac{2n + \alpha - 2\beta}{2 + \alpha - 2\beta} C(\alpha, n) |a_n| \, |b_n| \leq 1 ,$$

which implies $f * g \in T^*_\beta(\alpha)$. Similarly, we can show that $f * g \in T^*_\beta(\lambda)$. This completes the proof of Theorem 8.

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STRESZCZENIE

W pracy wprowadza się funkcje pregwiaździste rzędu α i typu β. Otrzymano nierówności dla współczynników, które charakteryzują funkcje pregwiaździste rzędu α i typu β, mające współczynniki ujemne. Otrzymano również twierdzenia o zniekształceniu i o pokryciu, a także wyznaczono promienie jednolistności i pregwiaździstości.

РЕЗЮМЕ

В данной работе введено презвездообразные функции ряда α и типа β. Полученные неравенства на коэффициенты характеризуют презвездообразные функции ряда α и типа β, которые имеют отрицательные коэффициенты. Получено теоремы искажения и покрытия, а также определено радиусы однолистности и звездообразности.