ANNALES
UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN—POLONIA
VOL. LII/LIII, 3 SECTIO AAA 1997/1998

Institute of Physics, Maria Curie-Skłodowska University,
20-031 Lublin, pl. M. Curie-Skłodowskiej 1, Poland

WIESŁAW A. KAMIŃSKI, ALEKSANDER D. LINKEVICH*

Correlation functions of nonlinear dynamical systems and methods of quantum field theory

Funkcja korelacji nieliniowych układów dynamicznych a metody kwantowej teorii pola

1. INTRODUCTION

As it is known, the nonlinear dynamical systems such as neural networks can possess a variety of types of behaviour including convergence to fixed points, persistent oscillations (limit cycles) and chaotic motions. In spite of great efforts to study such systems on the basis of functional analysis, differential geometry and topology and other mathematical sciences, fairly little knowledge has been obtained analytically, and the main research tool are still computer simulations. For this reason, in this paper we discuss the problem of developing analytic methods and look for support in the experience of physical studies. More precisely, we try to borrow methods constructed in the quantum field theory by Feynman, Dyson, Schwinger and others (see for introduction, e.g., [1–3]). The field-theoretic methods are successfuly applied to the problems of quantum statistical physics as well (see, e.g. [4–8]). The seminal paper [9] has opened the door for statistical theories for classical systems that possess the power close to that of the Schwinger–Dyson functional methods and the Feynman diagrammatic perturbation theory in the quantum field theory. Further this approach was mainly developed for the stochastic dynamical systems for which the average of observables is undertaken over noise [10–19]. In contrast, deterministic dynamical systems are the subject of the present paper and our final target is to elaborate such a technique that would enable us to investigate

* Department of Physics, Polatsk State University, Novo Polatsk, Belarus

deterministic chaos also in the neural systems. (Nevertheless, introducing some auxiliary noise into the system under consideration with its subsequent elimination is used in section 3).

We study a system whose state at the time t is described by the variables $\varphi_1(t)$,..., $\varphi_N(t) \in \mathbf{R}$ which evolve in time so as

$$\varphi_i(t) = F_i[\varphi(t)], \quad i = 1, \ldots, N \tag{1.1}$$

Here $F[\varphi]$ is a (nonlinear) function depending on the vector of the state variables $\varphi(\mathrm{t}) = (\varphi_1(\mathrm{t}), \ldots, \varphi_N(t))$. The quantities of interest are the correlation functions (or Green functions)

$$G_{j1 \ldots jm}(t_1, \ldots, t_m \mid m) = \langle \varphi_{j1}(t_1) \ldots \varphi_{jm}(t_m) \rangle \tag{1.2}$$

where the average is undertaken over the initial conditions (initial values of the state lying inside the basin of attraction of an attractor). They are of importance because they enable to conclude any type of the attractor.

As a specific example we will consider the known Lorenz system

$$\begin{aligned} \dot{x} &= \sigma(-x + y) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= -bz + xy \end{aligned} \tag{1.3}$$

where σ, r and b are some positive constants.

2. THE OPERATOR FORMALISM OF QUANTUM THEORY AND EQUATIONS FOR CORRELATION AND RESPONSE FUNCTIONS

Let us introduce the additional dynamical variables $\tilde{\varphi}_1(t), \ldots, \tilde{\varphi}_N(t) \in \mathbf{R}$ which satisfy both equations of motion

$$\tilde{\varphi}_i(t) = -\tilde{\varphi}_j(t) \frac{\delta F_j[\varphi(t)]}{\delta \varphi_i(t)}, \quad i = 1, \ldots, N \tag{2.1}$$

and the canonical commutation relations

$$[\tilde{\varphi}_i(t), \tilde{\varphi}_j(t)] = i\delta_{ij}, \quad i, j = 1, \ldots, \mathrm{N} \tag{2.2}$$

In eq. (2.1) and below summation over repeated indices is assumed. The appearance of the fields $\tilde{\varphi}$ in addition to the φ is caused by that, in contrast to the quantum systems for which the expectation values of products of the field operators φ and their Hermitian conjugates φ^{+} describe both correlations in a system and the response of the system to its external perturbations, no counterparts of the conjugated fields φ^{+} occur in the case of classical systems and consequently the response functions do not emerge automatically in the theory [9]. One can introduce the operator

$$H(t) = \tilde{\varphi}_j(t)\, F_j\,[\varphi(t)] \tag{2.3}$$

which plays the role of a non-Hermitian Hamiltonian in such a way that the equations of motion (1.1) and (2.1) recast into the cannonical form

$$\varphi_i(t) = i\,[H(t), \varphi_i(t)], \quad \tilde{\varphi}_i(t) = i\,[H(t), \tilde{\varphi}_i(t)] \tag{2.4}$$

It is convenient to combine the variables $\varphi_i(t)$ and $\tilde{\varphi}_i(t)$ into the two-component one $\Phi_i(t)$ so as

$$\Phi_i(t) = \begin{pmatrix} \varphi_i(t) \\ \tilde{\varphi}_i(t) \end{pmatrix}$$

Then the commutation relations (2.2) can be written in the form

$$[\Phi_i(t), \Phi_j(t)] = -i\sigma\delta_{ij}, \quad i, j = 1, \ldots, N, \quad \sigma = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{2.5}$$

and the equations of motions (2.4) so as

$$\Phi_i(t) = i\,[H(t), \Phi_i(t)], \quad i = 1, \ldots, N \tag{2.6}$$

For any polynomial functions $F_1[\varphi], \ldots, F_N[\varphi]$, the Hamiltonian $H(t)$ may be represented as

$$H(t) = \sum_k U^{(k)}_{i_1 \ldots i_k}\, \Phi_{i_1}(t) \ldots \Phi_{i_k}(t) \tag{2.7}$$

with the symmetrized coefficients $U^{(k)}_{i_1 \ldots i_k}$. From eqs.(2.5)–(2.7) it follows that

$$\sigma\dot{\Phi}_i(t) = \sum_k kU^{(k)}_{i_1 \ldots i_k} \quad \Phi_{i_2}(t) \ldots \Phi_{i_k}(t) \tag{2.8}$$

Thus the quantities $\Phi_i(t)$ can be regarded as dynamical variables in the Heisenberg picture. We can define them in the Schrodinger picture so as $\Phi_i^s = \Phi_i(t_0)$ where t_0 is an arbitrary time moment treated as the initial time value. The relationship between the two pictures is well known. So, the dynamical variables $\Phi_i(t)$ and Φ_i^s are connected by the transformation

$$\Phi_i(t) = \exp\{iH^s \cdot (t - t_0)\} \; \Phi_i^s \exp\{-iH^s \cdot (t - t_0)\}$$

which contains the operator (cf. eq.(2.7))

$$H^s = \sum_k U^{(k)}_{i_1 \ldots i_k} \quad \Phi_{i_1}^s \ldots \Phi_{i_k}^s$$

playing the role of the Hamiltonian of the system under consideration in the Schrodinger picture. It is easy to see that

$$H(t) = \exp\{iH^s \cdot (t - t_0)\} \; H^s \exp\{-iH^s \cdot (t - t_0)\} = H^s$$

Now we introduce the auxiliary source fields $\eta_i(t)$ and construct the operator [21]

$$S(t_2, \; t_1) = T \; \exp\left\{-i \int_{t_1}^{t_2} dt \;\; h(t)\right\} \tag{2.9}$$

where T is the usual chronological ordering operator which arranges the operators in order of increasing time from right to left and

$$h(t) = \eta_i(t) \; \Phi_i(t) \tag{2.10}$$

Using this operator we turn from the Heisenberg picture to the η–picture as follows:

$$\Phi_i^\eta(t) = S(0,t) \; \Phi_i(t) \; S(t,0) \tag{2.11}$$

Then one can easily derive the equations of motion for these dynamical variables. They are

$$\dot{\Phi}_i^\eta(t) = i[\mathrm{H}^\eta(t), \Phi_i^\eta(t)], \quad i = 1, \dots, N \tag{2.12}$$

where

$$H^\eta(t) = \eta_i(t)\,\Phi_i^\eta(t) + \sum_k U_{i_1 \dots i_k}^{(k)}\, \Phi_{i_1}^\eta(t) \dots \Phi_{i_k}^\eta(t) \tag{2.13}$$

is the Hamiltonian of the system with the additional external source $\eta(t)$. The commutation relations (2.5) are kept intact. This η-representation can be viewed as the Heisenberg picture for the generalized system described by the Hamiltonian (2.13). From eqs.(2.12) and (2.13) it follows that

$$\sigma\dot{\Phi}_i^\eta(t) = \eta_i(t) + \sum_k kU_{ii_2 \dots i_k}^{(k)}\, \Phi_{i_2}^\eta(t) \ \dots\ \Phi_{i_k}^\eta(t) \tag{2.14}$$

Now we are in position to deal with the Green functions (1.2) which are defined in terms of the two-component variables $\Phi_i(t)$ as

$$\begin{aligned} G_{i_1 \dots i_k}(t_1, \dots, t_k \,|\, k) &= \int d\Phi(t_0)\, P[\Phi(t_0)]\, T\{\Phi_{i_1}(t_1) \dots \Phi_{i_k}(t_k)\} \\ &\equiv \hat{P}T\{\Phi_{i_1}(t_1) \dots \Phi_{i_k}(t_k)\} \end{aligned} \tag{2.15}$$

Here $d\Phi(t_0) = d\Phi_1(t_0) \dots d\Phi_N(t_0)$, $P[\Phi(t_0)] = p[\varphi(t_0)] \cdot \delta[\tilde{\varphi}(t_0)]$, and $p[\varphi(t_0)]$ is the probability distribution density for the initial state $\varphi(t_0)$ of the system while $\delta[\tilde{\varphi}(t_0)] = \delta[\tilde{\varphi}_1(t_0)] \dots \delta[\tilde{\varphi}_N(t_0)]$ is the product of the Dirac δ-functions. The operator $\hat{P}$ symbolically denotes taking the average over the initial conditions as defined in (2.15). More precisely, eq.(2.15) defines a matrix function whose elements are both the correlation functions and response functions of the system.

Let us set the initial time $t_0 = 0$ and designate by t_m the maximum value of time moments under consideration. Generalize the definition (2.15) as follows

$$\begin{aligned} G_{i_1 \dots i_k}^\eta(t_1, \dots, t_k \,|\, k) &= e^{-W}\, T\{\hat{R}\,\Phi_{i_1}^\eta(t_1) \dots \Phi_{i_k}^\eta(t_k)\} = \\ &= e^{-W}\, \hat{P}T\{S(t_m, t_1)\, \Phi_{i_1}(t_1)\, S(t_1, t_2) \dots \Phi_{i_k}(t_k)\, S(t_k, 0)\} \end{aligned} \tag{2.16}$$

where

$$\hat{R} = \hat{P} S (t_m, 0), \quad e^{-W} = Tr\,\hat{R} \tag{2.17}$$

and we took into account eqs.(2.9) and (2.11). If we set the auxiliary external source $\eta\,(t)$ to zero then the generalized Green function G^η (2.16) is reduced to the desired usual one G (2.15). However, the presence of the source $\eta\,(t)$ enables to generate the G^η by functional differentiations of the functional W defined by eq.(2.17) and find also the G^η of the k-th order as a functional derivative of the G^η of the (k – 1)–th order. Acting by the operator $\hat{R}$ on eq. (2.14) we find

$$\sigma \frac{d}{dt_1} G^\eta_{i_1}(t_1 \mid 1) = \eta_{i_1}(t_1) + \sum_k k\, U^{(k)}_{i_1 i_2 \ldots i_k}\, G^\eta_{i_2 \ldots i_k}(t_1, \ldots, t_1 \mid k-1) \tag{2.18}$$

Taking the functional derivatives we obtain hence the hierarchy of equations for the generalized Green functions G^η [9, 21]. Each equation of this chain contains a new unknown Green function of higher order and therefore up to this stage we have benefited a little. The next step is to make use of some approximation when the Green functions of higher orders are expressed through the lower cumulants. A known scheme of such a kind is a Hartree–Fock or Gaussian approximation (see the discussion, e.g. [9] and references therein). The quantum field theory brings another procedure of constructing techniques for approximate calculation of the Green functions by making use of the so–called mass and charge renormalizations [1–3, 9, 21]. All these approximations were formulated and analysed in statistical physics for many particles problems and therefore they should be reexamined as applied to the nonlinear dynamical systems with small numbers of degrees of freedom. This important problem will be discussed elsewhere.

3. GENERATING FUNCTIONALS AND FEYNMAN DIAGRAMS FOR CORRELATION FUNCTIONS

Now we introduce noise into the system under consideration, i.e. in this section we will deal, instead of eq. (1.1), with the stochastic differential equations

$$\dot{\varphi}_i(t) = F_i[\varphi(t)] + \xi_i(t), \quad i = 1, \ldots, N \tag{3.1}$$

where ξ_i (t) is a Gaussian white noise with zero means and autocorrelation functions equal to$2\Gamma_{ij}$ $\delta(t - t')$ There are two reasons for doing this [22, 23]. First, real systems are never entirely free from the influence of noise and consequently a deterministic dynamics should, in principle, be considered always as a limit of a stochastic one as the noise amplitude tends to zero. Second, investigation of dynamical systems may be simplified by adding noise. Namely, introducing noise into the system can serve as a kind of regularization procedure. This role may be particularly significant for the systems with strange attractors. After derivation of a perturbation theory we will return to the deterministic case by taking the limit $\Gamma \to 0$.

Following [18], we introduce the generating functional

$$Z_\xi\,[\eta\,(t),\ \tilde{\eta}\,(t)] = \int D\varphi\,(t)\ \ D\,\tilde{\varphi}\,(t)\ \ \exp\Big\{\int dt\ [\eta_j\,(t)\ \varphi_j\,(t) + \\ + \tilde{\eta}_j(t),\ \tilde{\varphi}_j(t) + i\,\tilde{\varphi}_j(t)\ [\ \dot{\varphi}_j(t)\ - F_j[\varphi\,(t)] - \xi_j(t)]\Big\}\ J[\varphi\,(t)]\ p[\varphi\,(t_0)] \tag{3.2}$$

Here

$$J\,[\varphi\,(t)]\ = \exp\left(-\kappa\int d\,t\ \delta\,F_j\ [\varphi\,(t)]\ /\ \delta\,\varphi_j\ (t)\right)$$

is the Jacobian introduced to satisfy the normalization condition $Z_\xi\,[0, 0] = 1$. The quantity κ is the value of the step function $\Theta(x)$ at the zero value of its argument, i.e. $\kappa = \Theta(x)$. Usually the value $\kappa = 1/2$ is accepted and we follow this prescription as well (see [19] for more detail). The quantity $p[\varphi\,(t_0)]$ is the probability distribution density for the initial state of the system. We are mainly interested in behaviour of the system near an attractor. Therefore we will take, for simplicity, the uniform distribution of the initial states inside a neighbourhood D of the attractor (the region D should be located in the basin of attraction) and zero elsewhere. Hence integration in the formulae below is performed over paths (functions) for which $\varphi\,(t_0) \in D$.

After the average over the stochastic variable ξ we have the functional

$$Z_\xi\,[\eta\,(t),\ \tilde{\eta}\,(t)] = \int D\varphi\,(t)\ \ D\,\tilde{\varphi}\,(t)\ \ \exp\Big\{\int dt[\eta_j\,(t)\ \varphi_j\,(t) + \\ + \tilde{\eta}_j\,(t)\ \tilde{\varphi}_j\,(t)] - S\,[\varphi\,(t),\ \tilde{\varphi}\,(t)]\Big\} \tag{3.3}$$

containing the effective action of Martin–Siggia–Rose [9]

$$S\,[\varphi\,(t),\ \tilde{\varphi}\,(t)] = \int dt\, \Big\{ \tilde{\varphi}_j\,(t)\ \Gamma_{jk}\ \tilde{\varphi}\,(t) - i\,\tilde{\varphi}_j\,(t)\ [\dot{\varphi}_j\,(t) - \\ - F_j\,[\varphi\,(t)]] + \kappa\ \delta\, F_j\,[\varphi\,(t)]\ /\ \delta\,\varphi_j\,(t) \Big\} \tag{3.4}$$

Correlation functions can be found by differentiation of the above functional so as

$$G_{j_1 \ldots j_m}\,(t_1, \ldots, t_m\,|\,m) = Z^{-1}\,[0,0]\,\frac{\delta^m\, Z\,[\eta\,(t),\ \tilde{\eta}\,(t)]}{\delta\,\eta_{j_1}(t_1)\ldots\delta\,\eta_{j_m}\,(t_m)}\,|\,\eta=\tilde{\eta}=0 \tag{3.5}$$

Let us separate the functional $F[\varphi]$ on a linear term $K\varphi$ yielding a "free" dynamics of the system and the rest $W[\varphi]$ which describes interactions between the variables $\varphi_1, \ldots, \varphi_N$ and influences of the external signals (if any). The choice of the free dynamics term $K\varphi$ is affected by the attractor under consideration because the linearization of eqs.(3.1) should be carried out in a neighbourhood of a point from the region D around the attractor.

The action (3.4) is then written as $S = S_0 + S_W$ and the generating functional (3.3) can be represented in the form (see, e.g., [24] for a technique)

$$Z[\eta(t),\tilde{\eta}(t)] = \exp\left\{ -\frac{1}{2}\int dt ds \left[\frac{\delta}{\delta\varphi_j(t)} C_{jk}(t-s) \frac{\delta}{\delta\varphi_j(s)} + \frac{\delta}{\delta\varphi_j(t)} G_{jk}(t-s) \frac{\delta}{\tilde{\varphi}_k(s)} \right] \right\} \times \\ \times \exp\left\{ \int dt\ [\eta_j\,(t),\ \varphi_j\,(t) + \tilde{\eta}_j\,(t)\ \tilde{\varphi}_j\,(t)] - S_W\ [\varphi\,(t),\ \tilde{\varphi}\,(t)] \right\} \tag{3.6}$$

(We have omitted in the first exponent the terms that yield no contribution into the correlation functions (15)). This equation generates two types of contractions between the fields and consequently two types of lines in the diagrams. Namely, the quantity

$$G_{jk}\ (t,\ s) = \left\langle \varphi_j\ (t)\ i\,\tilde{\varphi}_k\,(s) \right\rangle_0 = {}_t\!\!\text{———}\!\sim\sim_s$$

is a propagator and the quantity

$$G_{jk}\ (t,\ s) = \left\langle \varphi_j\ (t)\ \varphi_k\,(s) \right\rangle_0 = {}_t\!\!\text{—————}_s$$

is a correlator. Let

$$A\varphi \equiv (\partial_t - K)\,\varphi = 0$$

be the equation of motion of the homogeneous linear conservative system in the deterministic limit. Then

$$G = A^{-1},\; C = A^{-1}(2\,\Gamma)A^{+\,-1}$$

Consider. eq. (3.6). Expanding the second exponent into the power series, we have the equation which can be calculated with the aid of the diagrammatic perturbation theory as follows.

The functional $S_W = [\varphi, \tilde{\varphi}]$ is diagrammaticaly depicted by a point (vertex), the factor $S_W^k = [\varphi, \tilde{\varphi}]$ corresponds to a diagram containing k such vertices. The operators $(\delta / \delta\varphi\, C\, \delta / \delta\varphi)$ and $(\delta / \delta\varphi\, G\, \delta / \delta\tilde{\varphi})$ produce lines connecting the pairs of the vertices. Every such a line is associated with the correlator C or propagator G. These lines adjoin to the vertices by means of all possible ways. Thus, action of the operators $(\delta / \delta\varphi\, C\, \delta / \delta\varphi)$ and $(\delta / \delta\varphi\, G\, \delta / \delta\tilde{\varphi})$ on the factor $S_W^k = [\varphi, \tilde{\varphi}]$ yields the set of diagrams which contain k vertices and an arbitrary number of the lines C and G connecting the vertices by all possible variants. Every vertex with $(m + n)$ lines is associated with the factor

$$\Lambda_{i_1 \ldots i_m\, j_1 \ldots j_n}(t_1, \ldots, t_m,\; s_1, \ldots, s_n) = \frac{\delta^{m+n}\, S_W\,[\varphi(t), \tilde{\varphi}(t)]}{\delta\,\varphi_{i_1}(t_1) \ldots \delta\,\varphi_{i_m}(t_m)\,\delta\,\tilde{\varphi}_{j_1}(s_1) \ldots \delta\,\tilde{\varphi}\, j_n(s_n)}$$

The arguments of this vertex function are convoluted with the corresponding arguments of the correlators and propagators associated with the lines.

To find the correlation functions with the aid of the diagrammatic perturbation theory, an infinite series of terms should be computed and summed up. This goal can partly be achieved by making use of the dimensional analysis and renormalization group approach (see, e.g., [25–27]). Indeed, if we consider the problem of finding the Fourier transform $G(w_1, \ldots, w_m \mid m)$ of the correlation function (1.3) then we can take the so-called renormalization point μ which is an arbitrary parameter whose dimension coincides with that of the variables $w_1, \ldots, w_m$. Then the dimensional analysis yields the equation

$$G\,(w_1, \ldots, w_m \,|\, m) = \mu^{\gamma} \cdot w_1^{D-\gamma} \cdot H\left(\frac{w_2}{w_1}, \ldots, \frac{w_m}{w_1}; \frac{w_1}{\mu}, \varsigma(\mu)\right)$$

with some numerical constants D and γ and some dimensionless function $\varsigma(\mu)$ which is commonly identified with an effective coupling constant of the theory $\alpha_s(Q)$ at an energy scale Q taken to be equal to the value μ. The function H is obviously dimensionless.

The last equation can easily be recast into the following one

$$[\mu\frac{\partial}{\partial\mu} + B\,(\varsigma)\frac{\partial}{\partial r} - \gamma]\;\widetilde{G}\,(w_1, \ldots, w_m;\ \mu,\ r \,|\, m)|_{r=\varsigma(\mu)} = 0$$

which has a typical form of the renormalization group equation. Here $\widetilde{G}\,(w_1, \ldots, w_m;$ $\mu, r \mid m)$ is such a function that reduces to the Fourier transform of the correlation function G when the variable r is taken to be equal to $\varsigma(\mu)$. $B(\varsigma)$ is a generalized Gell–Man–Low function and the parameter γ is known as an anomalous dimension.

4. CONCLUSION

In conclusion, we have developed in this work a new method for treatment of the nonlinear dynamic systems using the quantum field theory. Such an approach would help to search analytically a deterministic chaos behaviour in neural networks.

ACKNOWLEDGMENTS

The authors are thankful to V. I. Kuvshinov and N. M. Shumeiko for stimulating discussions. One of us (A.D.L.) is grateful to the Department of Theoretical Physics for kind hospitality and warm atmosphere during the stay in Lublin. The work was supported in part by the Maria Curie--Skłodowska University.

REFERENCES

[1] L. F. Abbott (1990) *J. Phys.*, A 23, 3835.
[2] D. J. Amit (1989), *Modelling Brain Functions* (Cambridge: Cambridge Univ. Press).
[3] R. Bausch, M. K. Janssen, M. Wagner (1976) *Z. Phys.*, B24, 113.
[4] M. A. Cohen, S. Grossberg (1983) *IEEE Trans. Syst., Man. Cybern.*, SMC-13, 815.
[5] C. De Dominicis, L. Peliti (1978) *Phys. Rev.*, B18, 353.

[6] R. FitzHugh (1961) *Biophys. J.*, 1, 445.
[7] D. J. Gross, (1976) in: *Methods in Field Theory*, (ed. by) R. Balian and J. Zinn–Justin (Amsterdam: North–Holland).
[8] J. J. Hopfield (1982) *Proc. Natl. Acad. Sci. USA*, 79, 2554.
[9] J. J. Hopfield (1984) *Proc. Natl. Acad. Sci. USA*, 81, 3088.
[10] M. K. Janssen (1976) *Z. Phys.*, B 23, 377.
[11] B. Jouvet (1977) preprint of the University of Paris.
[12] F. Langouche, D. Roekaerts, E. Tirapegui (1979) *Physica*, 95 A, 252.
[13] A. D. Linkevich (1992) *Proc. Seminar "Nonlinear Phenomena in Complex Systems", Polatsk, February 17–20, 1992*, 21–120.
[14] A. D. Linkevich (1993a) *Proc. Second Seminar "Nonlinear Phenomena in Complex Systems", Polatsk, February 15–17, 1993*, 383–392.
[15] A. D. Linkevich (1993b) *Proc. School–Seminar "Nonlinear Dynamical Systems", Braslav, June 27–July 3, 1993.*
[16] P. C. Martin, E. D. Siggia, H. A. Rose (1973) *Phys. Rev.*, A 8, 423.
[17] J. S. Nagumo, S. Arimoto, S. Yoshisawa (1962) *Proc. IRE*, 50, 2061.
[18] P. Peretto (1989) *The Modelling of Neural Networks* (Les Ulis: Editions de Physique).
[19] A. Peterman (1979) *Phys. Rep.*, 53, 157.
[20] R. Phythian (1975) *J. Phys.*, A8, 1423; (1976) *J. Phys.*, A 9, 269.
[21] H. D. Politzer (1974) *Phys. Rep.*, 14, 129.
[22] P. M. Stevenson (1981) *Ann. Phys.*, (N. Y.) 132, 383.
[23] A. N. Vasil'ev (1976) *Functional Methods in Quantum Field Theory and Statistics* (Leningrad: LGU) (in Russian).
[24] A. A. Vedenov (1988) *Modelling Elements of Thinking* (Moscow: Nauka) (in Russian).

STRESZCZENIE

W artykule przedstawiono system dynamiczny opisywany układem równań różniczkowych. Wykazano, że metody kwantowej teorii pola mogą być użyte i prowadzą do rozwiązań analitycznych, a zatem również do otrzymywania jawnej postaci funkcji korelacji.