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Analogies between ^{3}He Clusters and Atomic Nuclei

Analogie między klasterami ^{3}He a jądrami

Аналогии между ^{3}Не-кластерами и атомными ядрами

There is a growing interest in the physics of clusters (metal clusters, noble gas clusters etc...). Helium clusters are expected to be rather different from the clusters of heavier noble gases due to the fact that helium is a permanent liquid. In particular ^{3}He clusters are expected to exhibit a shell structure similar to the one existing in atomic nuclei. Many analogies between liquid ^{3}He and nuclear systems have been pointed out in the past: i) both systems are Fermi liquids with well established saturation properties; ii) they are both available in the degenerate limit; iii) they exhibit the occurrence of collective phenomena in the collisionless regime (zero sound in liquid ^{3}He and giant resonances in nuclei). The investigation of such collective phenomena is a good starting point for exploring the analogies between liquid ^{3}He and nuclear systems. In fact both zero sound and giant resonances are well treated in the framework of the mean field approach. Landau [1] first developed a theory for collective phenomena in liquid ^{3}He starting from the Vlasov equations and was able to predict the existence of zero sound anticipating its experimental discovery. In the case of atomic nuclei the standard theory for describing giant resonances is the linearized time dependent Hartree-Fock theory (RPA). The link between the time dependent Hartee-Fock theory and the Landau's theory for zero sound is now clear. In fact taking the macroscopic limit (long wave length limit) or, equivalently, the semiclassical limit of the TDHF equations, one straightforwardly obtains the Vlasov equations and hence the Landau's theory.

The above discussion and the analogy with the nuclear case suggest the possibility of using an effective interaction for carrying out self-consistent calculations in liquid ^{3}He in the framework of the Hartree-Fock theory, as currently done in nuclear physics. Such a possibility has been developed in [2] where the equation of state of liquid ^{3}He has been studied using an effective interaction similar to the ones employed in nuclear physics (Skyrme type interaction [3]). The interaction is of phenomenological type and gives rise to the following expression for the expectation value of the energy for a time reversal invariant state:

$$E = \int \left[\frac{\tau}{2m} (1 - \rho/\rho_c)^2 + \frac{1}{2} b \rho^2 + \frac{1}{2} c \rho^{2+\gamma} + A(\vec{\nabla} \rho)^2 \right] d\vec{r} \qquad (1)$$

where ρ and τ are the number density and kinetic energy density respectively. The term multiplying τ fixes the effective mass of the ^{3}He atom:

$$\frac{1}{2m^*} = \frac{1}{2m} (1 - \rho/\rho_c)^2 \qquad (2)$$

The parameters ρ_c, b, c and γ have been fixed to reproduce the experimental value of effective mass, ensity, binding energy and incompressibility at saturation. The predictions of the model are then in excellent agreement with experiments up to the solidification pressure (see Figs 1-3). In particular one should notice the clear improvement with respect to the predictions of the parabolic

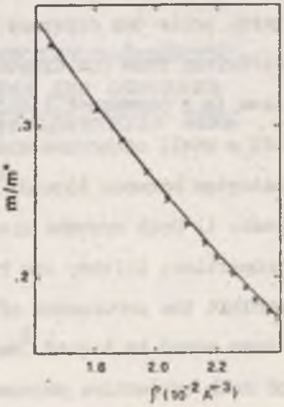

Fig.1. Effective mass of liquid ^{3}He as a function of the density.

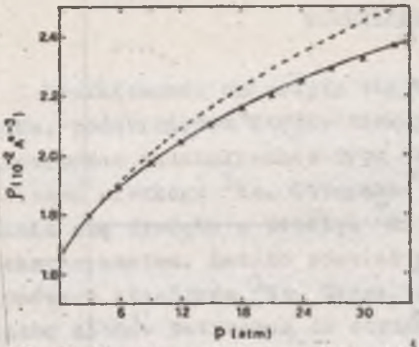

Fig.2. Equation of state of liquid ^{3}He. The full line is from eq.(1); the dashed line corresponds to the parabolic approximation. The crossed points correspond to experiments.

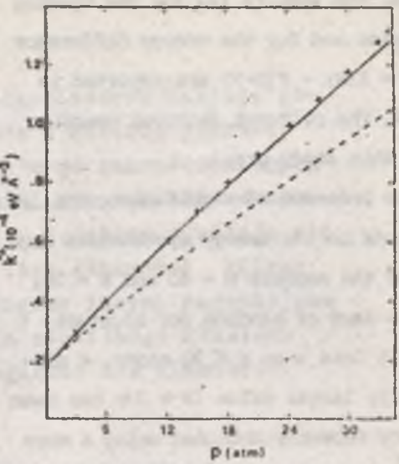

Fig.3. Incompressibility parameter $K^{-1}=\rho\frac{\partial P}{\partial\rho}$ of liquid ^{3}He.

approximation $E/N = E_0/N + 1/2\, k(\rho-\rho_0)^2\,\rho_0^3$ (dashed line) which is adequate only up to 6 atmospheres.

The surface parameter A entering eq.(1) is fixed to reproduce the experimental value of surface tension of liquid ^{3}He. Equation (1) can then be employed to investigate the structure of small clusters of ^{3}He atoms. The ground state of such systems is described as a Slater determinant built up with single particle wave functions given by the solution of the Hartree-Fock equation

$$-\frac{\hbar^2}{2}\vec{\nabla}\frac{1}{m^*(\rho)}\vec{\nabla}\,\psi_i + V\,\psi_i = \varepsilon_i\,\psi_i \qquad (3)$$

where

$$V = -\frac{\tau}{m\rho_c}\left(1-\rho/\rho_c\right) + b\rho + \frac{1}{2}c\,\rho^{1+\sigma}(2+\sigma) - 2A\nabla^2\rho \qquad (4)$$

is the 1-body self consistent density dependent potential. The order of single particle energies is expected to be the one given by potentials currently employed to describe saturating systems (Wood-Saxon, harmonic potential ...). The filling of major shells characterizing the harmonic oscillator potential is then expected toprovide particularly stable configurations. One then preicts [4] the following values for magic numbers: N= 8, 20, 40, 70, 112, 168 ... One can check that these values really correspond to magic numbers by evaluating the energy of clusters with N ± 1 atoms. The results [4] for the binding energies and for the energy difference D(N) = E(N) - E(N-1) are reported in Fig.4. The relevant features emerging from this study are:

a) the presence of significant oscillations in the energy systematics around the numbers N = 40 and N = 70;

b) the lack of binding for clusters with less than N ≅ 20 atoms. A slightly larger value (N ≅ 30) has been very recently obtained using a more accurate choice of the parameters of the effective interaction [5].

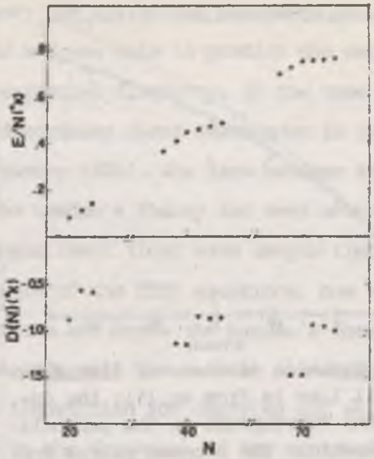

Fig.4. Energy systematics and single particle energies for ^{3}He clusters.

The above results for ^{3}He clusters differ from what is predicted to occur in the case of ^{4}He clusters [5-6], which are always bound and do not exhibit any shell structure.

Future investigations in this line of research might concern the magnetic properties of ^{3}He clusters, including the structure of fully polarized sytems. Clusters of mixed type (^{3}He + ^{4}He) could be also considered. In fact it is well known that ^{3}He atoms are deeply bound at the surface of ^{4}He systems. Of course future theoretical investigations will be strongly motivated by the availability of experimental information on helium clusters.

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STRESZCZENIE

Dyskutowano, opierając się na przybliżeniu Hartree'go-Focka, podobieństwa między ciekłym ^{3}He a materią jądrową. Zastosowano oddziaływanie typu Skyrme'a do samouzgodnionych obliczeń ciekłego ^{3}He. Otrzymane wyniki na prędkość rozchodzenia się dźwięku w ciekłym ^{3}He w $T = 0$ dobrze zgadzają się z eksperymentem. Badano również niektóre własności kilkuatomowych klasterów ^{3}He. Określono między innymi najmniejszą liczbę atomów potrzebną do otrzymania stabilnego klasteru oraz pokazano, że istnieją liczby magiczne dla klasterów.

РЕЗЮМЕ

На основе приближения Хартри-Фока рассматривается сходство между ^{3}He и ядерной матерей. При самосогласованных расчетах жидкого ^{3}He применялось взаимодействие типа Скирма. Полученные результаты на скорость распространения звука в жидком ^{3}He при $T = 0$ хорошо согласуются с экспериментом. Исследовались тоже некоторые свойства ^{3}He-кластеров из нескольких атомов. Между прочем - определено наименьшее число атомов необходимых для образования стабильного кластера и доказано что для кластеров существуют магические числа.