ANNALES
UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN — POLONIA
VOL. XLIII/XLIV, 13 SECTIO AAA 1988/1989

Instytut Fizyki UMCS

W.A. KAMIŃSKI*

DCX Ground State Transitions in Quasiparticle Random Phase Approximation Formalism

1. Introduction

The double charge exchange (DCX) reaction with pions is still attracting a good deal of attention (see ref. [1] the latest review of the subject). There was hope that special information about an affection of one nucleus by another would appear in the reaction. Unfortunately, the meson factories were built in the 1970's, any type of such correlations was masked by other complex interactions at highly accessible meson energies at that time. Only recently [2-4] as small as 50 MeV meson energies are available and nucleon correlations have begun to surface.

* Permanent address: Instytut Fizyki Uniwersytetu Marii Curie-Skłodowskiej, ul.Nowotki 10, 20-031 Lublin, Poland.

The double charge exchange process necessarily involves two neutrons. The ingoing pion π^+ has successive charge-exchange on a neutron. In a second step the produced pion π^0 exchanges the charge to another neutron to produce a proton and a pion π^-, which then leaves the nucleus and is detected. Theorists expect in general that the pions interact in the above picture only if the DCX process takes place on valence nucleons which are within 1 fm (about nucleon radius) of each other. In other words the strong short-range correlations to nuclear wave functions must be involved if one wants to reduce a considerable disagreement with the experiment [2]. In fact the theories including correlation effects come much closer to the data and a fairly satisfactory agreement is reported [3-4]. Another regularity of the reaction is the sensitivity to the role played by antisymmetry of nuclear wave functions, which allows the nucleons of the core to participate in an active way in the reaction [5]. The alleged role played by quarks in the DCX reaction has added elements of interest to the field [6].

In all reported calculations the strong limited shell model or even uncorrelated free nucleon wave functions were used. This limitation imposes applications to the light nuclei and/or to nuclei with a few nucleons out of the closed shell only. Bleszynski and Glauber's wave function of the ^{14}C nucleus is assumed to separate into a wave function for two valence neutrons in the p-shell orbit and a wave function for the core. The authors argue that the core plays a passive role and they omit its wave function in derivations. The two p-shell nucleons, of the total angular momentum zero, can only be in one of the two states, the spin singlet $^1S_0$ or the spin triplet $^3P_0$ state and final nuclear wave function is a free combination of these two states. For description of the calcium isotopes the same authors have used pure $f_{7/2}$ wave functions with the parametrized p-configuration admixture. More fundamental approach, the seniority scheme was proposed for description of the DCX reaction by Auerbach, Gibbs and Ginocchio [4], but in principle the model is only valid for nuclei with a few valence particles or holes of one kind, for example, for the calcium isotopes.

In the present paper we apply the random phase approximation based on the quasi-particles formalism (QRPA) to give a consistent approach to the DCX reaction in a wide mass region of nuclei. The motivation for doing results from the observation that the

QRPA formalism provides a very useful way of description of the beta and double beta decay processes [7-9]. Because in the DCX reaction the nuclear structure is as important as in the beta decay, one can expect similar usefulness of the QRPA. Moreover, further formal analogies between operators cause that both reactions show some similarities and have already been discussed in literature [10].

The article is arranged as follows. A form of the DCX operator for the p-wave pion in non-static and static limit is studied in Sec. 2. The basic features of the QRPA approach are reviewed in Sec. 3. Next section gives formulae for the DCX total amplitude in the ground state to ground state transition case. Summary and remarks (Sec. 5) and appendices close the paper.

2. The Double Charge Exchange Operator

We will consider the simplest local πNN interaction Lagrangian linear in the pseudoscalar-isovector pion field $\tilde{\varphi}$. The Lagrangian is a scalar-isoscalar quantity, the nucleon fields must therefore appear in a pseudoscalar-isovector combination. This leads to the well-known pseudoscalar coupling Lagrangian

$$\mathcal{L}_{PS}(x) = -g\bar{\Psi}(x) i\gamma_5 \tilde{\tau}\Psi(x)\cdot\tilde{\varphi} \quad . \tag{1}$$

The experimental value of the coupling constant g has been determined accurately from pion-nucleon and nucleon-nucleon scattering $g^2 \approx 180$. The Lorentz-invariant form of the πNN interaction deduced from the Lagrangian (1) is (e.g. [11])

$$\langle N(p')\,|\mathcal{L}_{PS}|N(p)\,,\pi^+(\mathbf{k})\rangle = ig\sqrt{2}\;\bar{u}(p')\,\tilde{\tau}\gamma_5 u(p)\cdot\tilde{\varphi} \quad . \tag{2}$$

In the above equations a tilde indicates an isovector character of the operators. Since we deal with nucleons which are not far off-shell, we use the standard expression for the nucleon spinors u to obtain

$$\bar{u}(p')\gamma_5 u(p) \;=\; \left(\frac{E+M}{2M}\cdot\frac{E'+M}{2M}\right)^{\frac{1}{2}} \xi^{\dagger}\left[\frac{\sigma\cdot\mathbf{p}'}{E'+M} \;-\; \frac{\sigma\cdot\mathbf{p}}{E+M}\right]\xi \quad . \tag{3}$$

Here ξ is the Pauli spinor for the nucleon, M - the nucleon mass and energy transfer $\omega = E' - E$. The total energy of nucleon fulfills the usual relativistic relation $E = E_p = (\mathbf{p}^2 + M^2)^{\frac{1}{2}}$. An analogous expression one can write for the quantities E' and p'. If we expand eq. (3) to the lowest order in $(\mathbf{p}' - \mathbf{p})$ and $(\mathbf{p}' + \mathbf{p})$ we find the result

$$\langle N(p')|\mathcal{L}_{PS}|N(p), \pi^+(\mathbf{k})\rangle = i\frac{g}{2M}\sigma\cdot\left\{(\mathbf{p}' - \mathbf{p}) - \frac{E' - E}{4M}(\mathbf{p}' + \mathbf{p})\right\}(\vec{\tau}\cdot\vec{\varphi}) \quad , \tag{4a}$$

$$= -\frac{f}{m_\pi}\sigma\cdot\left(\vec{\nabla}_\pi - \frac{\omega}{2M}\vec{\nabla}_N\right)(\vec{\tau}\cdot\vec{\varphi}) \quad , \tag{4b}$$

The second line involves a new interaction constant originated from the second form of the πNN interaction, the pseudovector coupling (PV)[1]. Its experimentally determined value is $f^2 \approx 1.005$. The operator $\vec{\nabla}_N = \frac{1}{2}\left(\vec{\nabla}_N - \overleftarrow{\nabla}_N\right)$ acts to the right and left on a single nucleon and $\mathbf{p}' - \mathbf{p} = \mathbf{k}$ is transfer of pion momentum.

The static non-relativistic reduction of the $\mathcal{L}_{PS}$ Lagrangian is easily shown to lead to the effective pion-nucleon Hamiltonian

$$H^{\pi}_{\pi NN} = -\frac{f}{m_\pi}(\sigma\cdot\nabla_\pi)(\vec{\tau}\cdot\vec{\varphi}) \quad . \tag{5}$$

The Hamiltonian (5) is linear in the pion momentum, and therefore it generates p-wave pion nucleon interaction only[2].

Eqs. (4) and (5) need some comments. At the most fundamental level the whole picture of the DCX reaction depends on the current choice of the πNN interaction operator. In the earliest calculations and many of the recent ones the simple static operator (5) has been used. This operator was obtained by keeping just the leading term in the non-relativistic limit of the pseudoscalar coupling (1), which is conventionally used to describe the πNN vertex. If we deal with some relativistic corrections introduced by some particular approximations, they can involve different correction terms into the original expression (3). There are at least three ways to switch on approximations : (i), the Foldy-Wouthuysen transformation, (ii), Galilean invariance of the non-relativistic expression and (iii), the

[1] One can show equivalence of the both types of couplings in the non-relativistic limit and we will not pay attention to the PV form of the interaction. For details see e.g. [11].

[2] Possible s-wave $\pi - N$ interaction is based on mechanisms completely different from those described above and we do not intend to discuss this point in the present paper.

higher-order non-relativistic reduction of eq. (3). The final forms differ by some factors [13]. Furthermore, the whole picture must be incomplete as the basic process πNN requires an off-shell leg and so must be embedded in some more complicated interactions. All these ambiguities suggest a need for a more consistent relativistic picture, but because of difficulties in describing nuclei relativistically, we limit our considerations to the non-relativistic static Hamiltonian (5) only.

A description of πNN scattering must correctly incorporate the contributions of second-order processes generated by absorption or emission of a pion on the nucleon. One has to calculate amplitudes of these processes and therefore we need the matrix elements connecting the vacuum and a one-pion state $|\pi_a(q)\rangle$ (e.g. [11]) :

$$\langle\pi_b(q')|H_{\pi NN}|0\rangle = i\frac{f}{m_\pi}\sigma\cdot\mathbf{q}'\tilde{\tau}_b e^{-i\mathbf{q}\cdot\mathbf{x}} \quad , \tag{6a}$$

$$\langle 0|H_{\pi NN}|\pi_a(q)\rangle = -i\frac{f}{m_\pi}\sigma\cdot\mathbf{q}\tilde{\tau}_a e^{i\mathbf{q}\cdot\mathbf{x}} \quad . \tag{6b}$$

The transition matrix T for the direct and crossed terms shown in Fig. 1 is obtained using the standard second-order perturbation theory with the operator $H_{\pi NN}$.

3. Quasiparticle Random Phase Approximation and the form of the $H_{\pi NN}$ Hamiltonian

The proton-neutron quasiparticle RPA formalism has been extensively discussed in literature [14-16], so we need only to present its basic feature here.

The pn-QRPA was developed for the description of charge-changing excitations, such as the Gamow-Teller transitions or charge exchange with pions. In these processes transitions from the 0^+ ground state of an even-even nucleus (Z,N) to states in the odd-odd nucleus (Z$\pm$1,N$\mp$1) are considered. The QRPA phonons with angular momentum J and its third component M are defined by the ansatz (e.g. [17]):

$$Q^{m\dagger}_{JM} = \sum_{pn}\left[X^m_{(pn)J}C(pnJM) - Y^m_{(pn)J}C^\dagger(pnJM)\right] \quad , \tag{7}$$

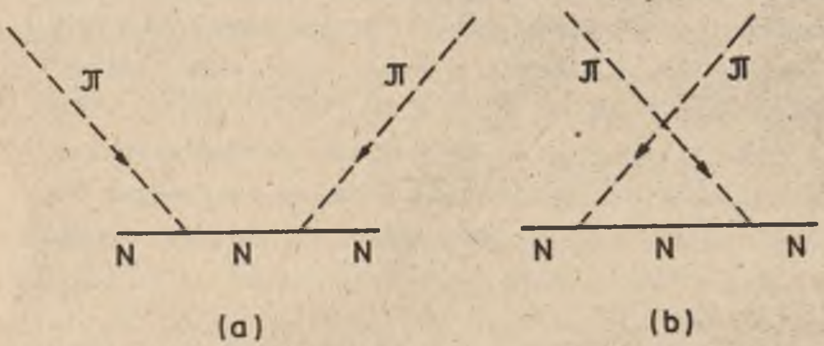


Figure 1. Illustration of the direct (a) and crossed (b) contributions.

where indices p and n distinguish between proton and neutron states. The proton-neutron pair-creation and annihilation operators are defined with the aid of the Clebsch-Gordan coefficients

$$C^{\dagger}(pnJM) = \sum_{m_p,m_n} (j_p m_p j_n m_n | JM) a^{\dagger}_{pm_p} b^{\dagger}_{nm_n} \tag{8}$$

and C(pnJM)=$[C^{\dagger}(pnJM)]^{\dagger}$. The forward- and backward-going amplitudes $X^{m}_{(pn)J}$ and $Y^{m}_{(pn)J}$ are obtained by solving the QPRA eigenproblem

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \Omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} \tag{9}$$

with normalization

$$\sum_{pn} \left[\left(X^{m}_{(pn)J} \right)^2 - \left(Y^{m}_{pn)J} \right)^2 \right] = 1 \quad . \tag{10}$$

The submatrices A and B are expressed explicitly

$$A^{JM}_{pn,p'n'} = \langle BCS | C(pnJM) H_{qp} C^{\dagger}(p'n'J'M') | BCS \rangle =$$

$$= \delta_{pp'}\delta_{nn'}\delta_{JJ'}\delta_{MM'}(\varepsilon_p + \varepsilon_n) -$$

$$-2g_{pp}\langle (pn)JM|G|(p'n')JM\rangle (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) -$$

$$-2g_{ph}\langle (pn^{-1})JM|G|(p'n'^{-1})JM\rangle (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) \quad , \qquad (11)$$

$$B^{JM}_{pn,p'n'} = \langle BCS|C(pnJM)\tilde{C}(p'n'JM)H_{qp}|BCS\rangle =$$

$$= -2g_{pp}\langle (pn)JM|G|(p'n')JM\rangle (u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) -$$

$$-2g_{ph}\langle (pn^{-1})JM|G|(p'n'^{-1})JM\rangle (u_p v_n v_{p'} u_{n'} + v_p u_n u_{p'} v_{n'}) \quad . \qquad (12)$$

Here H_{qp} is the nuclear Hamiltonian after the Valatin-Bogoliubov quasiparticle transformation. The quasiparticle energies ε_a (a= p or n) and the occupation amplitudes u_a, v_a are obtained by solving the BCS equations in some model space of a single-particle potential, the Wood-Saxon potential for example. The matrix elements of particle-particle $\langle (ab)JM|G|(a'b')JM\rangle$ and particle-hole $\langle (ab^{-1})JM|G|(a'b'^{-1})$ interaction are related to each other through the Pandya transformation and can be obtained from realistic or phenomenological forces[3]. The constants g_{pp} and g_{ph} parametrize the two-body matrix elements $\langle \quad |G| \quad \rangle$ in order to take into account the finite model space restriction always involved into calculations and a renormalization for finiteness of a nucleus. The submatrix A in eq. (11) is responsible for the mixing of various particle-particle states in the odd-odd nucleus reached by the charge-changing transition because its elements are dominated by the particle-hole interaction and quasiparticle energies. The contribution of the particle-particle interaction is smaller implying that this interaction has only a little effect on the mixing of particle-hole states in the (Z+1,N-1) intermediate nucleus. On the other hand, the elements of the submatrix B (eq. (12) consist of the both interaction terms of the same order. This indicates that the particle-particle interaction can significantly enhance the ground state correlations by increasing the backward-going amplitudes $Y^{m}_{(pn)J}$ [19].

[3] A realistic one-boson exchange potential was applied in papers [8, 9, 18] and a zero-range spin-isospin force in ref. [7], for example.

The supermatrix in eq. (9) has the dimension 2r, where r is the number od different possible proton-proton two-quasiparticle states of the angular momentum J and parity $\pi(\pi = \pm)$. The diagonalization of this matrix gives r eigenvectors for each J^π combination corresponding to the r positive eigenvalues Ω_l (l=1,2,...,r) and the same number negative eigenvalues, which we treat as nonphysical ones. The physical eigenvectors

$$|mJ^\pi M\rangle = Q^{m\dagger}_{JM}|RPA\rangle \tag{13}$$

with the RPA phonon operator $Q^{m\dagger}_{JM}$ given by eq. (7), can be constructed for two separate sets of states of the same intermediate nucleus (Z+1,N-1) depending on if the QRPA calculation is based on the parent (Z,N) or (Z+2,N-2) daughter nucleus ground states (see Fig. 2).

Both sets of intermediate states should be physically identical. In fact they result from the two different QRPA calculations which causes their mathematical nonequivalence. Particularly they fulfill the orthogonality relation approximately in the sense

$$\langle mJ_m|m'J_{m'}\rangle \approx 0 \quad for \quad |E^J_m - E^J_{m'}| \quad > \Delta E$$

and

$$\sum_{|E^J_m - E^J_{m'}| \, < \Delta E} |\langle mJ_m|m'J_{m'}\rangle|^2 \approx 1$$

with ΔE being of the order 1 MeV [19].

To describe the DCX reaction in the pn-QRPA model one has to expand the transition operator (5) in the quasiparticle operators $a^\dagger_p, a_p$ $(b^\dagger_n, b_n)$. We get (for details see Appendix A)

$$h_p(\mathbf{k}) = -\sqrt{12}i\frac{f}{m_\pi}\sum_{JM}\Big\{(-1)^J Y^*_{JM}(\Omega_k)\Big[\sum_{pn} G^J_{pn}(k)\mathcal{R}^{JM}_{pn}\Big]\Big\} \quad . \tag{14}$$

Here the QRPA part of the transition operator $\mathcal{R}^{JM}_{pn}$ can be expressed in the form:

$$\mathcal{R}^{JM}_{pn} = u_p v_n C^\dagger(pnJM) + v_p u_n \tilde{C}(pnJM) + u_p u_n D(pnJM) - v_p v_n \tilde{D}^\dagger(pnJM) \quad . \tag{15}$$

The $C^\dagger(pnJM)$ and $\tilde{C}(pnJM)$ operators have been defined in eqs. (8a)-(8b) and other two operators are expressed as follows

$$D^\dagger(pnJM) = \sum (j_p m_p j_n m_n|JM) a^\dagger_{pm_p}(-1)^{j_n+m_n} a_{n\tilde{m}_n} \quad ,$$

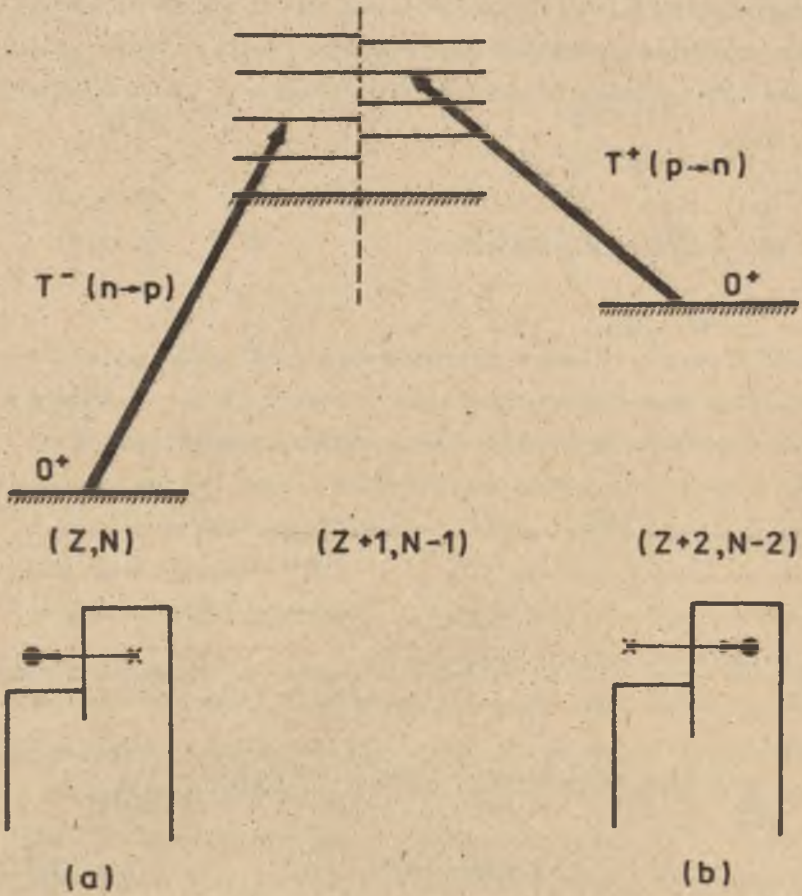


Figure 2. The QRPA model for the DCX reaction. One starts with the RPA proton-particle neutron-hole (a) or neutron-particle proton-hole (b) calculation to the intermediate (Z+1, N-1) nucleus states. The mode (a) corresponds to the ground state of the (Z,N) nucleus while mode (b) to the ground state of the (Z+2, N-2) nucleus. A transition from an unoccupied to an occupied single particle state is strongly stifled.

$$D(pnJM) = \left[D^\dagger(pnJM)\right]^\dagger \quad . \tag{16}$$

The tilde operation $\tilde{A}(abJM) = (-1)^{J+M}A(abJM)$ is used here to have for the annihilation operator the usual phase convention of spherical tensors. $Y_{JM}(\Omega_k)$ is the spherical harmonic of degree J [25] and an expression for the $G_{pn}^{JM}(k)$ form-factor can be found in Appendix A (eq. A6).

4. The QRPA Model for the DCX Reaction

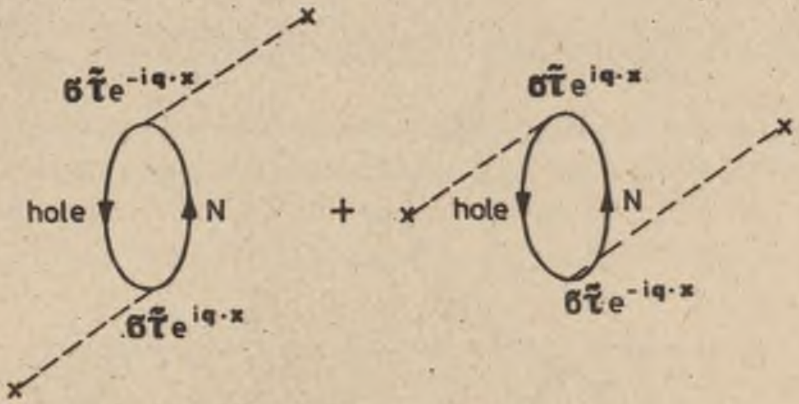
Treating the DCX reaction we assume a sequential mechanism. In this picture the incident pion π^+ of the beam exchanges its charge to a neutron. In the second step the produced pion π^0 exchanges the charge to another neutron to produce a proton and a pion π^-, which then leaves the nucleus and is detected. During this process spin-isospin dependent nuclear interactions mix the unperturbed particle-hole states giving a contribution to nucleon-hole pair excitations (Fig.3). In the RPA scheme the basic DCX process can be described by the two-fold action of the one-exchange QRPA operator h_p (14). Schematic picture of the mechanism is shown in Fig. 4.

In the second-order perturbation theory we can express the DCX transition amplitude as

$$F(\mathbf{k},\mathbf{k}') = \sum_{mm'} \frac{\langle f,0^+;\pi^-(\mathbf{k}')|h_p|mJM\rangle\langle mJM|m'JM\rangle\langle m'|h_p|i,0^+;\pi^+(\mathbf{k})\rangle}{E_i+\epsilon_k-\frac{E_m+E_{m'}}{2}}$$

$$+ \text{ crossed term.} \tag{17}$$

$|i,\pi^+(\mathbf{k})\rangle$ and $|f,\pi^-(\mathbf{k}')\rangle$ are the ground states of the parent and daughter nuclei; $\mathbf{k}$ and $\mathbf{k}'$ are in- and outgoing pion momenta and $\epsilon_k = (k^2 + +m_\pi^2)^{\frac{1}{2}}$ is the incident pion energy. Note that the summation in eq. (17) involves products of the transition matrix elements of the both QRPA mode calculations (compare Fig. 2 and remarks in Sec. 3). This trick allows to write eq. (17) symmetrically in both indices m and m′ because of quasi-orthogonality of the sets $\{|mJM\rangle\}$ and $\{|m'JM\rangle\}$. The procedure introduces an uncertainty in the energy denominator for which we involved an average value of the



(a) (b)

Figure 3. Diagrammatic representation of the nucleon-hole contribution in the lowest order: (a) – direct term, (b) – crossed term.

QRPA excitation energies E_m^J and $E_{m'}^J$, however this uncertainty is small compared to the mean value of the denominator.

Using eq. (6) and the explicit form of the h_p operator (eqs. (14)-(16) and (A3)) after cumbersome derivations one can find the final formula for the total amplitude F[4]

$$F(\mathbf{k},\mathbf{k}') = \sum_J F_J(\mathbf{k},\mathbf{k}') \quad , \tag{18a}$$

$$F_J(\mathbf{k},\mathbf{k}') = -\frac{1}{4\pi}\left(\frac{f}{m_\pi}^2\right) P_J(\cos\theta_{\mathbf{k}\mathbf{k}'}) \sum_{mm'} \frac{\langle mJ|m'J\rangle}{E_i + \epsilon_\pi - \frac{E_m^J + E_{m'}^J}{2}}$$

[4] See details in Appendix B

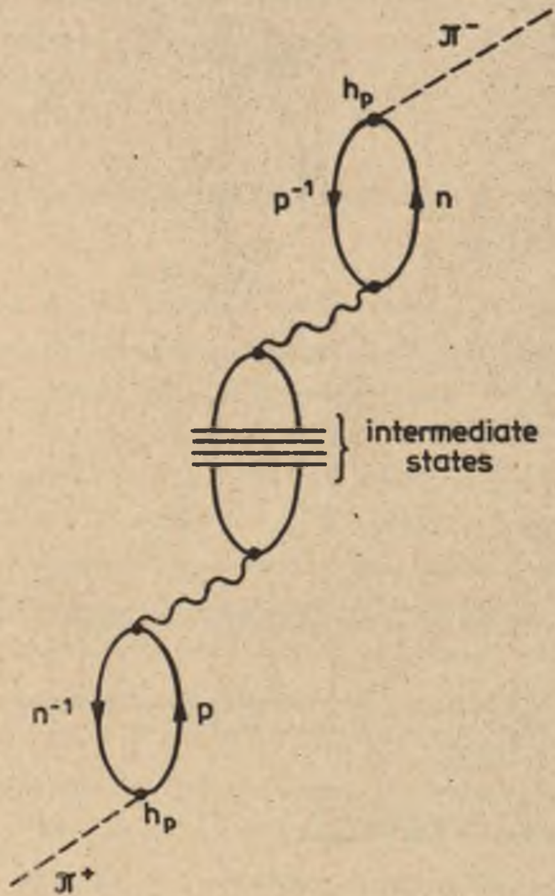


Figure 4. The DCX mechanism in the RPA model. Intermediate states of the (Z+1, N-1) nucleus are described in Sec. 3.

$$\cdot \left\{ \left[\sqrt{12} \sum_{pn} G^J_{pn}(k')(-1)^{j_p+j_n+J} \left(\bar{X}^m_{(pn)J}\bar{v}_p\bar{u}_n + \bar{Y}^m_{(pn)J}\bar{u}_p\bar{v}_n \right) \right] \right.$$

$$\left. \cdot \left[\sqrt{12} \sum_{pn} G^J_{(pn)J}(k) \left(X^{m'}_{(pn)J}u_p v_n + Y^{m'}_{(pn)J}v_p u_n \right) \right] \right\}_J \quad . \tag{18b}$$

Here $P_J(\cos\theta_{kk'})$ is the Legendre polynomial of the J^{th} order [20] and the overlap between two J^π states belonging to two different sets, $\langle mJM|m'JM\rangle$ is given by

$$\langle mJM|m'JM\rangle = \sum_{pn} \left(\bar{X}^m_{(pn)J} X^{m'}_{(pn)J} - \bar{Y}^m_{(pn)J} Y^{m'}_{(pn)J} \right) \langle 0^+_f|0^+_i\rangle \quad . \tag{19}$$

In eq. (19) the factor $\langle 0^+|0^+\rangle$ when approximated by the BCS states is

$$\langle 0^+_f|0^+_i\rangle = \prod_p (u_p\bar{u}_p + v_p\bar{v}_p) \prod_n (u_n\bar{u}_n + v_n\bar{v}_n) \quad . \tag{20}$$

In the above expressions all quantities with a bar correspond to the daughter (Z+2,N-2) nucleus and without a bar to the parent (Z,N) nucleus.

The crossed term (Fig. 1b) is treated similarly. The corresponding amplitude satisfies crossing invariance: the crossed term is obtained from the direct term by interchanging the in- and outgoing pion momenta as well as energies: $\mathbf{q} \longleftarrow -\mathbf{q}'$; $\epsilon_k \longleftarrow -\epsilon_{k'}$.

The scattering amplitude $F(\mathbf{k},\mathbf{k}')$ is connected directly with the differential cross section

$$\frac{d\sigma}{d\Omega}(\theta_{kk'}) = |F(\mathbf{k},\mathbf{k}')|^2 \quad . \tag{21}$$

The accuracy of the single and double charge exchange data requires the careful treatment of kinematic factors. The differential cross section (21) must be evaluated in the pion-nucleus centre-of-mass frame, which approximately coincides with the laboratory frame. The amplitudes (18) are defined in the πN centre-of-mass frame. It is therefore necessary to incorporate the kinematic factors relating these two frames of reference. This can be done in a manner of any textbook (e.g. [11]) and results in a slight change of angular momenta $\mathbf{k}$, $\mathbf{k}'$ and an angle $\theta_{kk'}$ [11,23].

5. Summary and Final Remarks

In the paper we have introduced formulae for the DCX transition amplitude within the pn-QRPA formalism. The static limit for the p-wave pion-nucleon effective interaction was used but in principle there are no difficulties to extend this approach onto relativistically corrected expression (4). Existing suggestions [23] and the author's preliminary estimations made for calcium isotopes [24] allow to admit that the recoil term is of no quantitative importance for p-wave pion-nucleon interaction.

An extension of the DCX description which one has to involve in the low pion energy region is an inclusion of the s-wave pion effective interaction [11, 22]. Suitable formulae for the double isobaric analog transitions can be found elsewhere [24] and for the ground state transitions in a subsequent paper.

The author would like to thank Professor Amand Faessler for suggesting the interesting problem of the DCX reaction with pions and for many helpful discussions. The fruitful advice and stimulating discussions on the QRPA formalism with Professor S.B.Khadkikar and Dr. J.Suhonen are also acknowledged.

The work was supported by the Bundesministerium für Forschung und Technologie.

Appendix A

We need to express the transition operator (5) in the second quantization formalism. Firstly using the relation (6) one can evaluate the pion operator part to have

$$\zeta_p(x) = \langle 0|H_{\pi NN}(x)|\pi^+(q)\rangle = -\sqrt{2}\, i\, \frac{f}{m_\pi}\sigma\cdot\mathbf{q}\; e^{i\mathbf{q}\cdot\mathbf{x}}\tau_+ \quad , \tag{A1}$$

where τ_+ is the nuclear isospin raising operator and we used pion plain wave expansion for the pion field operator $\vec{\varphi}$. Next we define the transition operator h_p from its density $h_p = \int d^3x \Psi_N^\dagger(\mathbf{x})\zeta_p(\mathbf{x})\Psi_N(\mathbf{x})$ and express it in terms of the creation and annihilation nucleon operators $c_p^\dagger, c_p$ $(c_n^\dagger, c_n)$ as

$$h_p(q) = -\sqrt{2}\, i\, \frac{f}{m_\pi}\sum_{pn}\left[\int d\xi d^3x \psi_p^*(\mathbf{x},\xi)\sigma\cdot\mathbf{q}\; e^{i\mathbf{q}\cdot\mathbf{x}}\Psi(\mathbf{x},\xi)\right] c_p^\dagger c_n \quad , \tag{A2}$$

or after transformation to the quasiparticles

$$h_p(\mathbf{q}) = -\sqrt{2}\, i\, \frac{f}{m_\pi} \sum_{pn,JM} \mathcal{F}^{JM}_{pn}(\mathbf{q})\mathcal{R}^{JM}_{pn} \quad . \qquad (A3)$$

The quasiparticle operator $\mathcal{R}^{JM}_{pn}$ is this of eq.(15) and $\mathcal{F}^{JM}_{pn}(\mathbf{q})$ is given by

$$\mathcal{F}^{JM}_{pn}(\mathbf{q}) = \sum_{(m)} (-1)^{j_n - m_n} (j_p m_p j_n - m_n | JM) \left[\int d^3x d\xi \psi^*_p(\mathbf{x},\xi) \sigma\cdot \mathbf{q} e^{i\mathbf{q}\cdot\mathbf{x}} \psi_n(\mathbf{x},\xi) \right] \quad . \qquad (A4)$$

In the above equations ξ means additional intrinsic spin coordinates. Cumbersome calculations allow to find more useful expression for $\mathcal{F}^{JM}_{pn}(\mathbf{q})$:

$$\mathcal{F}^{JM}_{pn}(\mathbf{q}) = \sqrt{4\pi}\,\sqrt{6}\, Y^*_{JM}(\Omega_q) G^J_{pn}(q) \quad , \qquad (A5)$$

where

$$G^J_{pn} = (-1)^{j_p + j_n} \hat{j}_p \hat{l}_p \hat{j}_n \hat{l}_n \sum_{l''} (-1)^{\frac{l_p + l_n - l''}{2}} (l_p 0 l_n 0 | l'' 0) R^{l''}_{pn}(q) \begin{Bmatrix} \frac{1}{2} & l_p & j_p \\ \frac{1}{2} & l_n & j_n \\ 1 & l'' & J \end{Bmatrix} \quad . \qquad (A6)$$

Here Y_{JM} is the spherical harmonic [25] and $\hat{j}_a = \sqrt{2j_a + 1}$. The symbols (|) and { } mean the Clebsch-Gordan coefficient and 6j symbol, respectively. The formulae for the $R^{l''}_{pn}(q)$ function can differ for different radial dependence of nucleon wave functions $\psi_p(\mathbf{x},\xi)(\psi_n(\mathbf{x},\xi))$. In the case of harmonic oscillator wave functions one get the analytical expression

$$R^{l''}_{pn} =$$

$$= \sqrt{n_p! n_n! (2n_p + 2l_p + 1)!! (2n_n + 2l_n + 1)!! 2^{l_p + l_n + - n_p - n_n}} \; q\, exp\left(-\frac{q^2 b^2}{4}\right)$$

$$\cdot \sum_{s,s'=0}^{n_p, n_n} \frac{(-2)^{s+s'} \left(\frac{1}{2}(l_p + l_n - l'') + s + s'\right)!}{s!(n_p - s)!(2l_p + 2s + 1)!! s'!(n_n - s')!(2l_n + 2s' + 1)!!} L^{l'' + \frac{1}{2}}_\beta (b^2 q^2 / 4) \qquad (A7)$$

The constant b is the harmonic oscillator length $b = (\hbar/M\omega)^{\frac{1}{2}} \approx 1.006 A^{\frac{1}{6}}$ fm and L_β means the Laguerre polynomial [20] with $\beta = \frac{1}{2}(l_p + l_n - l'') + s + s'$.

Appendix B

To express the amplitude $F(\mathbf{k},\mathbf{k}')$ (eq.(6)) in terms of the QRPA quantities one needs to calculate two matrix elements $\langle f,\pi^-|h_p|mJM\rangle$ and $\langle m'JM|h_p|i,\pi^+\rangle$. It can be done by using the quasiboson approximation [26]

$$\langle RPA|\left[C(abJM),C^\dagger(a'b'J'M')\right]|RPA\rangle \approx \delta_{aa'}\delta_{bb'}\delta_{JJ'}\delta_{MM'} \qquad (B1)$$

and the fact that the QRPA phonon annihilation operator Q^m_{JM} gives zero operating on the RPA vacuum. Finally one can receive

$$\langle RPA|\left[\mathcal{R}^{J'M'}_{p'n'},C^\dagger(pnJM)\right]|RPA\rangle = (-1)^{J'+M'}v_{p'}u_{n'}\delta_{pp'}\delta_{nn'}\delta_{JJ'}\delta_{M-M'} \quad , \qquad (B2)$$

$$\langle RPA|\left[\mathcal{R}^{J'M'}_{p'n'},\tilde{C}(pnJM)\right]|RPA\rangle = -(-1)^{J+M}u_{p'}v_{n'}\delta_{pp'}\delta_{nn'}\delta_{JJ'}\delta_{M-M'} \quad . \qquad (B3)$$

With the aid of the formulae (B1)-(B3) one finds expressions for both needed matrix elements

$$\langle f,0^+;\pi^-(\mathbf{k}')|h_p|mJM\rangle = -\sqrt{12}\, i\frac{f}{m_\pi}\sqrt{4\pi}Y_{JM}(\Omega_{k'})$$

$$\cdot\left[\sum_{pn}(-1)^{j_p+j_n}G^J_{pn}(k')\left(\bar{X}^m_{(pn)J}v_pu_n+\bar{Y}^m_{(pn)J}u_pv_n\right)\right] \qquad (B4)$$

and

$$\langle m'JM|h_p|i,0^+;\pi^+(\mathbf{k})\rangle = \sqrt{12}\, i\frac{f}{m_\pi}\sqrt{4\pi}(-1)^JY_{JM}(\Omega_k)$$

$$\cdot\left[\sum_{pn}G^J_{pn}(k)\left(X^{m'}_{(pn)J}u_pv_n+Y^{m'}_{(pn)J}v_pu_n\right)\right] \quad . \qquad (B5)$$

The quantities with and without bars correspond to the daughter and parent nucleus ground state, respectively.

Abstract: The quasiparticle random phase approximation (QRPA) was applied to the double charge exchange (DCX) reaction with pions $\pi^+ + {}_ZX_N \longrightarrow \pi^- + {}_{Z+2}X_{N-2}$. A case of the ground state to ground state transition is discussed. Final formulae for the

total amplitude and differential cross section are given within the p-wave $\pi - N$ effective interaction.

Streszczenie: W artykule zastosowano formalizm RPA do opisu reakcji z pionami podwójnej wymiamy ładunku na jądrach atomowych. Dyskutowany jest kanał reakcji, w którym końcowe jądro atomowe znajduje się w stanie podstawowym. Otrzymano wyrażenia na całkowitą amplitudę reakcji i różniczkowe przekroje czynne dla efektywnego oddziaływania pion-nukleon w przybliżeniu fali p.

References

[1]. *Proceedings of The Pion-Nucleus Double Charge Exchange Workshop* Los Alamos, January, 1985, eds. H.W.Baer and M.T.Leitch.

[2]. E.R.Siciliano, M.D.Cooper, M.B.Johnson, and M.T.Leitch, Phys.Rev. C **34** (1986) 267.

[3]. M.Bleszynski, R.J.Glauber, Phys.Rev. C **36** (1987) 681; E.Bleszynski, M.Bleszynski and R.J.Glauber, Phys.Rev.Lett. **60** (1988) 1483.

[4]. N.Auerbach, W.R.Gibbs and Joseph N.Ginocchio, W.B.Kaufman, Phys.Rev. C **38** 1277;
Joseph N.Ginocchio, Phys.Rev. C **40** (1989) 2168.

[5]. E.Oset, D.Strottman and G.E.Brown, Phys.Lett. B **73** (1978) 393.

[6]. G.E.Miller, Phys.Rev.Lett. **50** (1983) 1106.

[7]. P.Vogel and M.R.Zirnbauer, Phys.Rev.Lett. **57** (1986) 3148.

[8]. K.Muto and H.V.Klapdor, Phys.Lett. B **201** (1988) 420.

[9]. O.Civitarese, Amand Faessler and T.Tomoda, Phys.Lett. B **194** (1987) 11.

[10]. A.Fazely and L.C.Liu, Phys.Rev.Lett. **57** (1986) 968; T.Tomoda, Phys.Rev.Lett. **59** (1987) 2383.

[11]. T.Ericson and W.Weise, *Pions and nuclei* Clarendon Press, Oxford (1988).

[12]. J.M.Eisenberg, D.S.Koltun, *Theory of Meson Interactions with Nuclei*, John Wiley and Sons, Inc., New York-Chichester-Brisbane-Toronto (1980).

[13]. H.W.Fearing, Prog.Part.and Nuc.Phys., **12** (1981) 113.

[14]. J.A.Halbleib and R.S.Sorensen, Nucl.Phys. A **98** (1967) 542.

[15]. A.H.Huffman, Phys.Rev. C **2** (1970) 742.

[16]. D.Cha, Phys.Rev. C **27** (1983) 2269.

[17]. P.Ring, P.Schuck, *The Nuclear Many-Body Problem* Springer Verlag, New York -Heidelberg-Bonn (1980).

[18]. J.Suhonen, T, Taigel and Amand Faessler, Nucl.Phys. A **486** (1988) 91.

[19]. K.Grotz and H.V.Klapdor, Nucl.Phys. A **60** (1986) 395.

[20]. H.Bateman, A.Erdelyi, *Higher Transcendental Functions*, Mc Graw-Hill Book Co., Inc., New York-Toronto-London (1953).

[21]. E.Oset, W.Weise, Nucl.Phys. A **319** (1979) 47.

[22]. T.Karapiperis and M.Kobayashi, Phys.Rev.Lett. **54** (1985) 1230.

[23]. M.Brack, D.O.Riska, W.Weise, Nucl.Phys. A **287** (1977) 425.

[24]. W.A.Kaminski and Amand Faessler, Phys.Lett. B, submitted for publication.

[25]. D.A.Warshalovitsh, A.N.Moskalev, W.K.Khersonskij, *Quantum Theory of angular momentum* (in russian), Nauka, Leningrad (1975).

[26]. A.M.Lane, *Nuclear Theory*, W.A.Benjamin, Inc., New York-Amsterdam (1964).