ANNALES UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN-POLONIA
VOL.XLII,4 SECTIO AAA 1987

Instytut Fizyki UMCS

Zdzisław WROŃSKI, Danuta STACHÓRSKA

Electron Current of Metallic Electrode Placed in Plasma

Prąd elektronowy metalowej elektrody umieszczonej w plazmie

Электронный ток металлического зонда помещённого в плазме

1. INTRODUCTION

The theoretical determination of the electron current (J_e) of the metallic electrode, placed in ionised gas is an important problem of the plasma physics. A dependence of the current on the electron energy distribution $[F(\varepsilon)]$ is specially important. The widely known Druyvestein theory [1] is not fully satisfactory for resolving the problem. This theory does not explain the fact, observed in the experiment with the electrostatic probe, that electron current density $i = \frac{J_e}{S}$ decreases when the area surface S of the probe increases, e.g. see Herrmann and Klagge [2]. Attempts at an explanation of the experimental fact were often undertaken. The consideration of Swift [3] concerning the spherical probe are successful. However, the results of Swift theory can be used only for a low pressure with regards to accepted assumption.

In the present paper, the generalization of the Swift theory is described. In these consideration the problem of the calculation of the special integrals $[\theta(P, x)]$ is present. Values of many different integrals can be determined by integral $\theta(P, x)$. For the above reasons, integral is treated as the new special function. The values of $\theta(P, x)$ calculated with the computer are presented in the tabele.

2. GENERAL ANALYSIS OF THE PROBLEM

The metallic electrode, placed in the plasma, is the object of an intensive neutralization of charge carriers: the plasma becomes locally impoverished. When the electrode potential (V) in relation to the plasma is negative, the positive charge sheat surrounds the electrode. This sheat having the size (a) shields the electrical field of the electrode. The determination of the d is an open problem, e.g. see report [4]. In general, it is accepted that d is equal to the Debeye length $\lambda_d = \sqrt{\frac{\sigma_0 T_e T_i}{n_e e(T_e + T_i)}}$, σ_0 is here the permitivity of the free space, $T_e T_i$ are the electron and ion temperatures, n is the plasma density, e is electron charge. If the mean free path (λ_e) of the electrons is longer than the sheat size, the Druyvestein theory (modified by Medicus [5]) can be used for the determination of the electron current of the electrode. According to the above theory, the group $F_1(\varepsilon)\Delta\varepsilon$ of the electrons, placed at distance of d from the electrode, forms the current ΔJ_e defined as

$$\Delta J_e = gS\sqrt{\varepsilon}(1 - \frac{V}{\varepsilon}) \cdot F_1(\varepsilon) \cdot \Delta\varepsilon\,, \tag{1}$$

where $g = \sqrt{\frac{e^3}{8m}}$ and m is here the electron mass.

The advantage of the Druyvestein theory is, that it gives a simple relationship of $F_1(\varepsilon)$ with the second derivative $\frac{d^2 J_e}{dV^2}$ of the electron current,

$$\frac{d^2 J_e}{dV^2} = \frac{gSF_1(\varepsilon)}{\sqrt{\varepsilon}}\,. \tag{2}$$

The above formula is widely used in order to determine the electron energy distribution function $F_0(\varepsilon)$ of an undisturbed plasma. Using the above formula one accepts that $F_1(\varepsilon) = F_0(\varepsilon)$. However, this assumption is often incorrect. This assumption is insignificant if one uses the Druyvestein theory in the Swift modification. According to Swift, the plasma perturbation around the electrode has a more complex structure than that accepted in Druyvestein theory, see Figure 1.

Besides the region of the strong electrical field , the region of the weak field is present at which the plasma density is lower than that of the undisturbed plasma. The electron current passes through this second region in a diffusive manner. At distance of r from the spherical electrode, the group $[F(\varepsilon)\Delta\varepsilon]$ of the electrons forms the electron current

$$\Delta J_e = 4\pi r^2 eD\frac{d[F(\varepsilon)\Delta\varepsilon]}{dr}\,, \tag{3}$$

where $D = \frac{1}{3}\lambda_e\sqrt{\frac{2e\varepsilon}{m}}$. In case of the spherical electrode, a very useful assumption can be accepted that $F(\varepsilon) \to F_0(\varepsilon)$ as $r \to \infty$. When we use the above assumption then resolving equation (3), we obtain

$$F_1(\varepsilon) = F_0(\varepsilon) - \frac{\Delta J_e}{4\pi eD(a+d)}\,. \tag{4}$$

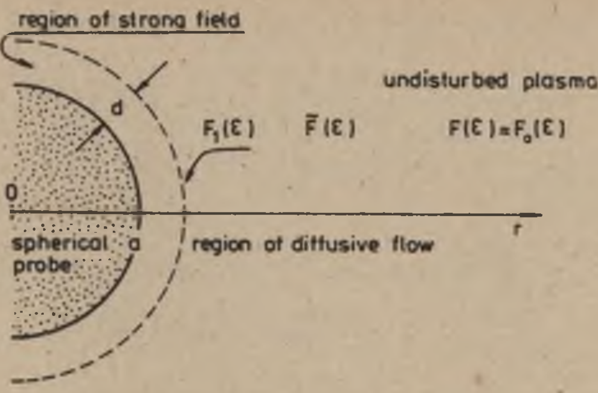

Fig. 1. Plasma stucture arround the spherical electrode

Substituting (4) into formula (1) and integrating over region $V \div \infty$ we obtain

$$J_e(V) = gS(P-1)\int_V^{\infty} \frac{\varepsilon - V}{\varepsilon - PV}\sqrt{\varepsilon}F_0(\varepsilon)d\varepsilon\,, \tag{5}$$

where $P = \dfrac{\psi}{2+\psi}$, $\psi = \dfrac{3a^2}{2\lambda_e(a+d)}$ and a is the electrode radius. On the basis of formula (5) a very important relationship of $F_0(\varepsilon)$ with $\dfrac{d^2J_e}{dV^2}$ can be determined as

$$\frac{d^2J_e}{dV^2} = gS\left[\frac{F(\varepsilon=V)}{\sqrt{\varepsilon}} - 2P(1-P)^2\int_V^{\infty}\frac{F(\varepsilon)\sqrt{\varepsilon^3}}{(\varepsilon-PV)^3}dV\right]. \tag{6}$$

Swift obtained analogous expressions. The presence of the integrals in formulae (5, 6) causes that the further solution of the problem is very difficult. Swift considering the relationship $\dfrac{d^2J_e}{dV^2} = f[F(\varepsilon)]$ determined the correction function for the Maxwellian and Druyvesteinian distributions of the electrons. Swift calculated suitable integrals in a rough manner, when he assumed that $\dfrac{a}{\lambda_e} < 0.6$. However, this assumption limits the use of the Swift solution in the experiment with the electrostatic probe.

The typical energy distributions $[F(\varepsilon)]$ of the plasma electron can be described by the formula:

$$F(\varepsilon) = \frac{n\alpha\sqrt{\varepsilon}}{\sqrt{T^3}}\exp-\left(\frac{\beta\varepsilon}{T}\right)^k \tag{7}$$

where α, β, k are the suitable constants dependent on the distribution type. Substituting (7) into (5, 6) we obtain

$$J_e(V) = \frac{gSn\alpha(1-P)}{\sqrt{T^3}} \int_V^\infty \left[\varepsilon - V(1-P) + \right. \\ \left. - \frac{PV^2(1-P)}{\varepsilon - PV}\right] \cdot \left[\exp -(\frac{\beta\varepsilon}{T})^k\right] d\varepsilon \,, \tag{8}$$

$$\frac{d^2 J_e}{dV^2} = \frac{gSn\alpha}{\sqrt{T^3}} \left\{ \left[\exp -(\frac{\beta V}{T})^k\right] - 2P(1-P)^2 \int_V^\infty \left[\frac{1}{\varepsilon - PV} + \right.\right. \\ \left.\left. + \frac{2PV}{(\varepsilon - PV)^2} + \frac{P^2V^2}{(\varepsilon - PV)^3}\right] \cdot \left[\exp - \left(\frac{\beta\varepsilon}{T}\right)^k\right] d\varepsilon \right\} . \tag{9}$$

In case of the Maxwellian distribution ($k = 1$, $\alpha = \frac{2}{\sqrt{\pi}}$, $\beta = 1$), the integrals of formula (8, 9) can be simply determined with the exponential-integral function $E_i(x) = \int_x^\infty \frac{1}{t} \cdot \exp -t \cdot dt$ see ref. [6]. For example the $J_e(V)$ can be described as

$$J_e(V) = gSn\frac{2}{\sqrt{\pi}}\sqrt{T}(1-P) \cdot \left(\exp - \frac{V}{T}\right) \left\{1 + \frac{P}{T} + \right. \\ \left. -P(1-P)\frac{V}{T}\left[\exp(1-P)\frac{V}{T}\right] \cdot E_i\left[(1-P)\frac{V}{T}\right]\right\} . \tag{10}$$

3. CASE OF THE DRUYVESTEINIAN ENERGY DISTRIBUTION OF ELECTRONS

For stable homogenous plasma (a model plasma), kept with the stable electrical field, the Druyvesteinian type of the electron energy distribution is characteristic. This distribution is the result of the solution of Boltzmann equation, if the independence of the electron collisions cross section on the electron energy is assumed, see Rutsher [7]. It is characteristic of the classical Druyvesteinian distribution that $k = 2$, $\beta^2 = \frac{2\Gamma(\frac{5}{4})}{3\Gamma(\frac{3}{4})} \cong 0.243$, $\alpha = \sqrt{\frac{2^5\Gamma^3(\frac{5}{4})}{3^3\Gamma^5(\frac{3}{4})}} \cong 0.565$. Often the simple Druyvesteinian distribution is used for which $k = 2$, $\beta = 0.5$, $\alpha = \frac{1}{\sqrt{2}\Gamma(\frac{3}{4})} \cong 0.577$ are characteristics $\Gamma(z) = \int_0^\infty t^{z-1} \cdot \exp -t \, dt$ is the gamma function, see ref. [6].

In case of the Druyvesteinian distribution the integrals of the expression (8, 9) cannot be determined by special functions known so far. However, they can be determined by the expression containing the error function erfc $(x) = \int_x^\infty \exp -t^2 \, dt$ see ref. [6], and the new special function $\theta(P, x) = \int_x^\infty \frac{1}{t - Px} \cdot \exp -t^2 \, dt$.

According to above, the formulae for J_e and for $\frac{d^2 J_e}{dV^2}$ can be described as

$$J_e(V) = gSn\alpha\frac{1}{\beta^2}\sqrt{T}(1-P)\left\{\frac{1}{2}\left[\exp-(\beta\frac{V}{T})^2\right]+ -(1-P)\beta\frac{V}{T}\operatorname{erfc}(\beta\frac{V}{T}) - P(1-P)(\beta\frac{V}{T})^2\theta(P,\beta\frac{V}{T})\right\}, \tag{11}$$

$$\frac{d^2 J_e}{dV^2} = \frac{gSn\alpha}{\sqrt{T^3}}\{[1-4P^2+3P^2+2P^4(1-P)(\beta\frac{V}{T})^2]\cdot[\exp-(\beta\frac{V}{T})^2]+ -4P^2(1-P)^2[P^2(\beta\frac{V}{T})^3-2P(\beta\frac{V}{T})]\operatorname{erfc}(\beta\frac{V}{T})+ -2P[(1-P)^2-5(1-P)^2(\beta\frac{V}{T})^2+P^3(\beta\frac{V}{T})^4]\theta(P,\beta\frac{V}{T})\} \tag{12}$$

The manner of reducing of integral $\int_V^\infty \frac{\exp-(\frac{\beta\varepsilon}{T})^2}{(\varepsilon-PV)^3}d\varepsilon$ to integral $\theta(P,x)$ is described in Appendix.

4. CALCULATION OF INTEGRAL $\theta(P,x)$

Introducing new variable $z=\frac{1}{t^2}$, integral $\theta(P,x)$ can be described as follows:

$$\theta(P,x)=\int_x^\infty \frac{\exp-t^2}{t-Px}dt=\int_o^{1/x}\frac{\exp-\frac{1}{z^2}}{z(1-zPx)}dz$$

This transformation permits to calculate value $\theta(P,x)$ by function RUMOAA (double precision) which is described in the reports [8, 9]. The use of the above function allows to calculate $\theta(P,x)$ with relative errors $< 1\cdot 10^{-9}$. The calculations were made with the computer R-22 ($\approx$ IBM-360). The computed values of $\theta(P,x)$ for $P=0.1, 0.2, \dots 0.9$ and $x=0.1, 0.2, \dots 3.0$ are presented in Table 1. Moreover, Figure 2 presents the dependence of $\theta(P,x)$on P and x.

In order to determine intermediate values $\theta_i(P_i,x_i)$ (for x_i, P_i absent in Table 1) linear approximation cannot be used, because the errors can be higher by several per cent. The shape of diagrams $\theta(P,x)=f(x)$ shows that the dependence can be approximated exponential type functions: $\theta(P,x)=A\exp-b\sqrt{x}$, $\theta(P,x)=A\exp-bx$, $\theta(P,x)=A\exp-bx\sqrt{x}$, at intervals $0.05\div 0.3$, $0.2\div 0.8$, $0.7\div 3$, respectively. When constants A and b are determined on the basis of $\theta_1(P,x_1)$ and $\theta_2(P,x_2)$ values, corresponding to x_1, $x_2=x_1+0.1$ arguments, then intermediate $\theta_i(P,x_i)$, corresponding to x_i, $(x_1<x_i<x_2)$ and determined according to the above approximation, is charged with an error smaller than 0.25%. In case of the calculation of intermediate values $\theta_i(P_i,x)$ (for intermediate P_i parameters, different from those given in Table 1) approximation $\theta(P,x)=\frac{A}{b-P}$ can be

used. When constants A and b are calculated on the basis of $\theta_1(P_1, x)$ and $\theta_2(P_2, x)$ then value $\theta_i(P_i, x)$ corresponding to P_i $(P_1 < P_i < P_1 + 0.1)$ and found according to the above approximation are charged with an error smaller than 0.25%.

The above degrees of the accuracy of the $\theta_i(P_i, x_i)$ determination are satisfactory from the practical point of view.

5. CASE OF A CYLINDRICAL ELECTRODE

Let us consider an element of an infinitely long cylindrical electrode having radius a and length l, see Figure 3a. The solution of appriopriate equation, corresponding to equation (3), gives

$$F(\varepsilon, r) - F_1(\varepsilon, a + d) = \frac{\Delta J_e}{2\pi l e D} \ln \frac{r}{a + d}, \tag{13}$$

where ΔJ_e is the diffusive current of electron group $F(\varepsilon)\Delta\varepsilon$. On the basis of formula (13), the relationship $F_1(\varepsilon) = f_1[F_0(\varepsilon)]$ cannot be simply determined under the condition that $F(\varepsilon, r = \infty) = F_0(\varepsilon)$, because $F(\varepsilon) \to \infty$ as $r \to \infty$. It is the principal difficulty of the use of Swift theory for cylindrical electrodes.

In report [10], the above problem was solved in a rough manner, where it was assumed that the Swift perturbation is located at the finite region $a+d\div$ several λ_e. However, this assumption requires grounds and it also requires an additional information about the plasma-electrode state. Moreover this procedure can be used only in case of the slim electrode, $(a << l, \lambda << l)$. This problem can be also solved by introduction of the „spherization" of the cylindrical electrode having finite length. Let us consider the cylindrical electrode having radius a and length l, see Figure 3b. Let us assume that the barrel-shaped surface, placed at distance x from the electrode surface and having the area $S_x = 2\pi[a^2 + (a + x)l + 2x^2 + \pi a x]$ is the geometric locus of points having identical plasma parameters. Surface S_x tends to the sphere as r tends to the infinity. Thus we can write

$$\Delta J_e = e 2\pi[a^2 + (a + x)l + 2x^2 + \pi a x] \frac{d[F(\varepsilon)\Delta\varepsilon]}{dx}. \tag{14}$$

Resolving equation (14) in limits $d \div \infty$, we obtain

$$F_1(\varepsilon) = F_0(\varepsilon) - \frac{\Delta J_e}{eD} \frac{1}{2\pi} \frac{1}{\sqrt{\Delta}} \ln \frac{4d + \pi a + l + \sqrt{\Delta}}{4d + \pi a + l - \sqrt{\Delta}},$$

where $\Delta = \pi(a + l)^2 - 8(a^2 - al)$.

Putting $F_1(\varepsilon)$ in formula (6) and integrating over interval $V \div \infty$, we obtain the expression for $J_e(V)$ which is identical to expression (8). The new formula for $J_e(V)$ has the new coefficients ψ_c, P_c, determined as:

$$P_c = \frac{\psi_c}{2 + \psi_c}, \quad \psi_c = \frac{3}{2} \frac{a^2 + al}{\lambda e \sqrt{\Delta}} \ln \frac{4d + \pi a + l + \sqrt{\Delta}}{4d + \pi a + l - \sqrt{\Delta}}.$$

The further use of the Swift theory for the cylindrical probe is the same as for the spherical probe.

6. EXAMPLE OF THE USE OF THE EXTENDED SWIFT THEORY

Herrmann et al. [11] presented the results of the probe measurement in the neon plasma for which the Druyvesteinian distribution is particularly characteristic. The electron concentration determined with the cylindrical probe by Herrmann was twice higher than that determined with the spherical probe. This discrepancy could be caused by small size of the spherical probe which was 0.6 mm in diameter. The slim cylindrical probe has the good contact with the plasma. The small spherical probe practically collects the electron current only with half of its surface. Another half is shaded by the probe leg. For the better result one should use a probe of a larger size.

Under the conditions of the Herrmann experiment ($P = 1.29$ torr) the mean free path of the electrons was ≈1 mm. If the probe of 1.2 mm in diameter is used, the ratio $\frac{a}{\lambda_e}$ equals 0.6. This value of $\frac{a}{\lambda_e}$ is the limit value permissible by Swift in his theory. In this case and at higher pressures the correcting function, determined with integral $\theta(P, x)$, ought to be used. Figure 4 gives the example of the use of the extended Swift theory.

It is seen that the discrepancy of the characteristics determined with the fundamental theory and those determined with the extended Swift theory is considerable if $\frac{a}{\lambda_e}$ is high. Moreover it can simply be deduced that the density of the electrode electron current decreases when the electrode radius increases. At higher gas pressures (several torr) the use of the extended Swift theory is necessary even for the small electrode as that having 1.2 mm in diameter.

7. CONCLUSION

The generalization of the Swift theory permits to use it at every values of the ratio of a electron mean free path and a probe radius. This generalization can be used for typical electron energy distributions: Maxwellian and Druyvesteinian. One should be stressed that the Swift theory is important because it takes to account the local impoverishment of the plasma occurring around the electrode.

Swift [3] has given one's attention to determine of the electrode potential in relation to the plasma. The results of this work can simply be incorporated in the Swift's considerations relative to the potential determination.

In order to correct the energy distributions, measured with an electrostatic probe, we have defined a new special integral. This integral can be useful in the solving of the other problems. In regards to the physical character of this report only the fundamental properties of the above integral are shown.

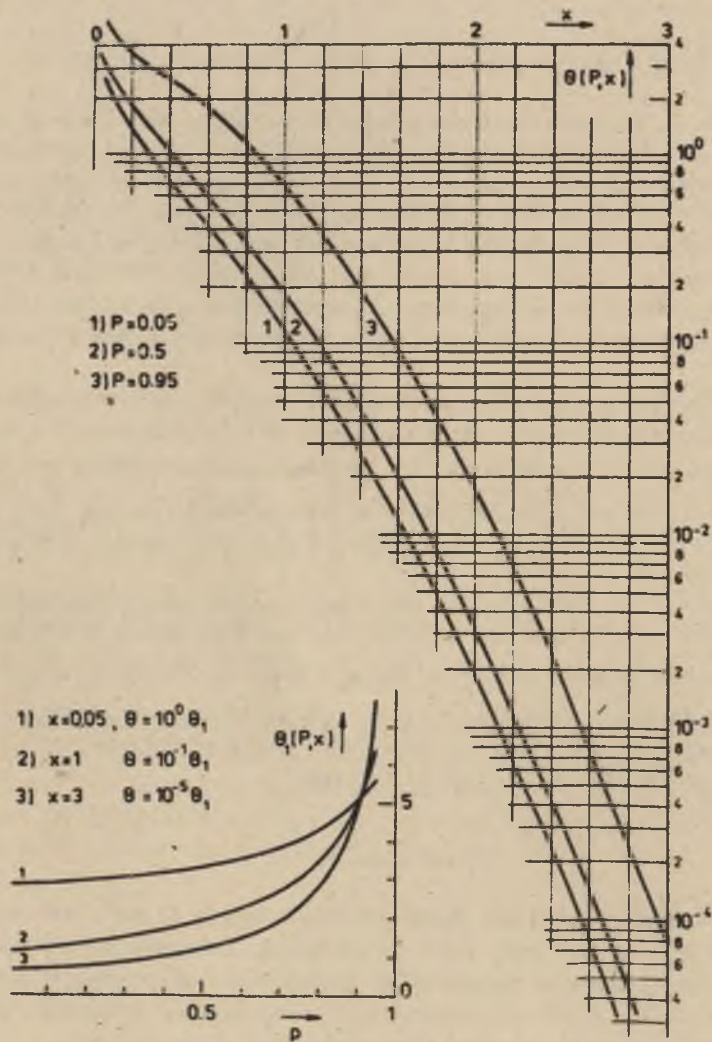

Fig. 2. Integral $\theta(P, x)$ as function of parameters P and arguments x

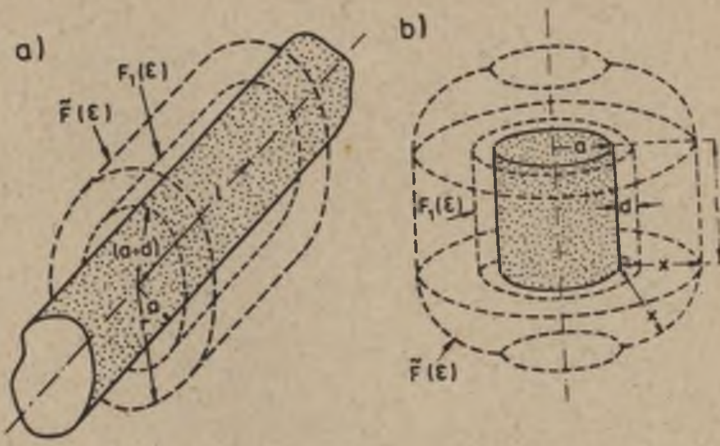

Fig. 3 Scheme of the use of the Swift theory for cylindrical electrodes

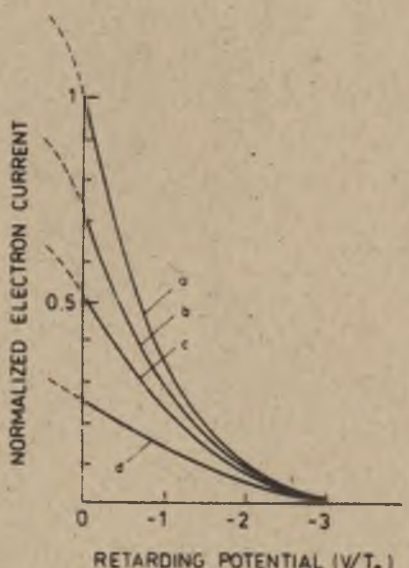

Fig. 4 The current-voltage characteristics of the spherical probe placed in the Druyvesteinian plasma. The characteristics were calculated according to the extended Swift theory for several values of ratio a/λ_e: a — ≈ 0, b — 1, c — 2, d — 5. Electron current $J(V)$ was normalized to value $J(0) = \frac{gSn\alpha\sqrt{T}}{2\beta^2}$ forecasted on the basis of the fundamental Druyvestein theory. According to Swift [3], $d = \lambda_e$ was assumed

Table 1. Values of integrals $\theta(P, x)$*

x,P	0.1		0.2		0.3		0.4	
0.1	2.1073	0	2.2075	0	2.3229	0	2.4581	0
0.2	1.4157	0	1.4976	0	1.5952	0	1.7111	0
0.3	1.0190	0	1.0881	0	1.1694	0	1.2669	0
0.4	7.5267	-1	8.0875	-1	8.7527	-1	9.5584	-1
0.5	5.6033	-1	6.0518	-1	6.5877	-1	7.2422	-1
0.6	4.1714	-1	4.5249	-1	4.9497	-1	5.4725	-1
0.7	3.0913	-1	3.3655	-1	3.6972	-1	4.1079	-1
0.8	2.2739	-1	2.4834	-1	2.7382	-1	3.0555	-1
0.9	1.6568	-1	1.8145	-1	2.0070	-1	2.2481	-1
1.0	1.1941	-1	1.3108	-1	1.4540	-1	1.6342	-1
1.1	8.5023	-2	9.3536	-2	1.0401	-1	1.1725	-1
1.2	5.9756	-2	6.5863	-2	7.3403	-2	8.2966	-2
1.3	4.1425	-2	4.5734	-2	5.1071	-2	5.7863	-2
1.4	2.8307	-2	3.1299	-2	3.5013	-2	3.9756	-2
1.5	1.9058	-2	2.1100	-2	2.3642	-2	2.6897	-2
1.6	1.2636	-2	1.4007	-2	1.5717	-2	1.7912	-2
1.7	8.2479	-3	9.1523	-3	1.0282	-2	1.1737	-2
1.8	5.2978	-3	5.8844	-3	6.6190	-3	7.5667	-3
1.9	3.3477	-3	3.7217	-3	4.1908	-3	4.7971	-3
2.0	2.0806	-3	2.3149	-3	2.6092	-3	2.9903	-3
2.1	1.2715	-3	1.4157	-3	1.5971	-3	1.8324	-3
2.2	7.6394	-4	8.5113	-4	9.6096	-4	1.1036	-3
2.3	7.6394	-4	5.0291	-4	5.6823	-4	6.5322	-4
2.4	2.6180	-4	2.9201	-4	3.3016	-4	3.7987	-4
2.5	1.4928	-4	1.6659	-4	1.8848	-4	2.1702	-4
2.6	8.3628	-5	9.3370	-5	1.0569	-4	1.2179	-4
2.7	4.6019	-5	5.1402	-5	5.8218	-5	6.7130	-5
2.8	2.4872	-5	2.7793	-5	3.1494	-5	3.6337	-5
2.9	1.3202	-5	1.4757	-5	1.6730	-5	1.9314	-5
3.0	6.8812	-6	7.6947	-6	8.7269	-6	1.0080	-5

* The reading manner of the $\theta(P, x)$ values is given in the following examples:

$$\theta(0.1, 0.1) = 2.1073 \cdot 10^{0} \qquad \theta(0.9, 3.0) = 4.6864 \cdot 10^{-5} .$$

0.5		0.6		0.7		0.8		0.9	
2.6206	0	2.8228	0	3.0880	0	3.4688	0	4.1329	0
1.8523	0	2.0306	0	2.2683	0	2.6157	0	3.2345	0
1.3870	0	1.5408	0	1.7448	0	2.0581	0	2.6206	0
1.0561	0	1.1861	0	1.3642	0	1.6334	0	2.1331	0
8.0650	-1	9.1416	-1	1.0636	0	1.2930	0	1.7272	0
6.1325	-1	7.0108	-1	8.2508	-1	1.0154	0	1.3848	0
4.6324	-1	5.3316	-1	6.3242	-1	7.8898	-1	1.0966	0
3.4635	-1	4.0117	-1	4.7978	-1	6.0539	-1	8.5664	-1
2.5601	-1	2.9823	-1	3.5933	-1	4.5807	-1	6.5688	-1
1.8668	-1	2.1880	-1	2.6541	-1	3.4158	-1	4.9899	-1
1.3456	-1	1.5829	-1	1.9320	-1	2.5084	-1	3.7180	-1
9.5530	-2	1.1285	-1	1.3853	-1	1.8132	-1	2.7244	-1
6.6825	-2	7.9250	-2	9.7779	-2	1.2895	-1	1.9627	-1
4.6038	-2	5.4788	-2	6.7920	-2	9.0208	-2	1.3896	-1
3.1223	-2	3.7277	-2	4.6415	-2	6.2048	-2	9.6688	-1
2.0839	-2	2.4953	-2	3.1195	-2	4.1957	-2	6.6089	-2
1.3684	-2	1.6429	-2	2.0615	-2	2.7884	-2	4.4373	-2
8.8377	-3	1.0636	-2	1.3393	-2	1.8211	-2	2.9261	-2
5.6124	-3	6.7701	-3	8.5519	-3	1.1686	-2	1.8949	-2
3.5040	-3	4.2355	-3	5.3662	-3	7.3668	-3	1.2048	-2
2.1503	-3	2.6042	-3	3.3084	-3	4.5615	-3	7.5221	-3
1.2968	-3	1.5733	-3	2.0039	-3	2.7741	-3	4.6102	-3
7.6846	-4	9.3388	-4	1.1922	-3	1.6568	-3	2.7737	-3
4.4739	-4	5.4455	-4	6.9672	-4	9.7162	-4	1.6380	-3
2.5586	-4	3.1185	-4	3.9984	-4	5.5947	-4	9.4947	-4
1.4372	-4	1.7540	-4	2.2533	-4	3.1627	-4	5.4014	-4
7.9289	-5	9.6884	-5	9.2468	-5	1.7552	-4	3.0156	-4
4.2954	-5	5.2546	-5	6.7737	-5	9.5623	-5	1.6522	-4
2.2849	-5	2.7980	-5	3.6133	-5	5.1133	-5	8.8831	-5
1.1933	-5	1.4628	-5	1.8914	-5	2.6837	-5	4.6864	-5

8. APPENDIX

(i) The integral $\int_x^\infty \frac{\exp -t^2}{(t-Px)^3}dt$ of the expression (9) was calculated as follows:

$$\int_x^\infty \frac{\exp -t^2}{(t-Px)^3}dt = \frac{\exp -x^2}{2x^2(1-P)^2} - \int_x^\infty \frac{t\cdot\exp -t^2}{(t-Px)^2}dt =$$

$$= \frac{\exp -x^2}{2x^2(1-P)^2} - \int_x^\infty \frac{\exp -t^2}{(t-Px)}dt - Px\int_x^\infty \frac{\exp -t^2}{(t-Px)^2}dt\,.$$

The next use of the integration by parts and of the substitution $t - Px = z$ gives

$$\int_x^\infty \frac{\exp -t^2}{(t-Px)^3}dt = \left[\frac{1}{2x^2(1-P)^2} - \frac{P}{1-P}\right](\exp -x^2 +$$

$$+2Px\ \mathrm{erfc}\,(x) - (1-2P^2x^2)\theta(P,x)$$

(ii) According to the above scheme, the integral defined as

$$\theta(P,x)_{\mu.\nu} = \int_x^\infty \frac{t^\mu \exp -t^2}{(t-Px)^\nu}dt$$

can be transformed to the expression containing integrals erfc (x) and $\theta(P,x)$. μ, ν are natural numbers and $\mu \le \nu$.

(iii) Putting $t^2 = z$, the integral $\theta(0,x)$ can be evaluated as

$$\theta(0,x) = \int_x^\infty \frac{1}{2}\frac{\exp -z}{z}dt = \frac{1}{2}E_1(x^2)\,.$$

REFERENCES

[1] Druyvestein M., *Physica* 1, 1003, (1934).

[2] Herrmann D., Klagge S., *Beitr. Plasma Phys.* 12, 45, (1972).

[3] Swift J. D., *Proc. Phys. Soc.* 79, 697, (1962).

[4] Swift J. D., Schwar M. I. R., Electrical Probes for Plasma Diagnostic, Illife Books, London 1970.

[5] Basu J., Sen C., *Japan J. Appl.Phys.* 32, 1081, (1973).

[6] Medicus G., *J. Appl. Phys.* 32, 2512, (1961).

[7] Janke E., Emde F., Losch F., Tafeln Heherer Funktionen, Stuttgart 1960

[8] Rutscher A., *Beitr. Plasma Phys.* 7, 43, (1967).

[9] Cylkowski Z., Alborithm 43, Evaluation of improper integral on finite interval, Zastosowania Matematyki 15, 1976.

[10] *Biblioteka Modułów Matematycznych*, Elwro, Wrocław, Poland 1981.

[11] Stachórska D., Wroński Z., *Proc.* XII ICPIG, Eindhoven 1975.

[12] Herrmann D., Pfau S., Kagel H., *Beitr. Plasma Phys.* 12, 1, (1972).

STRESZCZENIE

W świetle teorii Druyvesteina i Swifta badano prąd elektronowy sondy. Wprowadzenie nowej funkcji specjalnej pozwala określić prąd elektronowy sondy dla dowolnego stosunku promienia sondy do średniej drogi swobodnej elektronów. Wartości tej funkcji, obliczone numerycznie są zestawione w tabeli. Przedstawiono zastosowanie uogólnionej teorii Swifta dla sondy sferycznej. Ponadto pokazano, że nowa funkcja specjalna jest elementem rozwiązania innych nieelementarnych całek.

РЕЗЮМЕ

В рамках теории Дрювестейна и Свифта исследовался электронный ток зонда. Введение новой функдии позволяет определить электронный ток зонда при любом отношении радиуса зонда к свободному пробегу электронов. Значения этой функции, вычисленные с помощью ЭВМ, указаны в таблице. Показано применение обобщённой теории Свифта к сферическому зонду. Показано также использование новой специальной функции для решения других неэлементарных интегралов.

Złożone 31.VII.1987