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Model Equations for Waves Propagation in an Infinite Cylinder with Thin Walls of Elastic Rings

Równanie modelowe dla propagacji fal w nieskończonym cylindrze o cienkich i elastycznych pierścieniach

Модельные уравнения для распространения волн в бесконечном цилиндре с тонкими и упругими кольцами

1. INTRODUCTION

Compliant tubes containing a fluid have been studied since the time of Thomas Young [1] in connection with modelling the propagation of the arterial pressure pulse. Moodie [2] et al. has discussed the tube equations based upon a thin– walled shell theory for thethered tubes. The fluid has been assumed inviscid and a one–dimensional theory has been extracted by averaging quantities over the tube cross section. The other model has been employed to study the propagation of pressure and flow pulses along initially uniform tubes and their subsequent interaction with various junctions characteristic of the arterial system [3]. Wave propagation and shock formation in nonlinear elastic and viscoelastic fluid filled tubes for a Mooney–Rivilin material has been discussed [4]. Also radial motion of a non–linear viscoelastic tube has been studied [5]. Two–dimensional analysis was employed to study pulse propagation in thin–walled circularly cylindrical elastic tubes containing an inviscid and incompressible liquid [6]. A viscoelastic shell theory model for transcient pressure perturbations in fluid filled tubes has been presented and tested against experiments involving water filled latex tube [7].

In 1980 Lamb [8] showed using Taniuti–Wei's [9] method that in the absence of energy dissipation the fundamental set of equations describing liquid that is confined within an infinitely long circular cylinder with thin walls of elastic rings leads to the Korteweg–de Vries equation. For this case the same equation has been derived via the Lagrangian method [10]. On the contrary when dissipation has been taken into consideration Burgers equation has been derived [11, 12].

In the present paper we combine the above mentioned equations for a weak dissipation and dispersion case obtaining the Korteweg–de Vries–Burgers equation. Also the Korteweg–de Vries equation is derived via the derivative expansion method [13] in section 5. For a strong dispersion case the nonlinear Schrödinger equation is obtained via the derivative expansion method in section 3 and via the reductive Taniuti–Wei's method in section 4. Last section is devoted to the short summary.

2. FUNDAMENTAL SET OF EQUATIONS

We consider now the one–dimensional irrotational fluid waves of characteristic amplitude l and characteristic length λ in an infinitely long tube with thin walls of elastic rings and a diameter $2a$ to take into account dissipation of energy, nonlinearity and dispersion of medium on the assumption that $l << 2a << \lambda$. Then the fundamental set of equations may be written as follows

(i) the continuity equation

$$(\rho A)_t + (\rho V A)_x = 0\,, \tag{2.1}$$

(ii) the Euler's equation

$$V_t + VV_x + \frac{1}{\rho}p_x = \frac{\mu}{\rho}V_{xx} \tag{2.2}$$

(ii) the Newton's equation [9]

$$A_{tt} + \frac{E}{a^2\rho_m}A - \frac{2\pi a}{\rho_m h}p = \frac{\pi(Eh - 2aq)}{\rho_m h}\,, \tag{2.3}$$

(iv) the state equation

$$\rho \equiv \rho(p) \equiv d\cdot p\,, \tag{2.4}$$

where the following notation are used: ρ – liquid density, A – area of the crossection, V – liquid velocity, μ – viscosity coefficient, a – tube radius at the undisturbed uniform state, ρ_m – density of the tube material, E – Young's modulus in the circumferential direction, p – liquid pressure, q – outside pressure, h – thickness of the wall. The subscripts x and t imply partial differentiation.

3. THE NONLINEAR SCHRÖDINGER EQUATION. THE DERIVATIVE EXPANSION METHOD

We derive now the nonlinear Schrödinger equation via the derivative expansion method [13]. For simplicity the effect of damping is neglected and we assume that the fluid density is constant. In other context, Davey [14] has derived a nonlinear Schrödinger equation which is modified to allow dissipation.

Introducing new coordinates as follows

$$A \to \frac{1}{\pi a^2} A\,, \quad p \to \frac{2a}{Eh} p\,, \quad V \to \sqrt{\frac{2a\rho}{Eh}}\, V\,,$$

$$t \to \sqrt{\frac{E}{\rho_m a^2}}\, t\,, \qquad x \to \frac{2\rho}{\rho_m a h}\, x\,, \tag{3.1}$$

dimensionless equations are obtained, i.e.,

$$A_t + (AV)_x = 0\,, \tag{3.2}$$

$$V_t + VV_x + A_x + A_{ttx} = 0\,. \tag{3.3}$$

We define two dimensionless small parameters, namely:

$$\varepsilon = \frac{2a}{\lambda}\,, \tag{3.4}$$

$$\delta = \frac{l}{2a}\,, \tag{3.5}$$

ε and δ measure the weakness or dispersion and nonlinearity, respectively. The nonlinear Schrödinger equation is derived on the assumption that $\varepsilon = \delta$.

In equations (3.2) – (3.3), we introduce the multiple spatial and temporal scales

$$\begin{aligned} x_n &= \varepsilon^n x\,, \quad t_n = \varepsilon^n t\,, \quad n = 0, 1, 2 \ldots\,, \\ x_0 &= x\,, \qquad t_0 = t\,. \end{aligned} \tag{3.6}$$

The derivative operators are considered to be of the form

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon \frac{\partial}{\partial t_1} + \varepsilon^2 \frac{\partial}{\partial t_2} + \cdots\,, \tag{3.7a}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x_0} + \varepsilon \frac{\partial}{\partial x_1} + \varepsilon^2 \frac{\partial}{\partial x_2} + \cdots\,, \tag{3.7b}$$

$$\begin{aligned} \frac{\partial^3}{\partial x \partial t^2} &= \frac{\partial^3}{\partial t_0^2 \partial x_0} + \varepsilon \left(2 \frac{\partial^3}{\partial t_0 \partial t_1 \partial x_0} + \frac{\partial^3}{\partial t_0^2 \partial x_1} \right) + \\ &+ \varepsilon^2 \left(\frac{\partial^3}{\partial t_1^2 \partial x_0} + 2 \frac{\partial^3}{\partial t_0 \partial t_2 \partial x_0} + 2 \frac{\partial^3}{\partial t_0 \partial t_1 \partial x_1} + \frac{\partial^3}{\partial t_0^2 \partial x_0} \right) + \cdots\,. \end{aligned} \tag{3.7c}$$

The dependent variables V, A are expanded around the undisturbed uniform state into the asymptotic series in terms of the same parameter δ by writting

$$A = 1 + \delta A_1 + \delta^2 A_2 + \cdots\,, \qquad V = \delta V_1 + \delta^2 V_2 + \cdots\,, \tag{3.8}$$

Substitution of (3.7) and (3.8) into equations (3.2) and (3.3) yields a sequence of equations by equating the coefficients of like powers of ε:

$$A_{1t_0} + V_{1x_0} = 0\,, \tag{3.9}$$

ε :

$$V_{1t_0} + A_{1x_0} + A_{1t_0^2x_0} = 0\,, \tag{3.10}$$

$$A_{2t_0} + A_{1t_1} + V_{2x_0} + V_{1x_1} + A_1V_{1x_0} + V_1A_{1x_0} = 0\,, \tag{3.11}$$

ε^2 :

$$V_{2t_0} + V_{1t_1} + V_1V_{1x_0} + A_{2x_0} + A_{1x_1} + A_{2t_0^2x_0} + 2A_{1t_0t_1x_0} + A_{1t_0^2x_1} = 0\,, \tag{3.12}$$

$$\begin{aligned} &A_{3t_0} + A_{2t_1} + A_{1t_2} + V_{3x_0} + V_{2x_1} + V_{1x_2} + A_1(V_{2x_0} + V_{1x_1}) + \\ &+ A_2V_{1x_0} + V_1(A_{2x_0} + A_{1x_1}) + V_2A_{1x_0} = 0\,, \end{aligned} \tag{3.13}$$

ε^3 :

$$\begin{aligned} &V_{3t_0} + A_{3x_0} + A_{3t_0^2x_0} + V_{2t_1} + V_{1t_2} + V_1(V_{2x_0} + V_{1x_1}) + V_2V_{1x_0} + A_{2x_1} + \\ &A_{1x_2} + 2A_{2t_0t_1x_0} + A_{2t_0^2x_1} + A_{1t_1^2x_0} + 2A_{1t_0t_2x_0} + 2A_{1t_0t_1x_1} + A_{1t_0^2x_2} = 0\,. \end{aligned} \tag{3.14}$$

The linear equations (3.9) and (3.10) may be solved to get

$$A_1 = \phi(x_1, \ldots, t_1, \ldots)e^{i\theta} + c.c. + \alpha_1(x_1, \ldots, t_1, \ldots)\,, \tag{3.15}$$

$$V_1 = \frac{\omega}{k}\phi e^{i\theta} + c.c. + \beta_1(x_1, \ldots, t_1, \ldots)\,, \tag{3.16}$$

$$\theta \equiv kx_0 - \omega t_0\,, \tag{3.17}$$

$$\omega^2 = \frac{k^2}{1+k^2}\,, \tag{3.18}$$

where $c.c.$ stands for the complex conjugate to the proceeding term and is introduced for the reality of A_1 and V_1. ϕ is a complex function and α_1 and β_1 are real functions which describe interaction between slow mode and wave trains [15]. This problem, however, is not discussed in this paper. We assume that $\alpha_1 = \beta_1 = 0$.

Substituting equations (3.15) and (3.16) into the second order equations (3.11) and (3,12) we obtain

$$A_{2t_0} + V_{2x_0} + \left(\phi_{t_1} + \frac{\omega}{k}\phi_{x_1}\right)e^{i\theta} + c.c. + 2i\omega\phi^2e^{2i\theta} + c.c. = 0\,, \tag{3.19}$$

$$\begin{aligned} &V_{2t_0} + A_{2x_0} + A_{2t_0^2x_0} + \left(\frac{\omega}{k}\phi_{t_1} + \phi_{x_1} + 2\omega k\phi_{t_1} - \omega^2\phi_{x_1}\right)e^{i\theta} + c.c. + \\ &+ i\frac{\omega^2}{k}\phi^2e^{2i\theta} + c.c. = 0\,. \end{aligned} \tag{3.20}$$

This set of equations may be solved to get

$$A_2 = \frac{1}{2k^2}\phi^2 e^{2i\theta} + c.c. + \bar{b}(x_1,\ldots,t_1,\ldots)e^{i\theta} + c.c. + \bar{c}(x_1,\ldots,t_1,\ldots)\,, \qquad (3.21)$$

$$V_2 = \frac{\omega(1-2k^2)}{2k^3}\phi^2 e^{2i\theta} + c.c. + \bar{f}(x_1,\ldots,t_1,\ldots) + c.c. + \bar{g}(x_1,\ldots,t_1,\ldots) \qquad (3.22)$$

where $\bar{b}$, $\bar{f}$, $\bar{c}$, $\bar{g}$ are arbitrary functions which satisfy the following relation

$$i(\omega\bar{b} - k\bar{f}) = \phi_{t_1} + \frac{\omega}{k}\phi_{x_1} \qquad (3.23)$$

and ϕ satisfies the characteristic equation

$$\phi_{t_1} + \omega_k\phi_{x_1} = 0\,. \qquad (3.24)$$

Finally, from the third order equations (3.13) and (3.14) the nonlinear Schrödinger equation may be obtained

$$\begin{aligned} &2i\omega(1+k^2)\phi_{t_2} + \frac{i}{k}[\omega^2(1-k^2)+k^2]\phi_{x_2} - \frac{2\omega^2(1-k^2)}{k^2}|\phi|^2\phi + \\ &-k^2\phi_{t_1^2} + 2\omega k\phi_{t_1x_1} + i\omega(1+2k^2)\bar{b}_{t_1} + i\omega\bar{f}_{x_1} + ik(\bar{f}_{t_1} + \bar{b}_{x_1}) + \\ &-ik\omega^2\bar{b}_{x_1} = \omega(\bar{c}\omega + 2k\bar{g})\phi\,. \end{aligned} \qquad (3.25)$$

If we introduce coordinate transformation defined by

$$\xi_1 = x_1 - \omega_k t_1\,, \qquad \tau_1 = t_1 \qquad (3.26)$$

and

$$\xi_2 = x_2 - \omega_k t_2\,, \qquad \tau_2 = t_2 \qquad (3.27)$$

from (3.24) we obtain that $\frac{\partial}{\partial\tau_1} = 0$ and all terms containing $\bar{b}$ and $\bar{f}$ are eliminated because of (3.23) and dispersion relation (3.18). Equation (3.25) may be written in the more convenient form

$$i\phi_{\tau_2} + \frac{\omega^3(12k^6+35k^4+39k^2+9)}{4k^4(k^4+3k^2+3)}|\phi|^2\phi - \frac{3\omega}{2(1+k^2)^2}\phi_{\xi_1\xi_1} = C\phi\,, \qquad (3.28)$$

where C is an arbitrary constant to be determined from the boundary conditions.

4. THE NONLINEAR SCHRÖDINGER EQUATION. TANIUTI-WEI'S METHOD

In the previous section, we have derived the nonlinear Schrödinger equation via the derivative expansion method. Here, turning our interest to the same case of strongly dispersive waves, we develop similar analysis based on the reductive Taniuti–Wei's method. For this aim we expand quantities A, V into the following series

$$A = 1 + \sum_{n=1}^{\infty} \varepsilon^n \sum_{m=-\infty}^{\infty} A_m^{(n)}(\xi, \tau) e^{im(kx-\omega t)} ,$$

$$V = \sum_{n=1}^{\infty} \varepsilon^n \sum_{m=-\infty}^{\infty} V_m^{(n)}(\xi, \tau) e^{im(kx-\omega t)} , \tag{4.1}$$

where

$$A_m^{(1)} = V_m^{(1)} = 0 , \qquad m \neq \pm 1 , \tag{4.2}$$

$$U_m^{(n)} \equiv \begin{pmatrix} A_m^{(n)} \\ V_m^{(n)} \end{pmatrix} , \qquad U_m^{(n)*} \equiv U_{-m}^{(n)} , \tag{4.3}$$

$$\xi = \varepsilon(x - \lambda t) , \qquad \tau = \varepsilon^2 t . \tag{4.4}$$

The asterisk denotes the complex conjugate. Substitution of the expansion (4.1) into the fundamental set of equations (3.2), (3.3) yields the following equations for the n–th order terms

ε:

$$V_1^{(1)} = \frac{\omega}{k} A_1^{(1)} , \tag{4.5a}$$

$$\omega^2 = \frac{k^2}{1 + k^2} , \tag{4.5b}$$

$$-\lambda A_{1\xi}^{(1)} - i\omega A_1^{(2)} + V_{1\xi}^{(1)} + ikV_1^{(2)} = 0 , \tag{4.6a}$$

$$-\lambda V_{1\xi}^{(1)} - i\omega V_1^{(2)} + (1 - \omega^2 - 2\omega k\lambda) A_{1\xi}^{(1)} + ik(1 - \omega^2) A_1^{(2)} = 0 , \tag{4.6b}$$

ε^2 :

$$kV_2^{(2)} - \omega A_2^{(2)} + \omega (A_1^{(1)})^2 = 0 , \tag{4.6c}$$

$$-2\omega V_2^{(2)} - 2k(4\omega^2 - 1) A_2^{(2)} + \frac{\omega^2}{k} (A_1^{(1)})^2 = 0 , \tag{4.6d}$$

$$\begin{aligned} &A_{1\tau}^{(1)} - \lambda A_{1\xi}^{(2)} - i\omega A_1^{(3)} + V_{1\xi}^{(2)} + ikV_1^{(3)} + ikA_1^{(1)*} V_2^{(2)} + \\ &+ i\omega A_1^{(1)*} A_2^{(2)} + i(\omega A_0^{(2)} + kV_0^{(2)}) A_1^{(1)} = 0 , \end{aligned} \tag{4.7a}$$

$$\begin{aligned} &(\frac{\omega}{k} + 2\omega k) A_{1\tau}^{(1)} - \lambda V_{1\xi}^{(2)} - i\omega V_1^{(3)} + i\omega A_1^{(1)*} V_2^{(2)} + ikV_0^{(2)} V_1^{(1)} + \\ &+ (1 - \omega^2 - 2\omega k\lambda) A_{1\xi}^{(2)} + ik(1 - \omega^2) A_1^{(3)} + i(2\omega\lambda + k\lambda^2) A_{1\xi\xi}^{(1)} = 0 , \end{aligned} \tag{4.7b}$$

ε^3:

$$A_0^{(2)} = -\frac{3+k^2}{\omega^2(k^4+3k^2+3)}|A_1^{(1)}| + C_1 \,, \tag{4.7c}$$

$$V_0^{(2)} = -\frac{3+2k^2(2+k^2)}{\omega k(k^4+3k^2+3)}|A_1^{(1)}|^2 + C_2 \,. \tag{4.7d}$$

From equations (4.6a) – (4.6d), we obtain

$$\left(\frac{\omega}{k} - \lambda\right)A_{1\xi}^{(1)} - i\omega A_1^{(2)} + ikV_1^{(2)} = 0 \,, \tag{4.8}$$

$$\lambda = \omega_k \,, \tag{4.9}$$

$$A_2^{(2)} = \frac{1}{2k^2}(A_1^{(1)})^2 \,, \tag{4.10}$$

$$V_2^{(2)} = \frac{(1-2k^2)\omega}{2k^3}(A_1^{(1)})^2 \,. \tag{4.11}$$

A compatibility condition for the components of $V_1^{(3)}$ and $A_1^{(3)}$ is reduced to the nonlinear Schrödinger equation for the first order quantity

$$iA_{1\tau}^{(1)} + \frac{(4k^2-3)\omega}{4k^2(1+k^2)^2}|A_1^{(1)}|^2 A_1^{(1)} - \frac{3\omega}{2(1+k^2)^2}A_{1\xi\xi}^{(1)} =$$

$$= \frac{\omega^2}{2k^2}(\omega A_0^{(2)} + 2kV_0^{(2)})A_1^{(1)} \,. \tag{4.12}$$

Substituting (4.7c) and (4.7d) into (4.12), we obtain

$$iA_{1\tau}^{(1)} + \frac{\omega^3(12k^6+35k^4+39k^2+9)}{4k^4(k^4+3k^2+3)}|A_1^{(1)}|^2 A_1^{(1)} - \frac{3\omega}{2(1+k^2)^2}A_{1\xi\xi}^{(1)} = \hat{C}A_1^{(1)} \,, \tag{4.13}$$

where $\hat{C}$ is a new constant defined by

$$\hat{C} = \frac{\omega^2}{2k^2}(\omega C_1 + 2kC_2) \,. \tag{4.14}$$

C_1 and C_2 may be calculated from the boundary conditions.

5. THE KORTEWEG–DE VRIES EQUATION DERIVED VIA A DERIVATIVE EXPANSION METHOD

Our main purpose now is to apply a derivative expansion method in order to obtain the Korteweg–de Vries equation which describes the propagation of small amplitude and long waves. We introduce the multiple spatial and temporal scales

$$t_n = \varepsilon^n t \,, \qquad x_n = \varepsilon^n x \,, \qquad n = 1, 2, \dots \,. \tag{5.1}$$

The dependent variables V, A are expanded around the undisturbed uniform state by writing

$$V = \sum_{n=1}^{\infty} \delta^n V_n \,, \qquad A = 1 + \sum_{n=1}^{\infty} \delta^n A_n \,. \tag{5.2}$$

The Korteweg–de Vries equation is derived on the assumption that

$$\delta = \varepsilon^2 \,. \tag{5.3}$$

Substituting (5.1) and (5.2) into the fundamental set of equations (3.2), (3.3), we get a sequence of equations by equating the coefficients of like powers of ε. The first three sets of equations may be written in the followig form

ε^2 :

$$A_{1t_1} + V_{1x_1} = 0 \,, \tag{5.4a}$$

$$V_{1t_1} + A_{1x_1} = 0 \,, \tag{5.4b}$$

ε^3 :

$$A_{1t_2} + V_{1x_2} = 0 \,, \tag{5.5a}$$

$$V_{1t_2} + A_{1x_2} = 0 \,, \tag{5.5b}$$

ε^4 :

$$A_{2t_1} + A_{1t_3} + V_{2x_1} + (A_1 V_1)_{x_1} + V_{1x_3} = 0 \,, \tag{5.6a}$$

$$V_{2t_1} + V_{1t_3} + V_1 V_{1x_1} + A_{2x_1} + A_{1x_3} + A_{1x_1} t_1^2 = 0 \,. \tag{5.6b}$$

From equations (5.4) and (5.5), we find

$$V_1 = V_1(\xi_1 = x_1 - t_1) = A_1(\xi_1) \,, \tag{5.7}$$

$$V_1 = V_1(\xi_2 = x_2 - t_2) \,. \tag{5.8}$$

The fourth equations (5.6) lead to the following equation

$$\begin{gathered} V_{2x_1^2} - V_{2t_1^2} + A_{1t_3x_1} + (A_1 V_{1\xi_1})_{\xi_1} + (V_1 A_{1\xi_1})_{\xi_1} + V_{1\xi_1 x_3} + \\ V_{1\xi_1 t_3} + (V_1 V_{1\xi_1})_{\xi_1} + A_{1\xi_3\xi_1} + A_{1\xi_1^4} = 0 \end{gathered} \tag{5.9}$$

The second order terms may be removed if we assume that V_2 depends on x_1 and t_1 through ξ_1. Thus we have

$$2A_{1t_3} + 2A_{1x_3} + 3A_1 A_{1\xi_1} + A_{1\xi_1\xi_1\xi_1} = 0 \,. \tag{5.10}$$

Transforming to the coordinate system moving with the phase velocity equals to 1, i.e.,

$$\xi_3 = x_3 - t_3 \,, \qquad \tau = t_3 \,, \tag{5.11}$$

we obtain from (5.10) the Korteweg–de Vries equation

$$A_{1\tau} + \frac{3}{2} A_1 A_{1\xi_1} + \frac{1}{2} A_{1\xi_1\xi_1\xi_1} = 0 \,. \tag{5.12}$$

6. THE KORTEWEG–DE VRIES–BURGERS EQUATION

Our aim here is to obtain an approximate single model equation which describes the behaviour of small amplitude and long waves on assumption $\delta = \varepsilon^2$. For this purpose we apply a nonlinear perturbation Taniuti–Wei's method the idea of which is to introduce the following coordinate stretching

$$\xi = \sqrt{\varepsilon}\,(x - V_0 t)\,, \qquad \tau = \varepsilon\sqrt{\varepsilon}\,t\,. \tag{6.1}$$

Since we consider weakly nonlinear waves, we expand dependent variables around the undisturbed uniform state as power series in terms of the same parameter ε:

$$\begin{aligned} p &= q + \varepsilon p_1 + \varepsilon^2 p_2 + \ldots\,, \\ A &= A_0 + \varepsilon A_1 + \varepsilon^2 A_2 + \ldots\,, \\ V &= \varepsilon V_1 + \varepsilon^2 V_2 + \ldots\,, \end{aligned} \tag{6.2}$$

where $A_0 = \pi a^2$.

Here we make an assumption that viscosity coefficient is small and proportional to the parameter $\sqrt{\varepsilon}$

$$\mu = \sqrt{\varepsilon}\,\bar{\mu}\,. \tag{6.3}$$

For this case, thus we consider a balance between the nonlinearity, dispersion and dissipation of energy.

Introducing (6.1) – (6.3) into the fundamental set of equations (2.1) – (2.4) and equating all the coefficients of the various powers of ε, we obtain for the first power of ε:

$$A_1 = \frac{1}{g_1} p_1 = \frac{1}{g_2} V_1 \equiv \phi\,, \tag{6.4}$$

$$V_0^2 = \frac{A_0 S_1}{d(qS_2 + A_0 S_1)}\,, \tag{6.5}$$

where

$$S_1 \equiv \frac{E}{a^2 \rho_m}\,, \quad S_2 \equiv \frac{2\pi a}{\rho_m h}\,, \quad g_1 = \frac{S_1}{S_2}\,, \quad g_2 = \frac{(qS_2 + A_0 S_1)V_0}{qA_0 S_2}\,. \tag{6.6}$$

Finally, from ε^2, we obtain the Korteweg–de Vries–Burgers equation for ϕ:

$$\phi_\tau + \beta\phi\phi_\xi + M\phi_{\xi\xi} + \alpha\phi_{\xi\xi\xi} = 0\,, \tag{6.7}$$

where the nonlinear β, the dissipative M and the dispersive α coefficients are defined as follows:

$$\beta \equiv \frac{(A_0 g_2 + 2V_0)(qg_2 - V_0 g_1)}{qg_2 A_0 + (q + g_1 A_0)V_0}\,, \tag{6.8}$$

$$M \equiv \frac{g_2 \bar{\mu} A_0}{d[(q + g_1 A_0)V_0 + qg_2 A_0]}\,, \tag{6.9}$$

$$\alpha \equiv \frac{qV_0^4}{S_1[(q + g_1 A_0)V_0 + qg_2 A_0]}\,. \tag{6.10}$$

7. SUMMARY

Basing on the derivative expansion and the reductive Taniuti–Wei's methods, it was shown that the fundamental set of equations is reduced to the nonlinear Schrödinger equation or the Korteweg–de Vries and the Korteweg–de Vries – Burgers equations depending on if the system is strongly or weakly dispersive. The Korteweg–de Vries–Burgers equation is derived on the assumption that viscosity coefficient is small and proportional to the small parameter ε.

In a surface water waves context the nonlinear Schrödinger equation has been derived and studied in some details [16]. It is well known that this equation can be solved with the help of the inverse scattering transform [9], possess Painlevé test and Bäcklund transformation [17] to obtain an N–envelope soliton solution and periodic envelope one [18].

It is worth noticing that the two coefficients by the second and third terms in equations (3.28) and (4.13) are responsible for modulational stability of waves [16]. If their product is negative nonlinear wave solutions are modulationally stable. In our case it occurs for arbitrary value of k. So, waves in tubes are modulationally stable.

The Korteweg–de Vries–Burgers equation and its solutions has been discussed in some detail, see e.g. [19] and [20]. Also the Korteweg–de Vries equation has been modified to include both the dissipative and dispersive effects of viscous boundary layers [21]. In the frame of the two–dimensional theory of long gravity waves a model equation was derived which combines the Kadomtsev–Petviasvili with Burgers equation [22].

The Korteweg–de Vries equation (5.12) agrees in form with the same equation obtained via other methods [9, 10].

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STRESZCZENIE

W pracy zastosowano teorię fal nieliniowych, opartą na metodzie redukcji Taniuti–Wei i metodzie wielu skal dla wyprowadzenia nieliniowego równania Schrödingera, równania Kortewega-de Vriesa i równania Kortewega–de Vriesa–Burgersa. Równania opisują propagację fal w płynie wypełniającym cylinder o cienkich i elastycznych pierścieniach.

РЕЗЮМЕ

В работе применено теорию нелинейных волн, основанную на методах редукции Танюти-Веи и методе многих параметров, для получения нелинейного уравнения Шрёдингера, уравнения Кортевега–Де Фриса и уравнения Кортевега–Де Фриса-Бургерса. Эти уравнения описывают распространение волн в жидкости, заполняющей цилиндр с тонкими и упругими кольцами.

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