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Can We Get a Confinement in QCD from Higher Dimensions?

Jak otrzymać dielektryczny model confinementu z wyższych wymiarów?

Можно ли получить конфайнмент в хромодинамике при высших размерах?

Dedicated to Professor
Stanislaw Szpikowski on occasion
of his 60th birthday

1. INTRODUCTION

There has been a recent renaissance of the old idea by T. Kaluza [1] and O. Klein [2] i.e., the geometrical unification of gravity and other fundamental interactions using many dimensional manifold (5-dimensional in the original work by T. Kaluza). This idea consists of a unification of two major principles in Physics (local gauge invariance and local coordinate invariance) and reducing both principles to the second in a more than 4-dimensional world. The additional dimensions cannot be directly observed. In our approach, we propose some development of these ideas using nonriemannian geometry from Einstein's Unified Field Theory [3] (the so-called Einstein-Kaufman theory) 4, 5, 6 . In the old Kaluza-Klein approach there were not any "interference effects" between gravity and electromagnetism. This theory reproduces the Einstein and Maxwell equations in an already known form. In the nonabelian Kaluza-Klein theory [7, 8] (which unifies local nonabelian gauge invariance and local coordinate invariance principles) we face a fundamental problem with the value of the cosmological constant. The cosmological constant predicted by this theory is 10^{127} times bigger than upper limit from observational data. This forces us to abandon Riemannian geometry (the Levi-Civita connection) and to use some nonriemannian geometries defined on a multidimensional bundle manifold [9, 10] (a gauge bundle). Our approach consists of finding such a Kaluza-Klein theory, finding "interference effects" between gravity and gauge fields and their physical consequences.

Now we know that the local gauge invariance principle plays the fundamental role in elementary particle physics. The Weinberg-Salam-Glashow model of electroweak interactions and QCD (quantum chromodynamics) are constructed on the assumption of this invariance. The recent discovery of $W^{\pm}$ and Z^{0} bosons and some successes of QCD in describing jets of hadrons in high energy physics support this statement.

Thus investigations in the mathematical structure of a Kaluza-Klein theory can give a new light on a problem of unification of fundamental interactions, can explain some old puzzles of nuclear physics and predict new physical phenomena.

The most interesting problem which arises here occurs when this new Kaluza-Klein theory is considered as a realistic model of strong interactions. Thus we consider this theory as the source of the classical-dielectric model of confinement, supposing that the gauge group $G = SU(3)_c$ and adding spinor sources (quark fields). In this way a confinement idea emerges from the physics in higher dimensions with a geometrical interpretation. The lagrangian of the nonsymmetric Kaluza-Klein (Jordan-Thiry) theory, in the flat space limit, resembles the soliton bag model lagrangian.

2. THE NONSYMMETRIC KALUZA-KLEIN (JORDAN-THIRY) THEORY

In the last few years the nonsymmetric Kaluza-Klein 11, 12 theory has been constructed together with its extension to the nonsymmetric Jordan-Thiry theory (an additional scalar field Ψ). connected to the effective"gravitational constant", see classical works [13, 14, 15]. Nonabelian extensions of the nonsymmetric Kaluza-Klein and Jordan-Thiry theories have been found [16, 17].

It was possible to extend the nonsymmetric Kaluza-Klein theory to the case with material sources (including spin sources) and to include such phenomena as spontaneous symmetry breaking and Higgs' mechanism [18, 19] (with two critical points for a Higgs' potential). Thus we can consider the "interference effects" between electro-weak interactions (described by the geometrical version of Weinberg-Salam-Glashow model) and gravity. Simultaneously, this allows us to build a more realistic model of Grand Unification including gravitational field.

The linear version of the nonsymmetric Kaluza-Klein and Jordan-Thiry theories have been found [20].

The first exact solution in the 5-dimensional (electromagnetic) case has been obtained [21]. It was possible to find an extension of some earlier works to the case of the nonsymmetric Kaluza-Klein theory [22, 23, 24, 25, 26, 27] i.e. an introducing of fermion sources leading to the small "interference effects" (a dipole electric moment of fermion and PC breaking).

This will be very helpful to find a model of strong interactions, i.e. an extended QCD with "interference effects" between gravity and strong interactions.

The nonsymmetric Kaluza-Klein and Jordan-Thiry theories have a well defined geometry on a multidimensional manifold (5-dimensional in the electromagnetic case and $(n+4)$-dimensional in the nonabelian case, $n = \dim G$, where G is a gauge symmetry group). The geometry in this theory is a geometry from Einstein's Unified Field Theory [3, 5, 6] in the Kaufman version [4] This version is known as the Einstein-Kaufman theory. In some sense this geometry is a multidimensional extension of the Einstein-Kaufman geometry. This geometry is defined on the gauge manifold (manifold of a principal fibre bundle) and one calls it the Einstein geometry. The nonsymmetric Kaluza-Klein (or Jordan-Thiry) theory is a generalization of the Kaluza-Klein (or Jordan-Thiry) theory and Einstein Unified Field Theory.

These theories realize a true unification of gravitational and gauge fields in the following sense: they not only unify a local gauge invariance principle and a local coordinate invariance principle, but they provide "interference effects" between gravitational and gauge fields (electromagnetic field in the 5-dimensional case) as well. There are the following "interference effects":

1) The additional term in the lagrangian for the electromagnetic field equals $2(g^{[\mu\nu]} F_{\mu\nu})^2$ (or gauge field equals $2\cdot l_{ab}(g^{[\mu\nu]} H^a_{\mu\nu})(g^{[\alpha\beta]} H^b_{\alpha\beta})$), where $F_{\mu\nu}$ is the strength of the electromagnetic field and $H^a_{\mu\nu}$ is the strength of Yang-Mills' field.
2) The new energy-momentum tensor for an electromagnetic field (gauge field).
3) The existence of two field strength tensors for the electromagnetic (gauge) field, i.e., $F_{\mu\nu}$, $H_{\mu\nu}$ ($H^a_{\mu\nu}$, $L^a_{\mu\nu}$).
4) The source in the second pair of Maxwell's (Yang-Mills') equations, i.e., a current j_μ (j^a_μ).
5) The polarization of vacuum $M_{\mu\nu} = -\frac{1}{4\pi}(H_{\mu\nu} - F_{\mu\nu})$, ($M^a_{\mu\nu} = -\frac{1}{4\pi}(L^a_{\mu\nu} - H^a_{\mu\nu})$) with an interpretation as the torsion in the 5th dimension (in higher dimensions in Yang-Mills' case).
6) The additional term in the equation of motion for a test particle (additional term for a Lorentz-force term in the electromagnetic case) as appears in the modified Kerner-Wong equation.
7) The cosmological constant depending on a dimensionless constant μ with an asymptotic behaviour

$$\varrho(\mu) \sim \frac{\text{const}}{\mu^2} \quad \text{(for large } \mu\text{)}$$

This constant in general is a rational function of μ, i.e.

$$\varrho(\mu) = \frac{P_m(\mu)}{Q_{m+2}(\mu)}$$

Thus we can avoid some problems with the enormous cosmological constant which appears in the classical approach, when μ is chosen as a root of the polynomial P_m, or becomes sufficiently large [16, 17]. The constant μ is simultaneously a coupling constant between a skewon field $g_{[\mu\nu]}$ and a Yang-Mills' field in the linear approximation [20].

In the case of the nonsymmetric Jordan-Thiry theory we get some additional effects:

1) The lagrangian for a scalar field Ψ.
2) The energy-momentum tensor for the scalar field Ψ.
3) Additional scalar forces in the equation of motion for a test particle (generalized Kerner-Wong equation).

The scalar field Ψ is connected to the effective "gravitational constant" by:

$$G_{eff} = G_N\, e^{-(n+2)\Psi}$$

where G_N is a Newton constant and $n = \dim G$, G is a gauge symmetry group. This field seems to be massive, with short range behaviour (Yukawa-like behaviour) [12, 17]. In this way, there are no problems with the weak equivalence principle.

Let us give some details of the nonsymmetric Kaluza-Klein (Jordan-Thiry) theory [11, 15, 16, 17].

The lagrangian in the nonsymmetric Jordan-Thiry (Kaluza-Klein) theory is a scalar curvature for the connection defined on the (n+4)-dimensional manifold (bundle manifold) with Einstein geometry. It has the following form:

$$\sqrt{-g}\, R(W) = \sqrt{-g}\Big\{\overline{R}(\overline{W}) + e^{(n+2)\Psi}\widetilde{R}(\widetilde{\Gamma}) + 8\pi\, e^{-(n+2)\Psi} L_{YM} + {}$$
$$+ L_{scal}(\Psi)\Big\} + \partial_\mu K^\mu \qquad (2.1)$$

where $R(W)$ is the Moffat-Ricci curvature scalar for the connection $W^A{}_B$ (on (n+4)-dimensional manifold), $\widetilde{R}(\widetilde{\Gamma})$ is the Moffat Ricci curvature scalar for the connection $\widetilde{\omega}^a{}_b$ on the group G (gauge symmetry group)

$$L_{YM} = -\frac{1}{8\pi}\, l_{ab}\,(2H^a\cdot H^b - L^{a\mu\nu} H^b{}_{\mu\nu}) \qquad (2.2)$$

$$H^{a} = g^{[\mu\nu]} H^{a}_{\mu\nu} \tag{2.3}$$

$$L^{a\mu\nu} = g^{\alpha\mu} g^{\beta\nu} L^{a}_{\alpha\beta} \tag{2.4}$$

$$L^{d}_{\gamma\beta} = -L^{d}_{\beta\gamma} \tag{2.5}$$

$$l_{dc}\, g_{\mu\beta}\, g^{\gamma\mu} L^{d}_{\gamma\alpha} + l_{cd}\, g_{\alpha\mu}\, g^{\mu\gamma} L^{d}_{\beta\gamma} = 2l_{cd}\, g_{\alpha\mu}\, g^{\mu\gamma} H^{d}_{\beta\gamma} \tag{2.6}$$

$R(W)$ is the lagrangian in the nonsymmetric Jordan-Thiry theory, L_{YM} plays the role of the lagrangian for the Yang-Mills' field. $\tilde{R}(\tilde{\Gamma})$ plays the role of the cosmological constant and $\bar{R}(\bar{\Gamma})$ is the lagrangian of the gravitational field in the nonsymmetric theory of gravitation. $L_{scal}(\Psi)$ plays the role of the lagrangian for the scalar field Ψ.

$L^{a}_{\mu\nu}$ plays the role of the second tensor of the Yang-Mills' (gauge) field strength.

Equation (2.6) expresses the relationship between both tensors $H^{a}_{\mu\nu}$ and $L^{a}_{\mu\nu}$. This relationship is linear with respect to $H^{a}_{\mu\nu}$ and $L^{a}_{\mu\nu}$ and nonlinear with respect to $g_{\mu\nu}$.

$$L_{scal}(\Psi) = (m\tilde{g}^{(\gamma\nu)} + ng^{[\mu\nu]}\, g_{\delta\mu}\, \tilde{g}^{(\delta\gamma)})\Psi_{,\nu}\Psi_{,\gamma} \tag{2.7}$$

where

$$m = l^{[dc]} l_{[dc]} - n(n-1) \tag{2.8}$$

$$l_{ab} = h_{ab} + \mu K_{ab} = C^{c}_{ad}\, C^{d}_{bc} + \mu\, C^{c}_{ab}\, \mathrm{Tr}\left[(X_a)^2\right] \tag{2.9}$$

Also, μ is a dimensionless constant, C^{c}_{ab} are structure constant of the Lie algebra of the group G, X_a are generator of this algebra. Tr is understood here in the sense of the representation of an enveloping algebra of the Lie algebra of the gauge symmetry group G.

The field Ψ is related to the effective gravitational "constant" which now is a function of space-time. In the electromagnetic case $G = U(1)$ we have similarly [10, 16]

$$\sqrt{-g}\, R(\overline{W}) =$$
$$= \sqrt{-g}\left\{ \overline{R}(\overline{W}) + e^{-3\Psi} \left((2g^{[\mu\nu]} F_{\mu\nu})^2 - H^{\mu\nu} F_{\mu\nu} \right) + L_{scal}(\Psi) \right\} + \partial_\mu X^\mu \tag{2.10}$$

where

$$L_{scal}(\Psi) = g^{[\nu\mu]} g_{\delta\nu}\, \tilde{g}^{(\alpha\delta)}\, \Psi_{,\mu} \cdot \Psi_{,\alpha} \tag{2.11}$$

is a lagrangian for the scalar field Ψ.

$$H^{\mu\nu} = g^{\alpha\mu} g^{\beta\nu} \cdot H_{\alpha\beta} \tag{2.12}$$

$$g_{\delta\beta}\, g^{\gamma\delta} H_{\gamma\alpha} + g_{\alpha\delta}\, g^{\delta\gamma} H_{\beta\gamma} = 2g_{\alpha\delta}\, g^{\delta\gamma} F_{\beta\gamma} \tag{2.13}$$

$$H_{\beta\gamma} = -H_{\gamma\beta} \tag{2.14}$$

$H^a_{\mu\nu}$ and $F_{\mu\nu}$ are respectively the strength of Yang-Mills' and the electromagnetic field. $H_{\mu\nu}$ is a second tensor of the electromagnetic field strength. The field Ψ is related to the effective gravitational "constant" via:

$$G_{eff} = G_N\, e^{-3\Psi} \tag{2.15}$$

$g_{\mu\nu}$ is a nonsymmetric, real tensor defined on a space time such that

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]} \tag{2.16}$$

$$g_{\alpha\beta}\, g^{\gamma\beta} = g_{\beta\alpha}\, g^{\beta\gamma} = \delta^\gamma_\alpha \tag{2.17}$$

where the order of indices is important.

This tensor satisfies usual compatibility conditions and induces on a space-time a non-symmetric connection from the Einstein-Kaufman theory [4]. The tensor l_{ab} satisfies similar comptiability conditions and induces similar connection on the group G [16, 17]. For $\tilde{g}^{(\alpha\gamma)}$ we have

$$g_{(\alpha\gamma)}\, \tilde{g}^{(\alpha\beta)} = \delta^\beta_\gamma \tag{2.18}$$

If we put $\Psi = 0$ we get the nonsymmetric Kaluza-Klein theory. From the Palatini variational principle for (2.1) we get field equations (see [11 , 15 , 16 , 17] for details). Thus we get the theory which unifies gravity, gauge fields and scalar forces. The gravitational field in this theory is described by a non-symmetric, real tensor $g_{\mu\nu}$ (and a scalar field Ψ), which connets it with Moffat's theory of ravitation (one of the most important alternative theory of gravitation - see [28], for a review). The nonsymmetric Kaluza-Klein (Jordan-Thiry) theory has been previously designed as a unification of Moffat's theory of gravitation and electromagnetic (or Yang-Mills') field. However, there are some differences.

First of all Moffat and his co-workers are extensively using the Einstein-Strauss theory [6] in a hypercomplex-hermitian version [29], but not the Einstein-Kaufman. The Einstein-Strauss theory cannot be extended in any simple way to higher dimensions, even in the 5-dimensional (electromagnetic) case. It is a hard task also to incorporate spin sources in Einstein-Strauss theory. In both cases, we meet a fundamental physical problem. The lagrangian becomes hypercomplex (not real). In our case we do not have these problems because everything is real. In the case of the nonsymmetric Jordan-Thiry theory, we effectively get the scalar-tensor theory of gravitation in the nonsymmetric version. The scalar field behaves very well in the linear approximation. It has been proved that we could avoid tachyons and ghosts in the particle spectrum of the theory (if we put $m > 0$). In the case of classical Jordan-Thiry theory, the scalar field is a ghost (a negative kinetic energy). This new version of the Kaluza-Klein theory is capable of removing singularities from the solution of coupled gravitational and Yang-Mills' equations even in the case of spherical symmetry. Such solution has been found in the electromagnetic case [21]. It is well known that in the case of Einstein-Maxwell equations we cannot get any non-singular, localizable, stationary solutions (the so-called Hilbert-Levi-Civita-Thiry-Einstein-Lichnerowicz-Pauli - theorem, see [30 ,31 ,32 ,33]). This result has been recently extended to the case of nonabelian gauge fields [34].

Recently, R.B. Mann (see R.B. Mann, "Exact solutions of an algebraically extended Kaluza-Klein theory", Harvard University preprint HUTP-83/A055, Cambridge, January 1984) found eight classes of spherically symmetric and stationary solutions in the nonsymmetric Kaluza-Klein theory. These solutions are more general than this from [21]and some of them have no singularities in gravitational and electromagnetic fields. Some of these solutions possess a nonzero magnetic field and nonzero $g_{[23]} = f \neq 0$. The nonsingular solutions are parametrized by: fermion charge l^2, electric charge Q and a new constant u_0. This constant is related to $g_{[23]}$ similarly as l^2 to $g_{[14]}$. It plays a similar role for $g_{[\mu\nu]}$ as a magnetic charge for $F_{\mu\nu}$. We recall that the first exact solution found in 21) has no singularity in an electric field and a finite energy. However, it has a weak singularity in $g_{(\alpha\beta)}$. In this case we put $g_{[23]} = 0$. It seems that we can extend these solutions without any problems to the nonabelian case.

Thus we can look for models of elementary particles as exact solutions of field equations.

In the theory there are two field strength for the electromagnetic (Yang-Mills') field - $F_{\mu\nu}$, $H_{\mu\nu}$ ($H^a_{\mu\nu}$, $L^a_{\mu\nu}$). The first is built from $(\vec{E}, \vec{B})$, $((\vec{E}^a, \vec{B}^a))$ the second from $(\vec{D}, \vec{H})$ $((\vec{D}^a, \vec{H}^a))$. The relations between both tensors are given by Eqs. (2.6) and (2.13).

According to modern ideas [35,36, 37] the confinement of color could be connected to dielectricity of the vacuum (dielectric model of confinement). Due to the so-called antiscreening mechanism, the effective dielectric constant is equal to zero. This means that the energy of an isolated charge goes to infinity. Now there are also so-called classical-dielectric model of confinement (see Lehman, H., and Wu Tsai Tsu, "Classical models of confinement", Preprint DESY83-086, September 1983). The confinement is induced by a special kind of dielectricity of the vacuum such that $\vec{E} \neq 0$ and $\vec{D} = 0$ ($\vec{E}^a \neq 0$, $\vec{D}^a = 0$). In this case we do not have a distribution of charge. This is similar to the electric-type of Meissner effect.

It is easy to see that in our case (the nonsymmetric Kaluza-Klein theory) the dielectricity is induced by the nonsymmetric tensors $g_{\mu\nu}$ and l_{ab}. If $g_{[\mu\nu]} = 0$, $\vec{D} = \vec{E}$ and $\vec{B} = \vec{H}$.

The gravitational field described by the nonsymmetric tensor $g_{\mu\nu}$ behaves as a medium for an electromagnetic field (gauge field). The condition $\vec{E} \neq 0$, $\vec{D} = 0$ $(\vec{E}^a \neq 0, \vec{D}^a = 0)$ can be satisfied in the axial, stationary case for $F_{\mu\nu}$, $H_{\mu\nu}$, $(H^a_{\mu\nu}, L^a_{\mu\nu})$, $g_{\mu\nu}$. Thus it is interesting to find an exact solution with axial symmetry for the nonsymmetric Kaluza-Klein theory with fermion sources for $G = SU(3)$. This could offer us a model of a hadron.

The axially symmetric, stationary case seems to be very interesting from more general point of view. We have in General Relativity very peculiar properties of stationary, axially-symmetric solutions of the Einstein-Maxwell equations. These solutions describe the gravitational and electromagnetic fields of a rotating charged mass. Thus we get the magnetic field component. Asymptotically (these solutions are asymptotically flat) the magnetic field behaves as a dipole field. We can calculate the gyromagnetic ratio at infinity, i.e., the ratio of the magnetic dipole moment and the angular momentum moment. It is worth noticing that we get the anomalous gyromagnetic ratio 38 i.e., the gyromagnetic ratio for an electron (for a charged Dirac particle). We cannot interpret the Kerr-Newman solution as a model of the fermion for we have a singularity. In the nonsymmetric Kaluza-Klein theory we can expect completely nonsingular solutions. We can also expect the asymptotic behaviour of the Einstein-Maxwell theory. Thus it seems that we probably will get the solutions with anomalous gyromagnetic ratio. Such a solution could be treated as a model (classical) of 1/2 spin particle. In the nonabelian case $(G = SU(3)_c \times U(1)_{em})$ this could offer us a model of a charged barion (i.e., proton), where the skewon field $g_{[\mu\nu]}$ induces a confinement of color. In this way, the skewon field $g_{[\mu\nu]}$ plays a double role:

1) additional gravitational interactions (from Moffat's theory of gravitation),
2) a strong interaction field connected to the confinement problem.

It has been proved by R.B. Mann and J.W. Moffat [39,40] that the skewon field $g_{[\mu\nu]}$ has zero spin. In a linear approximation it is the so-called generalized Maxwell field (an abelian gauge field). Thus it is natural to expect an exchange of some spin zero particles in the nuclear-nucleon potential for low and intermediate energies. We do not observe such particles. However, we cannot fit experimental data for nucleon-nucleon interaction without the mysterious σ-(spin zero) particles [41].

It happens that we need two such particles to fit the data. In our proposal, they are connected to the skewon field $g_{[\mu\nu]}$ and to the scalar field Ψ from the nonsymmetric Jordan-Thiry theory. The reason we do not detect such particles directly seems to be clear now. They are confined, because they are actually a cause of confinement. The scalar field from the nonsymmetric Jordan-Thiry theory induces an additional dielectricity of the vacuum (see lagrangians for scalar field Ψ and for Yang-Mills' field in Eqs. (2.1),(2.2), (2.7). Let us notice that a function of the scalar field Ψ appears as a factor before the Yang-Mills' lagrangian in Eq. (2.1). This has some important consequences: the effective gravitational "constant" depends on Ψ and in the flat space limit $g_{\mu\nu} = \eta_{\mu\nu}$ the lagrangian resembles bosonic part of the soliton bag model lagrangian if we put

$$e^{-10\Psi} = 2\left(1 - \frac{\sigma}{\sigma_0}\right); \quad \sigma_0 = \text{const.} \tag{2.19}$$

for $n = 8$, $G = SU(3)$ (see [42]).
One finds

$$\Psi = -\frac{1}{10} \ln\left(1 - \frac{\sigma}{\sigma_0}\right) - \frac{\ln 2}{10} \tag{2.20}$$

and in the flat space limit one easily gets

$$\begin{aligned} L = &-\frac{1}{4}\left(1 - \frac{\sigma}{\sigma_0}\right)\left(h_{ab} + \mu^2 K^c_b K_{ca}\right)H^a_{\mu\nu} H^{b\mu\nu} + \\ &+ \frac{\sigma_0 \varrho(\mu)}{16\pi(\sigma_0 - \sigma)} + \frac{m\sigma_0^2}{100(\sigma_0 - \sigma)^4}\eta^{\mu\nu}\sigma_{,\mu}\sigma_{,\nu} \end{aligned} \tag{2.21}$$

The full lagrangian (2.1) is more general and it contains a gravitational field.

Friedberg and Lee (see [43]) consider the soliton bag model with a more general factor $K(\sigma)$.

$$L = -\frac{1}{4} K(\sigma) h_{ab} H^{a\mu\nu} H^{b}_{\mu\nu} - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) \tag{2.22}$$

They consider that the scalar field σ is a new dynamical field with self-interaction given by $U(\sigma)$. The quantity K is a dielectric constant which depends on σ. It is interesting to observe many similarities between (2.22) and our lagrangian from the nonsymmetric Jordan-Thiry theory i.e. (2.1). Thus in our model we have in the flat space limit an effective dielectric constant.

$$K_{eff} = 4\, e^{-10\Psi} \tag{2.23}$$

It is interesting to notice that the scalar field Ψ enters into the effective gravitational "constant and into the effective dielectric "constant" in the flat space limit.

We recall that in a full nonsymmetric Jordan-Thiry theory (curved nonriemannian space-time) we have the following symmetry for the scalar field [12, 17]

$$\Psi \longrightarrow \Psi' = f(\Psi) \tag{2.24}$$

where f is an arbitrary function. In this way the formulae (2.19) and (2.23) can be treated as transformations for a scalar field in the nonsymmetric Jordan-Thiry theory. Thus we can connect a bosonic part of some soliton bag model lagrangians via Eq. (2.24) in the nonsymmetric Jordan-Thiry theory. In this way we see some possibilities of connecting gravitational and strong interactions via the nonsymmetric Kaluza-Klein (Jordan-Thiry) theory. This is a little in the spirit of an idea of strong gravity [44]. In this approach, there are two metric (symmetric) tensors. It is easy to see that in the nonsymmetric Kaluza-Klein (Jordan-Thiry) theory there are two metric (symmetric) tensors $g_{(\alpha\beta)}$ and $f_{\alpha\beta}$ such that

$$f_{\alpha\beta}\, g^{(\alpha\gamma)} = \delta^{\gamma}_{\beta} \; ; \; g_{\alpha\beta}\, g^{\alpha\gamma} = g_{\beta\alpha}\, g^{\gamma\alpha} = \delta^{\gamma}_{\beta} \qquad (2.25)$$

and it is easy to see that if $g_{[\alpha\beta]} = 0$, then $f_{\alpha\beta} = g_{(\alpha\beta)}$.

Thus we propose the lagrangian of the nonsymmetric Jordan--Thiry theory as the bosonic part of the lagrangian of strong interactions. Why? It seems that something is missing in the QCD lagrangian. We have the following objectives:

1) σ -particle (which we mentioned before),
2) an exact solution with color radiation (this means color at infinity - no confinement) found by J. Tafel and A. Trautman [45].

Thus it seems that the QCD lagrangian is incomplete in the bosonic part. In our proposal, we replace the QCD lagrangian by the lagrangian from nonsymmetric-nonabelian Jordan-Thiry theory for $G = SU(3)_c$. In this way we can get a dielectric model of confinement and a soliton bag model-like lagrangian [42, 43] (see also DeTar, C.E., Donoghue, "Bag models of hadrons", UUHEP83/3, UMHEP-117 - preprint 1983).

Thus we propose the following program of investigation:

1) Find exact solutions of the nonsymmetric Kaluza-Klein and Jordan-Thiry theory in abelian and nonabelian cases with and without fermion sources in the case of spherical and axial symmetry, using inverse scattering, and Riemann invariants methods.
2) To find an effective interaction of two axially symmetric solutions, exactly, or, using some numerical methods in the case of $G = SU(3)_c$, with fermion sources. This could be similar to the nucleon-nucleon interaction in the Skyrme model. The solution should be treated as particles using a collective coordinate method.
3) To find wave-like solutions of the field equations in the abelian and nonabelian cases. This could, in the electromagnetic case, offer a solution which could be treated as a kind of electromagneto-gravitational wave (nonlinear wave) with nontrivial interactions between all fields. The objec-

tive of this hope is related to points [4] and [5] in the list of "interference effects" (we recall that the displacement current in the classical Maxwell equations leads us to the nontrivial interaction between the electric and magnetic field - the reason d'être of the wave solutions for Maxwell equations. However, this is only a historical remark). By a nontrivial interaction, we mean that the flow of energy is possible from one field to the second in a quasiperiodic way.

There are also some proposals concerning cosmology:

1) To find a cosmological solution of I Bianchi-type in the nonsymmetric Kaluza-Klein theory with material sources [18]. We expect completely nonsingular solutions in the presence of an electromagnetic field.

2) To find a new (or old) inflationary scenario for the Universe from the nonsymmetric-nonabelian Kaluza-Klein theory. In [19] has been proved that from the nonsymmetric Kaluza-Klein theory we could get a Higgs' potential with two critical points. This offers phase transitions in early cosmology and could give Guth's new (or old) inflationary scenario without the Coleman-Weinberg theory.

It is also interesting to do some research under the formal structure of the nonsymmetric Kaluza-Klein and Jordan-Thiry theories. They are:

1) A rigorous treatment of the nonsymmetric tensor $l_{ab} = h_{ab} + uK_{ab}$ defined on the algebra of matrices (enveloping algebra of the Lie algebra of the gauge symmetry group).
2) An extension of the nonsymmetric Kaluza-Klein and Jordan-Thiry theories including supergravity and supersymmetry (some ideas how to do it can be found in [46], [48]).
3) Studies under a spontaneous compactification of an n-dimensional submanifold of an (n + 4)-dimensional manifold with Einstein geometry (a global or/and local compactification).

CONCLUSIONS

In this paper we propose the lagrangian of the nonsymmetric-nonabelian Jordan-Thiry theory as the bosonic part of the lagrangian of strong interactions. In this way the QCD lagrangian would be extended, including the skewon field $g_{[\mu\nu]}$ and the scalar field Ψ. Both fields $g_{[\mu\nu]}$ and Ψ play double roles: 1) as a part of gravitational interactions, 2) as a part of a strong interaction field. The existence of $g_{[\mu\nu]}$ and Ψ could explain (in principle) the σ-particles in a nucleon-nucleon potential and a confinement of color via the classical-dielectric model of confinement. It is possible on the level of the nonsymmetric Jordan-Thiry theory to connect some soliton bag models, via transformation of the scalar field Ψ.

We propose a program of research which consists in finding exact solutions in this theory. These solutions could be treated as models of particles (generalized Skrymions [47,48]). Our approach seems to be more realistic, because, we include to the lagrangian gauge and gravitational fields. In the Skryme model we have to deal with an effective model of strong interactions. This model, despite many spectacular successes, has some problems. For example, a mass difference between nucleon and Δ^{++} particle. Moreover, the interactions between two skrymions can give a qualitatively good description of a nucleon-nucleon potential (see Rho, M., "Pion interactions within nuclei", SPhT/CEN Saclay - preprint 1984, p. 54 (from Skrymions to Paris potential)). In this way we could approach some classical nuclear phenomenology (see Thomas, A. W., "Chiral symmetry and the bag model: a new starting point for nuclear physics", TH3368-CERN TRIPP-82-29 - preprint July 1982)).

One could search axially symmetric, stationary solutions in the nonsymmetric Kaluza-Klein (Jordan-Thiry) theory using formalism presented in [49]. Finally, we conclude that some of E. Witten's ideas [50] can be employed for the nonsymmetric Kaluza-Klein (Jordan-Thiry) theory.

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STRESZCZENIE

W niniejszej pracy prezentujemy niesymetryczne teorie Kaluzy-Kleina i Jordana-Thiry jako interesującą propozycję fizyki w wyższych wymiarach. Pokazujemy, jak otrzymać dielektryczny model confinementu z "efektów interferencyjnych" w tych teoriach. Postulujemy, że stare problemy fizyki jądrowej, tzn. σ-cząstki, mogą być związane z polem skośnie-symetrycznym (skewon) $g_{[\mu\nu]}$ i Ψ w niesymetrycznej teorii Jordana-Thiry.

Р Е З Ю М Е

В данной работе приведены несимметрические теории Калюцы-Клейна и Жордана-Тири как интересные предложения физики при высших размерах. Указывается, как получить диэлектрическую модель конфайнмент на основе "интерференционных эффектов" в этих теориях. Полагается, что давние вопросы ядерной физики, т.е. б-частицы, могут быть связаны с косо-симметричным (skewon) полем $\varepsilon_{[\mu\nu]}$ и Υ в несимметричной теории Жордана-Тири.