ANNALES
UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN—POLONIA
VOL. XL/XLI, 9 SECTIO AAA 1985/1986

Instytut Fizyki Teoretycznej
Uniwersytet Warszawski

J. DOBACZEWSKI*

On the SU(6) Dynamic Symmetry in Nuclei

O dynamicznej symetrii Su(6) w jądrach

О динамической SU(6)-симметрии в ядрах

Dedicated to Professor
Stanisław Szpikowski on occasion
of his 60th birthday.

The Interacting Boson Model (IBM) [1] has been recently receiving a large interest in nuclear physics. The microscopic foundations of the model still remain, however, an open question. The basic assumptions of the simplest version of the model can be expressed as follows:

1) The set of quadrupole collective states of a given nucleus can be identified with a basis of the symmetric representation of the SU(6) group.
2) The quadrupole collective Hamiltonian can be approximated by a scalar and time-even operator quadratic in the SU(6) generators.

* Supported in part by the Ministry of Science and High Education under contract no. CPBP01.09.

The first assumption can be called a kinematic one because it defines the Hilbert space of the collective quadrupole states. The second assumption is of a dynamic nature since it formulates an approximation for the nuclear many-body Hamiltonian projected onto the collective space. In the present study I will discuss the first assumption and try to search for conditions under which such a space of collective states can be found in the Fock space for a fermion system.

In the IBM model one usually visualizes the SU(6) symmetric representation by using a set of six boson operators, five of them forming a quadrupole spherical tensor (d^+_μ) and the sixth being a scalar (s^+). However, this visualization is not the main point of the model. In fact it merely constitutes a calculational tool allowing to express the Hamiltonian and the collective states in an explicit way. Abstract symbols for the su(6) generators could have been used as well, allowing for the same group theoretical methods to be used when solving the Schrödinger equation. In this respect a direct search for the SU(6) dynamic symmetry in the fermion space is more fundamental than a microscopic determination of the boson operators.

The interpretation of a boson as an approximated pair of fermions, which has motivated the introduction of the IBM, is used in the model only to fix the SU(6) symmetric representation for a given nucleus. The representation containing N_B-boson states, where N_B is half of the number of valence fermions, is usually used. This prescription is neither absolutely necessary to reproduce the experimental data nor it is uniquely defined, especially for deformed nuclei where the concept of the valence shell has no clear-cut meaning. In what follows I will use the boson operators only as an illustration, focusing attention directly on a determination of the SU(6) representation space.

Let the states of the SU(6) symmetric representation be visualized by:

$$|\Psi_j) = (d^+_\mu)^{n_d}(s^+)^{n_s}|0) \quad , \quad n_d+n_s = N_B \quad , \tag{1}$$

where $|0)$ is the boson vacuum and the index j comprises all necessary labels used to number the states of the representation. All states of the representation can be obtained by a successive application of the operators $d^+_\mu s$ on the state

$$|\Psi_0) = (s^+)^{N_B}|0) \tag{2}$$

(the highest weight state). The representation space is thus defined by the highest weight state, having the property

$$(d^+_\mu s)^+|\Psi_0) = 0 \quad , \tag{3}$$

and by the algebraic relations between the "shift" operators $d^+_\mu s$:

$$[\, d^+_\mu s \, , \, d^+_\nu s \,] = 0 \quad , \tag{4a}$$

$$[\, d^+_\mu s \, , \, (d^+_\nu s)^+ \,] = d^+_\mu d_\nu - \delta_{\mu\nu} s^+ s \quad , \tag{4b}$$

$$[\, d^+_\mu d_\nu - \delta_{\mu\nu} s^+ s \, , \, d^+_\kappa s \,] = \delta_{\nu\kappa} d^+_\mu s + \delta_{\mu\nu} d^+_\kappa s \quad . \tag{4c}$$

If a set of the fermion collective states is to be identified with a symmetric SU(6) representation space one has to find a fermion highest weight state $|\Psi_0\rangle$ (denoted by the angle bracket) and a fermion shift operator $\hat{F}^+_{2\mu}$ fulfilling the conditions:

$$\hat{F}_{2\mu}|\Psi_0\rangle = 0 \quad , \tag{5}$$

$$[\, \hat{F}^+_{2\mu} \, , \, \hat{F}^+_{2\nu} \,] = 0 \quad , \tag{6a}$$

$$[\, \hat{F}^+_{2\mu} \, , \, \hat{F}_{2\nu} \,] = \hat{G}_{\mu\nu} \quad , \tag{6b}$$

$$[\, \hat{G}_{\mu\nu} \, , \, \hat{F}^+_{2\kappa} \,] = \delta_{\nu\kappa} \hat{F}^+_{2\mu} + \delta_{\mu\nu} \hat{F}^+_{2\kappa} \quad . \tag{6c}$$

Equation (5) defines the highest weight state, eq. (6b) is the definition of the $\hat{G}_{\mu\nu}$ operators and eqs. (6a) and (6c) are the commutation relations of the su(6) algebra formed by the 35 operators: $\hat{F}^+_{2\mu}$, $\hat{F}_{2\mu}$ and $\hat{G}_{\mu\nu}$. Let us notice that the shift operators, and the other generators of the su(6) algebra, do not change the fermion number N_F.

The search for the dynamic SU(6) symmetry in a nucleus should proceed in the following way:

1) Find a model for the shift operators $\hat{F}^{+}_{2\mu}$. One is certainly not able to write down an exact operator transforming one collective state into another one (say the ground state of a spherical nucleus into the first excited 2^+ state). The shell model arguments (based on the Tamm-Dancoff or RPA methods) can however be used to obtain a reasonable guess.
2) Check (or assure if the guess for the shift operators contains some freedom) the su(6) commutation relations, eqs. (6a) and (6c). Again this probably cannot be achieved in an exact way. One should however keep in mind that these relations constitute the base of the dynamic SU(6) symmetry in nuclei and their strong violation by "reasonable" shift operators should be considered as a strong argument to abandon the idea of such a symmetry at all. In what follows I will try to fulfil these relations exactly, because it seems plausible that an approximate algebra closure can take place only for a set of operators not very different from an exact algebra.
3) Having constructed the su(6) algebra in the Fock space one can split this space into a sum of the irreducible representations of SU(6). This in principle can be achieved by diagonalizing the Cartan subalgebra consisting of the $\hat{G}_{\mu\mu}$ operators and analyzing the resulting eigenvalues. One should note that the entire Fock space will thus be decomposed into the sum of the SU(6) representations while we are interested only in one candidate for the collective space. Two conditions can be used to select the proper representation: i) a mean energy (for example the trace of the nuclear many-body Hamiltonian calculated in the collective representation) should be small [2], ii) the nuclear Hamiltonian should weakly couple the collective and any other representation. None of the two conditions is easy to be implemented. One should however keep in mind that the choice of the collective representation is in principle the matter of a derivation; for the chosen shift operators and nuclear Hamiltonian one should check whether the collective space is well approximated by a symmetric representation - and if it is so - which symmetric representation is the best candidate.
4) The projection of the nuclear Hamiltonian on the collective

SU(6) representation should be identified with the collective Hamiltonian. Such a projection can always be in the chosen representation expressed as a function of the su(6) generators. Hence again it is a matter of a derivation whether this function can be approximated by an expression quadratic in the generators.

The scheme presented by the steps 1)-4) is certainly very difficult to be realized and definite answers cannot be expected to come out very soon. However, a microscopic justification of the IBM should probably follow the presented outline or should refer to it when a necessary approximations are done.

THE ANGULAR MOMENTUM

As it was mentioned in the previous section, the fermion $\hat{F}^{+}_{2\mu}$ generators are assumed to form a quadrupole spherical tensor. By definition, the shift operators should thus fulfil the following condition:

$$\hat{R}(\Omega)\hat{F}^{+}_{2\mu}\hat{R}^{+}(\Omega) = \sum_{\nu=-2}^{2} D^{2}_{\nu\mu}(\Omega)\hat{F}^{+}_{2\nu} \quad , \tag{7}$$

where $\hat{R}(\Omega)$ is the rotation operator,

$$\hat{R}(\Omega) = \exp(-i\phi\hat{J}_z)\exp(-i\theta\hat{J}_y)\exp(-i\psi\hat{J}_z) \quad , \tag{8}$$

which depends on the Euler angles $\Omega = \{\phi,\theta,\psi\}$ and on the physical* angular momentum operator $\vec{J}$, and $D^{\lambda}_{\nu\mu}$ is the Wigner function. Equation (7) is equivalent to the following commutation relation:

$$[\hat{J}_{1\nu}, \hat{F}^{+}_{2\mu}] = \sqrt{6}\,(2\mu 1\nu|2\mu+\nu)\hat{F}^{+}_{2\mu+\nu} \quad , \tag{9}$$

where $\hat{J}_{1\nu}$ are the spherical covariant components of the physical angular momentum operator which form the so(3) algebra:

---

*The name "physical" is used to stress that the $\vec{J}$ operator is the generator of the real rotations in the three-dimensional space.

$$[\hat{J}_{1\mu}, \hat{J}_{1\nu}] = -\sqrt{2}\,(1\mu 1\nu|1\mu{+}\nu)\hat{J}_{1\mu+\nu} \quad . \qquad (10)$$

An angular momentum operator can also be constructed inside the su(6) algebra [1]. For this let us replace the $\hat{G}_{\mu\nu}$ operators, eq. (6b), by:

$$\hat{P}_{\lambda\mu} = ([\hat{F}_2^+, \hat{\tilde{F}}_2])_{\lambda\mu} = (\hat{F}_2^+ \times \hat{\tilde{F}}_2)_{\lambda\mu} - (-1)^{\lambda}(\hat{\tilde{F}}_2 \times \hat{F}_2^+)_{\lambda\mu} \qquad (11)$$

for $\lambda = 0,1,2,3,4$, where $\hat{\tilde{F}}_{2\mu} = (-1)^{\mu}\hat{F}_{2-\mu}$ and the symbol $(\times)_{\lambda\mu}$ denotes the vector coupling to the angular momentum λ and the projection μ. A little calculation allows to show that the vector operator $\hat{I}_{1\nu} = \sqrt{10}\,\hat{P}_{1\nu}$ generates the so(3) subalgebra of the su(6),

$$[\hat{I}_{1\mu}, \hat{I}_{1\nu}] = -\sqrt{2}\,(1\mu 1\nu|1\mu{+}\nu)\hat{I}_{1\mu+\nu} \quad . \qquad (12)$$

and that it rises and lowers the magnetic indices of the $\hat{F}^+_{2\mu}$, $\hat{\tilde{F}}_{2\mu}$ and $\hat{P}_{\lambda\mu}$ operators exactly in the same way as the physical angular momentum operator does:

$$[\hat{I}_{1\nu}, \hat{F}^+_{2\mu}] = \sqrt{6}\,(2\mu 1\nu|2\mu{+}\nu)\hat{F}^+_{2\mu+\nu} \quad , \qquad (13)$$

This observation plays a crucial role in the following discussion, so it is worthwhile to stress the origin of the two different angular momenta: i) the physical angular momentum operator $\hat{J}_{1\nu}$ enters into the analysis of the problem because the su(6) generators form spherical tensors with respect to the physical rotations, ii) the angular momentum operator $\hat{I}_{1\nu}$ is one of the generators of the su(6) algebra and its commutation relations are a consequence of the su(6) commutation relations. Depending on the choice of the $\hat{F}^+_{2\mu}$ operators, the $\hat{I}_{1\nu}$ operator is or is not equal to the physical angular momentum operator $\hat{J}_{1\nu}$.

It is clear that instead of considering the su(6) algebra alone one has to deal with a larger algebra which also includes the physical angular momentum operator. Defining the operator $\hat{K}_{1\nu}$,

$$\hat{K}_{1\nu} = \hat{J}_{1\nu} - \hat{I}_{1\nu} \quad , \qquad (14)$$

one can easily show that it fulfils the so(3) commutation relations, analogous to those of eqs. (10) and (12), and commutes with the su(6) generators. The algebra one has to consider is thus equal to the direct sum so(3) ⊕ su(6) where so(3) is generated by the $\hat{K}_{1\nu}$ operators and su(6) by the $\hat{F}^+_{2\mu}$, $\hat{F}_{2\mu}$ and $\hat{P}_{\lambda\mu}$ operators. The physical angular momentum operator is equal to the sum of the $\hat{K}_{1\nu}$ and $\hat{I}_{1\nu}$ operators, which commute with each other, and the eigenstates of $\hat{J}^2$ can be obtained by the vector coupling of the $\hat{K}^2$ and $\hat{I}^2$ eigenstates. We have thus arrived to the same construction scheme as used by other authors [3-5], who introduced arbitrary pseudo-orbital (K) and pseudo-spin (I) angular momenta. Of course the final goal is to use the model angular momentum I to measure the physical angular momentum J. This means that the interesting representations of the SO(3) ⊗ SU(6) group are those which are the scalar ones with respect to the angular momentum K.

THE SINGLE-PARTICLE SU(6) SHIFT OPERATORS

Let me first study the simplest approximation for the shift operators, namely the approximation by single-particle operators:

$$\hat{F}^+_{2\mu} = \sum_{kl} F^{(2\mu)+}_{kl} a^+_k a_l \quad , \tag{15}$$

where the indices k and l number a suitable finite set of the single-particle states and $F^{(2\mu)+}_{kl}$ is a matrix. This approximation includes all possible solutions of the particle-hole Tamm-Dancoff or RPA methods. Because the bifermion operators $a^+_k a_l$ form the unitary algebra,

$$[a^+_k a_l \, , a^+_m a_n] = \delta_{lm} a^+_k a_n - \delta_{kn} a^+_m a_l \quad , \tag{16}$$

the matrix commutation relation

$$[[F^{(2\mu)+}, F^{(2\nu)}], F^{(2\kappa)+}] = \delta_{\nu\kappa} F^{(2\mu)+} + \delta_{\mu\nu} F^{(2\kappa)+} \tag{17}$$

is the sufficient and necessary condition for the commutation relation (6c) to be fulfilled in the Fock space. This means that every matrix representation of the SU(6) group will give a closed su(6) algebra of the single-particle operators in the Fock space. By listing the irreducible SU(6) representations and multiplying them by the SO(3) representations one can obtain all possible so(3) ⊕ su(6) single-particle algebras. In verifying their applicability to the justification of the IBM one should follow the following steps:

1) Take an SU(6) matrix representation and find all possible values of the angular momentum I in this representation.
2) Couple all values of I to a chosen value of the angular momentum K to obtain the possible physical angular momentum values and their multiplicities.
3) Check whether the obtained set of the physical angular momentum values is identical with a set constituting the valence shell for a given range of the proton and neutron numbers.
4) Find all possible SO(3) ⊗ SU(6) representations in the many-fermion space constructed from the found set of the single-particle states.
5) Pick out all scalar (K=0) representations.

By following above procedure one can verify that <u>non</u> of the matrix SU(6) representations can provide a correct set of the single-particle angular momenta and/or that the symmetric SU(6) representations are missing in the corresponding Fock spaces. This constitutes a proof that the shift operators $\tilde{F}^{+}_{2\mu}$ of the hypothetical SU(6) dynamic symmetry <u>cannot</u> be single-particle operators.

Let me present this proof for the simplest case when the chosen SU(6) matrix representation is the one given by the Young diagram □ . The angular momentum I can then be equal to 0 or 2. By choosing the angular momentum K equal to 1/2, one obtains the sequence of single-particle physical spins: 1/2, 3/2, 5/2. An obvious example of such sequence is found in the nuclear s-d shell. In this particular case the angular momentum I can be identified with the orbital angular momentum and K with the intrinsic nucleon spin. Another example is given by the $p_{1/2}$, $p_{3/2}$ and $f_{5/2}$ orbits where the I and K angular momenta have no physical interpretation.

Denoting the $(K=1/2) \otimes \Box$ representation by $(1/2)\otimes[1]$, where [1] is the partition corresponding to the Young diagram $\Box$, one can list all allowed $SO(3) \otimes SU(6)$ representations for arbitrary fermion number N_F, see table I. One can see that the scalar (K=0) representations of SO(3), occurring for even values of N_F, are not the symmetric representations of SU(6) (apart from $N_F=2$ where however $N_B=2$ too) and cannot thus be considered as IBM collective quadrupole spaces.

Coupling the representation $\Box$ with $K = 3/2$ one would obtain the physical angular momenta 1/2, 3/2, 3/2, 5/2 and 7/2. The double-occurence of the 3/2 value makes it difficult to apply this case to the nuclear shell model. The same type of argument can also be used to disregard still higher values of K or bigger SU(6) matrix representations as $\Box\Box$ (I = 0,0,2,2,4) or $\begin{smallmatrix}\Box\\ \Box\end{smallmatrix}$ (I = 1,2,3), where the multiple j-values are very common. For any assumed matrix SU(6) representation one obtains two unwanted results: i) the physical single-particle angular momentum values occur more than once in a given $SO(3) \otimes SU(6)$ matrix representation and ii) in the Fock space there are no symmetric SU(6) representations which would fulfil the requirements of the IBM. Hence one has to end up with the conclusion that the SU(6) dynamic symmetry in nuclei cannot be built upon the single-particle shift operators $\hat{F}^{+}_{2\mu}$.

After stating this rigorous result one should remark that the assumption which have led to it is quite restrictive. The shift operators have been assumed to be equal to single-particle operators when acting on <u>all</u> states of the Fock space. In fact this requirement should be reduced to a very limited subspace - namely the very subspace of the collective quadrupole states which we just want to describe:

$$\hat{F}^{+}_{2\mu} = \Big(\sum_{kl} F^{(2\mu)+}_{kl} a^{+}_{k} a_{l} \Big)\hat{P} + \hat{F}'^{+}_{2\mu}(1-\hat{P}) \quad , \qquad (18)$$

where $\hat{P}$ is the corresponding (scalar) projection operator and $\hat{F}'^{+}_{2\mu}$ is an arbitrary quadrupole operator. Such shift operator is not a single-particle operator and one cannot calculate the necessary commutators from the algebraic properties of the bifermion operators $a^{+}_{k}a_{l}$. In looking for the microscopic

Table I

Decomposition of the $j = 1/2, 3/2, 5/2$ nuclear shell into the SO(3) ⊗ SU(6) representations. N_F denotes the fermion number and k is the value of the angular momentum K (see text).

N_F	(k) ⊗ [SU(6) partition]
1	$(\frac{1}{2}) \otimes [1]$
2	$(0) \otimes [2]$
	$(1) \otimes [11]$
3	$(\frac{1}{2}) \otimes [21]$
	$(\frac{3}{2}) \otimes [111]$
4	$(0) \otimes [22]$
	$(1) \otimes [211]$
	$(2) \otimes [1111]$
5	$(\frac{1}{2}) \otimes [221]$
	$(\frac{3}{2}) \otimes [2111]$
	$(\frac{5}{2}) \otimes [11111]$
6	$(0) \otimes [222]$
	$(1) \otimes [2211]$
	$(2) \otimes [21111]$
	$(3) \otimes [111111]$

justification of the IBM one is thus forced to deal explicitly with operators much more complicated than the single-particle ones. From the mathematical point of view one has to search for the su(6) closed algebras not in the $su(\sum_i(2j_i+1))$ algebra but in its enveloping algebra [6], which is much larger and complicated and much less studied than the bifermion algebra itself.

THE SU(6) SHIFT OPERATORS BASED ON THE GINOCCHIO MODEL

There exists one example of the closed su(6) algebra in the fermion Fock space, which is generated by the non-single-particle shift operators and has symmetric representations in the Fock space. Its construction is based on the Ginocchio [4] model in which one defines the monopole $(\hat{S}^+)$ and the quadrupole $(\hat{D}^+_\mu)$ pairs of fermions in such a way that the operators $\hat{S}^+$, $\hat{D}^+_\mu$, $[\hat{S}^+,\hat{D}_\mu]$, $[\hat{S}^+,\hat{S}]$ and $[\hat{D}^+_\mu,\hat{D}_\nu]$ together with their hermitian conjugations form the so(8) algebra. The generator

$$\hat{I}_{1\nu} = \frac{1}{4}\sqrt{10}\,([\hat{D}^+,\hat{\tilde{D}}])_{1\nu} \qquad (19)$$

acts in the so(8) algebra in the same way as the physical angular momentum $\hat{J}_{1\nu}$ and thus the operator

$$\hat{K}_{1\nu} = \hat{J}_{1\nu} - \hat{I}_{1\nu} \qquad (20)$$

commutes with all the so(8) generators and forms the so(3) subalgebra. Instead of the SO(8) group one has thus to consider the direct product SO(3) ⊗ SO(8).

In the single-particle basis the pair operators can be expressed as:

$$\hat{S}^+ = \frac{1}{2}\sum_{kl} S^+_{kl}a^+_k a^+_l \;, \qquad (21a)$$

$$\hat{D}^+_\mu = \frac{1}{2}\sum_{kl} D^{(\mu)+}_{kl} a^+_k a^+_l \;, \qquad (21b)$$

where S^+ and $D^{(\mu)+}$ are antysymmetric matrices. By using the doubled dimension of the fermion single-particle space [7],

$$c = \begin{pmatrix} a \\ a^+ \end{pmatrix} \quad , \quad c^+ = (a^+ , a) \quad , \tag{22}$$

one can represent these operators in the form

$$\hat{S}^+ = \frac{1}{2} \sum_{KL} \mathcal{S}^+_{KL} c^+_K c_L \quad , \tag{23a}$$

$$\hat{D}^+_\mu = \frac{1}{2} \sum_{KL} \mathcal{D}^{(\mu)+}_{KL} c^+_K c_L \quad , \tag{23b}$$

where the capital indices K and L assume twice as much values as the ordinary single-particle indices k and l, and the matrices $\mathcal{S}^+$ and $\mathcal{D}^{(\mu)+}$ read

$$\mathcal{S}^+ = \begin{pmatrix} 0 & S^+ \\ 0 & 0 \end{pmatrix} \quad , \quad \mathcal{D}^{(\mu)+} = \begin{pmatrix} 0 & D^{(\mu)+} \\ 0 & 0 \end{pmatrix} \quad . \tag{24}$$

Similarly, all other generators of the so(8) algebra can be represented by matrices of doubled dimensions. Every such matrix $\mathcal{A}$, fulfilling the condition

$$\mathcal{F}\mathcal{A}\mathcal{F} = -\mathcal{A}^T \quad \text{where} \quad \mathcal{F} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \tag{25ab}$$

is in one-to-one correspondence with a bifermion operator $\hat{A} = \frac{1}{2} \sum_{KL} \mathcal{A}_{KL} c^+_K c_L$. It is easy to show that the commutation relations between the bifermion operators $\hat{A}$ are equivalent to the respective relations between the matrices $\mathcal{A}$. Every matrix representation of the so(3) ⊕ so(8) algebra will thus generate the corresponding so(3) ⊕ so(8) algebra in the Fock space, provided the matrix representation consists of matrices fulfilling condition (25a).

The Ginocchio model is based on the $(k) \otimes [\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}]$ matrix representation of SO(3) ⊗ SO(8), where k is a value of the angular momentum K and the SO(8) representation is denoted by its highest weight vector. Among the representations of the SO(3) ⊗ SO(8) group in the Fock space there is the representation $(0) \otimes [(\frac{1}{2}\Omega)000]$ $(\Omega=2(2k+1))$, which is schematically presented in fig. 1 for $k=2$ $(\Omega=10)$. This representation contains states with various fermion numbers from $N_F=0$ to $N_F=2\Omega$. There are $(N_F/2+5)!/(N_F!5!)$ independent states for $N_F \le \Omega$ and $(\Omega-N_F/2+5)!/((\Omega-N_F/2)!5!)$ independent states for

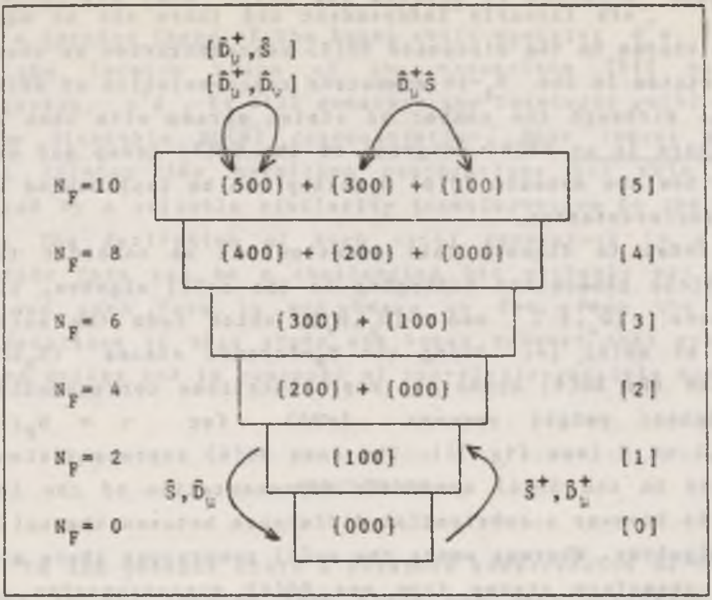

Fig. 1. A schematic visualization of the lower part ($N_F \leq \Omega$) of the SO(3) ⊗ SO(8) representation (0) × [5000]. Six boxes symbolize the symmetric SU(6) representations with the corresponding partitions indicated on the right and the fermion numbers indicated on the left. In each box the constituting SO(6) representations are denoted by their partitions. The arrows denote: i) the action of the SO(8) generators $\hat{S}^+$ and $\hat{D}^+_\mu$ ($\hat{S}$ and $\hat{D}_\mu$), which transfer states up (down) between different SU(6) representations, ii) the action of the SO(6) generators $[\hat{D}^+_\mu, \hat{D}_\nu]$ and $[\hat{D}^+_\mu, \hat{S}]$, which transfer states inside one SO(6) representation, and iii) the action of the $\hat{D}^+_\mu \hat{S}$ operators which transfer states from one SO(6) representation to another one and are missing in the SO(8) algebra.

$N_F > \Omega$. This means that all states $(\bar{S}^+)^{n_s}(\bar{D}^+_\mu)^{n_d}|0\rangle$ for $2(n_s+n_d)=N_F$ are linearly independent and there are as many N_F-fermion states in the discussed SO(8) representation as there are boson states in the N_B-th symmetric representation of SU(6) for $N_B = 2N_F$. Although the number of states agrees with that of the IBM, there is no SU(6) subgroup of the SO(8) group and one may wander how the dynamic SU(6) symmetry can be implemented in the SO(8) representation.

In order to discuss this question let us consider the single-particle generators belonging to the so(8) algebra, i.e the operators $[\bar{D}^+_\mu, \bar{S}]$ and $[\bar{D}^+_\mu, \hat{D}_\nu]$, which form the so(6) subalgebra of so(8) [4]. Among the N_F-fermion states $(N_F \leq \Omega)$ one can find the SO(6) symmetric representations corresponding to the highest weight vectors $(\sigma 00)$ for $\sigma = N_F/2$, $N_F/2-2,\ldots,1$ or 0 (see fig. 1). The same SO(6) representations can be found in the SU(6) symmetric representation of the IBM [1]. There is however a substantial difference between the so(8) and su(6) algebras. Whereas among the su(6) generators there are such which transform states from one SO(6) representation to another one, for example the shift operator $d^+_\mu s$, the corresponding operator in the fermion space $\bar{D}^+_\mu \bar{S}$ is not an so(8) generator and it does not form a closed algebra when added to the fermion so(6) algebra.

In order to convince ourselves that there exists a fermion shift operator, which forms the su(6) algebra when commuted with the so(6) generators, we have to invoke the theory of boson expansions. It is known [8-10] that the fermion states of the Ginocchio model admit the boson mapping, which is achieved by representing the pair creation operators $\bar{S}^+$ and $\bar{D}^+_\mu$ in terms of boson creation operators s^+ and d^+_μ:

$$\bar{S}^+ \quad \longleftrightarrow \quad s^+(\Omega + 2N_B) - \left(\sum_{\nu=-2}^{2} \tilde{d}^+_\nu d^+_\nu - s^+ s^+\right)s \quad , \qquad (26a)$$

$$\bar{D}^+_\mu \quad \longleftrightarrow \quad d^+_\mu(\Omega + 2N_B) + \left(\sum_{\nu=-2}^{2} \tilde{d}^+_\nu d^+_\nu - s^+ s^+\right)\tilde{d}_\mu \quad , \qquad (26b)$$

Every fermion state obtained by a multiple action on the fermion vacuum with the pair creation operators is mapped onto the boson state obtained by a corresponding action on the boson vacuum with the boson images (26). The orthogonality of fermion states and hermitian conjugation of fermion operators are not preserved, but they can be restored by an appropriate similarity transformation [11]. Since the mapping is faithful, there must exist a <u>fermion image</u> of the boson shift operator $d^{+}_{\mu}s$. Together with the fermion image of the boson-like [11] hermitian conjugation, $s^{+}d_{\mu}$, it will generate the fermionic su(6) algebra in the Ginocchio SO(8) representation. Both images are not mutual fermion-like hermitian conjugations but this can be restored by a suitable similarity transformation in the fermion space. The derivation of such su(6) generators in a closed algebraic form can be a challenging but probably not an easy task and such form is not known so far. From the general considerations of this study one knows however that such su(6) algebra exists and is composed of non-single-particle operators.

CONCLUSION

In the present study a possible construction of the su(6) algebra in the fermion Fock space has been discussed. It is argued that in order to find a microscopic justification of the Interacting Boson Model one should construct the fermion-number-conserving shift operator $\hat{F}^{+}_{\mu}$, playing in the fermion su(6) algebra a role analogous to the $d^{+}_{\mu}s$ generator of the boson realization of su(6), and that the boson operators itself need not to have a microscopic significance. It is shown however that such shift operator <u>cannot</u> be a single-particle operator. An example of a non-single-particle su(6) shift operator is provided by the Ginocchio model but a closed algebraic form of it is difficult to unveil.

REFERENCES

[1] A r i m a A., I a c h e l l o F.: Ann.Phys. 1976, 99, 253; Ann.Phys. 1978, 111, 201; Ann.Phys. 1978, 115, 325; Ann.Phys. 1979, 123, 468.

[2] K l e i n A., V a l l i e r s M.: Phys.Lett. (1981) 98B, 5.

[3] H e c h t K. T., A d l e r A.: Nucl.Phys. 1969, 137, 129.

[4] G i n o c c h i o J. N.: Ann.Phys. 1980, 126, 234.

[5] W u C.-L., F e n g D.H., C h e n X.-G., C h e n J.-Q., G u i d r y M. W.: Phys.Lett. 1986, 168B, 313.

[6] B a r u t A. O. R ą c z k a R.: Theory of Group Representations and Applications, Polish Scientific Publishers, Warszawa, 1980.

[7] R i n g P., S c h u c k P.: The Nuclear Many-Body Problem, Springer-Verlag, New York, 1980.

[8] G e y e r H. B., H a h n e F. J. W.: Nucl.Phys. 1981 A363, 45.

[9] A r i m a A., Y o s h i d a N., G i n o c c h i o J.N. Phys.Lett. 1981, 101B, 209.

[11] D o b a c z e w s k i J.: Nucl.Phys. 1982, A380, 1.

[12] D o b a c z e w s k i J.: Nucl.Phys. 1981, A369, 237.

STRESZCZENIE

W nawiązaniu do modelu oddziałujących bozonów (IBM) dyskutowana jest możliwość skonstruowania fermionowej algebry su(6) i odpowiadających jej reprezentacji symetrycznych w przestrzeni Focka dla fermionów. Podany jest dowód, że algebra taka nie może się składać z operatorów jednocząstkowych.

Р Е З Ю М Е

В связи с моделью взаимодействующих бозонов (IBM) анализируется возможность построения фермионной алгебры SU(6) и соответствующих ей представлений, симметричных для фермионов в пространстве Фока. Доказывается, что алгебры того типа нельзя составить из одночастичных операторов.