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Isospin Oscillations in Σ-Hypernuclei

Oscylacje izospinowe w Σ-hiperjądrach

Изоспиновые осцилляции в Σ-гиперядрах

One of the various symmetry groups considered by Professor Stanisław Szpikowski in his works on nuclear structure, is the SU_2 isospin group which plays an important role both in nuclear and particle physics. The isospin symmetry is not exact. It is broken by electromagnetic forces and mass differences, which leads to several effects. Here, I shall describe one such effect: the isospin oscillations in Σ hypernuclei.

Let us consider the states of the ${}^{12}_{\Sigma}C$ hypernuclei produced in the (K^-, π^-) strangeness exchange reactions on ${}^{12}C$. Because of the presence of the Σ's symmetry (Lane) potential, the pure charge states, ${}^{12}_{\Sigma^+}C({}^{11}B + \Sigma^+)$ and ${}^{12}_{\Sigma^0}C({}^{11}C + \Sigma^0)$, are not true eigenstates, i.e., the Σ hyperons do not preserve their charge identity, and the true eigenstates are superpositions of the two charge states. On the other hand, because of the breaking of the isospin symmetry, the eigenstates of the isospin T are also not true eigenstates.

I apply the single particle (s.p.) picture and restrict myself to the substitutional $(p3/2,\ p3/2^{-1})_{\Sigma N}$ states. I assume that the nuclear cores C (${}^{11}B$ and ${}^{11}C$) are rigid (undisturbed by Σ), and that their isospin

$T^C = 1/2$. The nuclear ΣC interaction $V_{\Sigma C}$ is represented by a s.p. potential,

$$V_{\Sigma C} = v(r) + iw(r) + v_t(r)\, \vec{T}^C \vec{t}^\Sigma \quad , \tag{1}$$

where the imaginary potential w describes the absorption due to the $\Sigma N \rightarrow \Lambda N$ process.

I work in the charge basis and write the wave function Ψ of ${}^{12}_{\Sigma}C$ as

$$\begin{aligned} \Psi &= \Psi_{\Sigma^+} + \Psi_{\Sigma^0} \\ \Psi_{\Sigma^+} &= R(^{11}B)\,\chi^{(+)}\,\varphi_{\Sigma^+}(\vec{r}) \ , \\ \Psi_{\Sigma^0} &= R(^{11}C)\,\chi^{(o)}\,\varphi_{\Sigma^0}(\vec{r}) \ , \end{aligned} \tag{2}$$

where $\vec{r}$ is the relative $C\Sigma$ position vector, $R(C)$ is the normalized wave function of C, $\chi^{(+)} = \chi(^{11}B)\,\chi(\Sigma^+)$ and $\chi^{(o)} = \chi(^{11}C)\,\chi(\Sigma^o)$ are the isospin functions in the Σ^+ and Σ^o channels. By inserting expressions (2) into the Schrödinger equation for Ψ, and projecting the resulting equation onto $\langle R\chi^{(+)}|$ and $\langle R\chi^{(o)}|$ (here, the approximation $R(^{11}B) \cong R(^{11}C)$ is used), one gets

$$\begin{aligned} \left\{-(\hbar^2/2\mu_{\Sigma^+})\Delta + v + iw + V_c - \tfrac{1}{2}v_t - E_{\Sigma^+} + i\Gamma/2\right\}\varphi_{\Sigma^+} &= -v_t\,\varphi_{\Sigma^0}/\sqrt{2} \ , \\ \left\{-(\hbar^2/2\mu_{\Sigma^0})\Delta + v + iw - E_{\Sigma^0} + i\Gamma/2\right\}\varphi_{\Sigma^0} &= -v_t\,\varphi_{\Sigma^+}/\sqrt{2} \ , \end{aligned} \tag{3}$$

where V_c is the Coulomb potential, μ_Σ is the C reduced mass, $-\Gamma/2$ is the imaginary part of the energy eigenvalue with Γ interpreted as the width of the state, and

$$\begin{aligned} E_{\Sigma^+} &= -B_{\Sigma^+} = M({}^{12}_{\Sigma}C) - M(^{11}B) - M(\Sigma^+) \ , \\ E_{\Sigma^0} &= -B_{\Sigma^0} = M({}^{12}_{\Sigma}C) - M(^{11}C) - M(\Sigma^o) = E_{\Sigma^+} - 4.57\ \text{MeV} \end{aligned} \tag{4}$$

where B_{Σ^+} (B_{Σ^0}) is the binding energy of Σ^+ (Σ^o) in ${}^{12}_{\Sigma}C$.

For the p3/2 orbits considered here, I write $\psi_\Sigma(\vec{r}) = Y_{1m}(\hat{r})\, u_\Sigma(r)/r$. I denote by $p_{\Sigma^+}(r)$ and $p_{\Sigma^o}(r)$ the probability of finding respectively Σ^+ and Σ^o in the unit interval Δr:

$$p_{\Sigma^+}(r) = |u_{\Sigma^+}(r)|^2, \quad p_{\Sigma^o}(r) = |u_{\Sigma^o}(r)|^2. \tag{5}$$

I assume that Ψ is normalized, i.e.,

$$\int dr \left[p_{\Sigma^+}(r) + p_{\Sigma^o}(r) \right] = 1 .$$

To discuss the isospin T of the state Ψ, I decompose $\chi^{(+)}$ and $\chi^{(o)}$ into eigenstates χ^T of T = 1/2 and 3/2 (with the third component fixed, $T_3 = -1/2$),

$$\chi^{(+)} = \sqrt{1/3}\, \chi^{3/2} - \sqrt{2/3}\, \chi^{1/2} ,$$
$$\chi^{(o)} = \sqrt{2/3}\, \chi^{3/2} + \sqrt{1/3}\, \chi^{1/2} , \tag{6}$$

and obtain the corresponding decomposition of $\Psi = \sum_T \Psi_T$,

$\Psi_T = R\chi^T \varphi_T(\vec{r}) = R\chi^T Y_{1m}(\hat{r})\, u_T(r)/r$

with

$$u_{1/2} = \sqrt{1/3}\, u_{\Sigma^o} - \sqrt{2/3}\, u_{\Sigma^+}$$
$$u_{3/2} = \sqrt{2/3}\, u_{\Sigma^o} + \sqrt{1/3}\, u_{\Sigma^+} \tag{7}$$

The probability of finding in the unit interval Δr our system in the isospin T state is: $p_T(r) = |u_T(r)|^2$.

Let me now present results obtained by solving eqs (3).

The form used for v and v_t was:

$$v(r) = V\, \varrho(r)/\varrho_o , \quad v_t(r) = V_t\, \varrho(r)/\varrho_o \tag{8}$$

where $\varrho_o = 0.166\ \mathrm{fm}^{-3}$ is the equilibrium density of nuclear matter, the nuclear core density $\varrho(r) = \varrho(o)\left\{1 + \alpha(r/a)^2\right\} \exp\left\{-(r/a)^2\right\}$ with a = 1.69 fm and α = 0.811 determined by electron scattering on

^{11}B [1], $V = -30$ MeV, and $V_t = 5$ MeV (the value calculated by Dover and Gal [2] with Model D of the Nijmegen barion-barion interaction). For V_c, I used the Coulomb potential of a uniform charge distribution with the same rms (2.42 fm) as $\varrho(r)$. The absorptive potential $w(r)$ was obtained (exactly as in [3]) in the local density approximation from the semi-classical expression (modified by Pauli blocking and binding effects) for w in nuclear matter.

Among the exponentially decaying solutions of eq. (3), I shall restrict myself to discuss the state which goes over into the state of $^{12}_{\Sigma^0}C$ when $v_t \to 0$. For this state one gets $E_{\Sigma^0} = 2.84$ MeV and $\Gamma = 0.9$ MeV. The corresponding value of $\Delta M = M(^{12}_{\Sigma}C) - M(^{12}C) = 274.5$ MeV is in a qualitative agreement with the CERN data [4]: $\Delta M(^{12}_{\Sigma^0}C) = 275$ MeV, and with the recent KEK data [5]: $\Delta M(^{12}_{\Sigma}C)_B = 277.6$ MeV.

The probabilities p_Σ and p_T are shown in Fig. 1. The most striking feature are the oscillations of p_T beyond the nuclear core. To discuss these isospin oscillations, I write the expressions for p_T, obtained with the help of eqs (7) and (5):

$$p_{3/2}(r) = p_{\Sigma^+}(r)/3 + 2\,p_{\Sigma^0}(r)/3 + A(r)\cos\Phi(r) ,$$

$$p_{1/2}(r) = 2p_{\Sigma^+}(r)/3 + p_{\Sigma^0}(r)/3 - A(r)\cos\Phi(r) , \tag{9}$$

where

$$\Phi(r) = \arg\left\{ u_{\Sigma^0}(r) / u_{\Sigma^+}(r) \right\} ,$$

$$A(r) = 2\sqrt{2\,p_{\Sigma^+}(r)\,p_{\Sigma^0}(r)}\,/\,3 . \tag{10}$$

To simplify the discussion, I restrict myself to such big values of r, that (for $\Sigma = \Sigma^+, \Sigma^0$)

$$u_\Sigma \sim A_\Sigma \exp(ik_\Sigma r) , \tag{11}$$

where A_Σ are complex constants determined in the process of solving eqs (3), and the complex momenta k_Σ are defined by

$$k^2_\Sigma = 2\mu_\Sigma(E_\Sigma - i\Gamma/2)\,\hbar^2 . \tag{12}$$

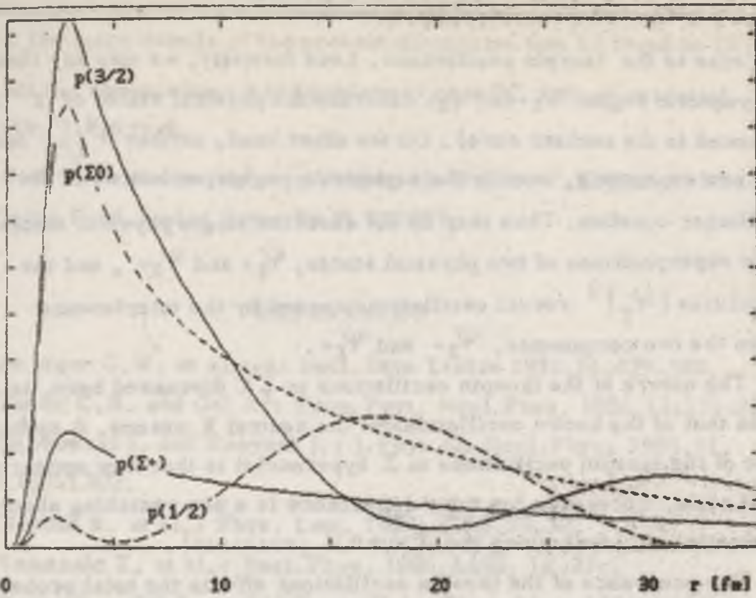

Fig. 1. The distribution of charge and isospin in ${}^{12}_{\Sigma}C$.

Eqs (11) lead to the following asymptotic expressions:

$$\Phi \sim \mathrm{Re}\left\{k_{\Sigma^o} - k_{\Sigma^+}\right\} r + \delta$$

$$A \sim (2\sqrt{2}/3)\, |A_{\Sigma^o} A_{\Sigma^+}| \exp\left[-\,\mathrm{Im}\left\{k_{\Sigma^o} + k_{\Sigma^+}\right\} r\right] , \qquad (13)$$

where $\delta = \arg(A_{\Sigma^o}/A_{\Sigma^+})$. By inserting expressions (13) into eqs (9), one sees that the probabilities p_T in the asymptotic region oscillate with the wave length $\lambda = 2\pi/\mathrm{Re}\left\{k_{\Sigma^o} - k_{\Sigma^+}\right\}$, and with the amplitude A. Since A is (exponentially) decreasing with r, the oscillations are damped. This is exactly the behaviour shown in Fig. 1.

Another way of explaining the isospin oscillations is to transform system (3) of equations for Ψ_{Σ^+} and Ψ_{Σ^o}, coupled by v_t, into a system of equations for $\Psi_T (T = 1/2, 3/2)$. The coupling due to v_t disappears in the equations for Ψ_T, but a new coupling appears due to the Coulomb interaction and mass differences. In contradistinction to the coupling v_t between Ψ_{Σ^+} and Ψ_{Σ^o}, which vanishes outside the nuclear core, the constant mass difference term couples $\Psi_{1/2}$ with $\Psi_{3/2}$ everywhere, thus giving rise to the isospin oscillations. Less formally, we may say that in the asymptotic region φ_{Σ^+} and φ_{Σ^o} describe the physical states of Σ^+ and Σ^o (bound to the nuclear core). On the other hand, neither $\varphi_{1/2}$ nor $\varphi_{3/2}$ are separately, even in the asymptotic region, solutions of the Schrödinger equation. Thus they do not describe single physical states, but are superpositions of two physical states, φ_{Σ^+} and φ_{Σ^o}, and the probabilities $|\varphi_T|^2$ reveal oscillations caused by the interference between the two components, φ_{Σ^+} and φ_{Σ^o}.

The nature of the isospin oscillations in $^{12}_{\Sigma}C$ discussed here, is the same as that of the known oscillations of the neutral K mesons. A special feature of the isospin oscillations in Σ hypernuclei is that they appear in a bound state. Necessary for their appearance is a non vanishing absorptive potential w (describing the $\Sigma N \rightarrow \Lambda N$ processes).

The occurance of the isospin oscillations affects the total probability $P_T = \int dr\, p_T$ of finding $^{12}_{\Sigma}C$ in the isospin T state. The result of the present calculation, $P_{3/2} = 1 - P_{1/2} = .68$, differs from the value $P_{3/2} = .99$ suggested by Dover et al. [6]. The problem of a direct experimental detection of the isospin oscillations in $^{12}_{\Sigma}C$ is difficult because they occur outside the nuclear core. It would be probably easier to detect the effect of the coupling of the two charge states in Σ atoms (e.g. in the $^{11}C + \Sigma^-$ atomic system coupled with the $^{11}B + \Sigma^o$ system).

So far the isospin oscillations in Σ hypernuclei is a hypothetical phenomenon and its relevance in the physical phenomena may be questioned. Namely, there is one difficulty here, which should be mentioned. Since for the state of $^{12}_{\Sigma}C$ discussed $E_{\Sigma} > 0$, the state is of the type of the "unstable bound states embedded in the continuum" (UBS) discussed by Gal et al.[7]. Recently, the identification of the UBS with the peaks in the π distribution observed in the (K^-, π) reactions has been critisized by Morimatsu and Yazaki [8]. Their work, however, involves an approximate expression for the formation cross sections in terms of an approximate Green function. Furthermore, their criticism does not apply to bound states, and the properties of the Σ hypernuclear state discussed here do not change drastically when E_{Σ} become negative - the situation expected in hypernuclei heavier than $^{12}_{\Sigma}C$.

A few more details of the present discussion may be found in [9].

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STRESZCZENIE

Pokazano w pracy możliwość oscylacji radialnego rozkładu izospinu w Σ-hiperjądrach dla przypadku stanów ${}^{12}_{\Sigma}C$.

Р Е З Ю М Е

Возможность осцилляций в радиальном распределении изоспина в Σ-гиперядрах демонстрируется в случае подстановочных состояний в ${}^{12}_{\Sigma}C$.