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On the Relationship between the Source of the Pion Field in a Nucleus and Certain Quasispin Algebras

O związku między źródłem pola pionowego w jądrze i pewnymi algebrami quasispinowymi

О связи между источником пионного поля и некоторыми квазиспиновыми алгебрами

1. Introduction

In 1971 Migdal[1] suggested that a pion condensate may exist in nuclear matter under certain conditions, like a sufficiently large nuclear density and a sufficiently strong coupling between the pion and the nucleon fields. In a theory where the nucleons are described by a Hartree-Fock ("HF") state and the pions by a classical field $\underset{\sim}{\Phi}(\vec{x},t)$, the pion field satisfies a Klein-Gordon equation with a source term containing the single particle wave functions $\varphi_\alpha(\vec{x},t)$ of the nucleons. If we are interested in stationary states of the nucleus (or nuclear matter), $\underset{\sim}{\Phi}$ and φ_α are time-independent functions and the equation for $\underset{\sim}{\Phi}(\vec{x})$ reads:

$$\left[\Delta - m_\pi^2\right] \underset{\sim}{\phi}(\vec{x}) = g_\pi \vec{\nabla} \cdot \underset{\sim}{\vec{\rho}}^{\,SI}(\vec{x}) \tag{1.1}$$

Here, m_π is the pion mass and g_π is the pion–nucleon coupling constant in the pseudo-vectorial coupling model. The quantity $\underset{\sim}{\vec{\rho}}^{\,SI}(\vec{x})$ is a "spin–isospin density" defined as:

$$\underset{\sim}{\vec{\rho}}^{\,SI}(\vec{x}) = \sum_\alpha \varphi_\alpha^+(\vec{x}) \vec{\sigma} \underset{\sim}{\tau} \varphi_\alpha(\vec{x}) \tag{1.2}$$

where $\vec{\sigma}$ and $\underset{\sim}{\tau}$ are the Pauli matrices acting in spin and isospin space respectively. The single particle (s.p.) wave functions $\varphi_\alpha(\vec{x})$ satisfy a set of HF equations (see e.g. ref.2) which contain a term describing the coupling of the nucleons with the pion field. If one eliminates the pion field from equ. (1.2) and substitutes it into the HF equations, this coupling term yields the Hartree-potential arising from the one–pion exchange ("OPE") interaction. Speaking of a "pion–condensate" means that the source term $g_\pi\ \vec{\nabla} \cdot \underset{\sim}{\vec{\rho}}^{\,SI}(\vec{x})$ is to be large and thus produces an appreciable (classical) pion field strength in the nucleus. As a result of many investigations we know by now that this situation does not exist in ordinary (ground) states of finite nuclei nor in nuclear matter of normal density.[3,4] On the other hand, we think that there may be specific nuclear excitations where the pion field is appreciably larger than in ordinary states where $\underset{\sim}{\vec{\rho}}^{\,SI}(\vec{x})$ is zero or almost zero (and thus also $\underset{\sim}{\Phi}(\vec{x})$).[2] The crucial quantity in this context is the spin–isospin density $\underset{\sim}{\vec{\rho}}^{\,SI}(\vec{x})$. If the s.p. states φ_α are pure neutron or proton states, only the 3–component $\vec{\rho}_3^{\,SI}$ in isospin space may be unequal zero and thus only a π^0–field may arise. If we restrict ourselves to symmetric nuclei (N=Z), this is certainly so. If for an even–even symmetric nucleus the nucleons occupy pairwise time reversed s.p. states, the source term is seen to be strictly zero, since the contribution of time–reversed pairs of orbitals cancels in (1.2). If, however, we occupy pairwise states φ_α and $\tilde{K}\varphi_\alpha$, where $\tilde{K}$ is defined by (K_0 = complex conjugation)

$$\tilde{K} = \sigma_2 \tau_2 K_0 \tag{1.3}$$

the orbitals φ_α and $\bar{K}\varphi_\alpha$ contibute coherently to the spin–isospin density due to the property:

$$\tilde{K}\,\vec{\sigma}\,\tau_3\,\tilde{K}^{+} = \vec{\sigma}\,\tau_3 \tag{1.4}$$

The time reversal operator $\hat{K}$ is given by:

$$\hat{K} = i\sigma_2 K_0 \tag{1.5}$$

where the factor i is put in for convenience. Therefore, $\bar{K}$ is an operator, which in addition to reverting the time, also changes a neutron into a proton a.v.v.. If a "$\bar{K}$-pair" is occupied (i.e. a pair of orbitals $\varphi_\alpha, \bar{K}\varphi_\alpha$) but <u>not also</u> its time reversed image $(\hat{K}\varphi_\alpha, \hat{K}\bar{K}\varphi_\alpha)$, then a finite contribution to the spin–isospin density arises. The number of such "unsaturated $\bar{K}$-pairs" determines the strength of the spin–isospin density $\vec{\rho}_3$ ^{SI} and thus of the importance of the pion field in the nuclear state.[5)]

The $\bar{K}$-pairs are related to the spin–isospin lattice schemes which were proposed by various authors[6)], but also to much earlier work[7)] dedicated to the problem of incorporating np–pairing in the ordinary nuclear BCS theory which considers only pairing between neutrons and between protons.

The main features of the ordinary BCS theory can be represented in the simple quasi–spin model of Lipkin. In the next chapter we want to show that the $\bar{K}$-pairs can be used as building blocks of a very similar quasi–spin algebra. We shall show that the expectation value of certain generators of this group provides a useful measure of the pion source strength in very much the same way as the paring tensor determines the gap in the ordinary BCS theory.

2. The quasi-spin algebras

We denote the spin- and isospin-operators of particle i as usual by

$$\vec{s}(i) = \frac{1}{2}\,\vec{\sigma}(i) \tag{2.1}$$

and

$$\underset{\sim}{t}(i) = \frac{1}{2}\,\underset{\sim}{\tau}(i) \tag{2.1'}$$

where $\vec{\sigma}(i)$ and $\underset{\sim}{\tau}(i)$ are the Pauli matrices acting in the spin and isospin space of particle i respectively. Pauli matrices acting in the same particle space satisfy the relations:

$$\sigma_j\,\sigma_k = \sigma_l \tag{2.2}$$

and equally for the τ_j where j,k and l are any cyclic permutation of the cartesian components 1,2,3. We define raising and lowering operators by:

$$s_{\pm} = s_1 \pm i\,s_2 \tag{2.3}$$

$$t_{\pm} = t_1 \pm i\,t_2 \tag{2.3'}$$

and projection operators on spin up and down as well as on protons and neutrons by:

$$s_{\uparrow} = \frac{1}{2} + s_3\,; \quad s_{\downarrow} = \frac{1}{2} - s_3 \tag{2.4}$$

$$t_p = \frac{1}{2} + t_3\,; \quad t_n = \frac{1}{2} - t_3 \tag{2.4'}$$

The raising and lowering operators satisfy the well-known relations:

$$[s_+, s_-] = 2\,s_3 \tag{2.5}$$

$$[s_3, s_{\pm}] = \pm\,s_{\pm} \tag{2.6}$$

and equally for the corresponding operators in isospin space. We now define the following two sets of "quasi-spin operators" acting in the space of A-particle states:

$$S_{\pm}^{I} := \sum_{i=1}^{A} s_{\mp}(i)\,t_{\pm}(i) \tag{2.7}$$

$$S_3^{I} := \frac{1}{2}\sum_{i=1}^{A}\left[t_3(i) - s_3(i)\right] = \frac{1}{2}\sum_{i=1}^{A}\left[t_p(i)s_\downarrow(i) - t_n(i)s_\uparrow(i)\right] \tag{2.8}$$

$$S_\pm^{II} := \sum_{i=1}^{A} s_\pm(i)\, t_\pm(i) \tag{2.9}$$

$$S_3^{(II)} := \frac{1}{2}\sum_{i=1}^{A}\left[t_3(i) + s_3(i)\right] = \frac{1}{2}\sum_{i=1}^{A}\left[t_p(i)s_\uparrow(i) - t_n(i)s_\downarrow(i)\right] \tag{2.10}$$

Using the equations (2.5) and (2.6) one easily checks that the following commutation relations hold:

$$\left[S_+^{(I)}, S_-^{(I)}\right] = 2S_3^{(I)} \tag{2.11}$$

$$\left[S_3^{(I)}, S_\pm^{(I)}\right] = \pm S_\pm^{I} \tag{2.12}$$

and equally for the S^{II}–operators. Furthermore, one finds that commutators between I– and II–operators vanish. The operators $\vec{S}^{I}$ and $\vec{S}^{II}$ thus form two commuting quasi–spin algebras. The interesting aspect of these quasi–spin operators is that their building blocks are products of spin and isospin operators acting in the same particle space. The expectation values of these operators may thus serve as a measure of correlations between the directions in spin and isospin space.

In second quantization the operators have the following form:

$$S_+^{I} = \sum_i a^+_{i-\frac{1}{2}\frac{1}{2}}\, a_{i\frac{1}{2}-\frac{1}{2}}\ ; \qquad S_-^{I} = \left(S_+^{I}\right)^+ \tag{2.13}$$

$$S_3^{I} = \frac{1}{2}\sum_i\left[a^+_{i-\frac{1}{2}\frac{1}{2}}\, a_{i-\frac{1}{2}\frac{1}{2}} - a^+_{i\frac{1}{2}-\frac{1}{2}}\, a_{i\frac{1}{2}-\frac{1}{2}}\right] =: \frac{1}{2}\left[\hat{p}_\downarrow - \hat{n}_\uparrow\right] \tag{2.14}$$

$$S_+^{II} = \sum_i a^+_{i\frac{1}{2}\frac{1}{2}}\, a_{i-\frac{1}{2}-\frac{1}{2}}\ ; \qquad S_-^{II} = \left(S_+^{II}\right)^+ \tag{2.15}$$

$$S_3^{II} = \frac{1}{2}\sum_i\left[a^+_{i\frac{1}{2}\frac{1}{2}}\, a_{i\frac{1}{2}\frac{1}{2}} - a^+_{i-\frac{1}{2}-\frac{1}{2}}\, a_{i-\frac{1}{2}-\frac{1}{2}}\right] =: \frac{1}{2}\left[\hat{p}_\uparrow - \hat{n}_\downarrow\right] \tag{2.16}$$

Here, the symbol $a^+_{im_sm_t}$ is the creation operator for a particle with spin projection m_s and isospin projection m_t ($m_t = +\frac{1}{2}$ is a proton, $m_t = -\frac{1}{2}$ is a neutron). The operators $\hat{p}_\downarrow, \hat{p}_\uparrow, \hat{n}_\downarrow, \hat{n}_\uparrow$ which are introduced in the equations (2.14) and (2.16) count protons with spin down and up and neutrons with spin down and up, respectively.

The second quantized form of the generators (2.13) — (2.16) suggests the existence of two other quasi-spin algebras which contain the missing bilinear combinations of creation and annihilation operators. Indeed one finds still the following two closed algebras:

$$\mathcal{B}^{I}_{+} = \sum_i a^+_{i\frac{1}{2}-\frac{1}{2}}\, a^+_{i-\frac{1}{2}\frac{1}{2}}\ ; \qquad \mathcal{B}^{I}_{-} = (\mathcal{B}^{I}_{+})^+ \tag{2.17}$$

$$\mathcal{B}^{I}_{3} = \frac{1}{2}\sum_i\left[a^+_{i\frac{1}{2}-\frac{1}{2}}\, a_{i\frac{1}{2}-\frac{1}{2}} - a_{i-\frac{1}{2}\frac{1}{2}}\, a^+_{i-\frac{1}{2}\frac{1}{2}}\right] = \frac{1}{2}\left[\hat{n}_{\uparrow} + \hat{p}_{\downarrow} - N_0\right] \tag{2.18}$$

$$\mathcal{B}^{II}_{+} = \sum_i a^+_{i\frac{1}{2}\frac{1}{2}}\, a^+_{i-\frac{1}{2}-\frac{1}{2}}\ ; \qquad \mathcal{B}^{II}_{-} = (\mathcal{B}^{II}_{+})^+ \tag{2.19}$$

$$\mathcal{B}^{II}_{3} = \frac{1}{2}\sum_i\left[a^+_{i\frac{1}{2}\frac{1}{2}}\, a_{i\frac{1}{2}\frac{1}{2}} - a_{i-\frac{1}{2}-\frac{1}{2}}\, a^+_{i-\frac{1}{2}-\frac{1}{2}}\right] = \frac{1}{2}\left[\hat{p}_{\uparrow} + \hat{n}_{\downarrow} - N_0\right] \tag{2.20}$$

which again satisfy the commutation relations (2.11) and (2.12). Operators of different algebras commute with each other. N_0 is the number of quantum states i of the model.

The algebras of the $\vec{S}^{I}$– and $\vec{S}^{II}$–operators generate unitary transformations which are a subgroup of $SU(4)$. Another subgroup of $SU(4)$ is the product $SU(2) \times SU(2)$ of unitary transformations in spin– and isospin–space.

The S^I– and S^{II}– generators could be used as building blocks for a model Hamiltonian simulating Gamow–Teller modes in $N \neq Z$ nuclei.

We performed HF calculations in $^{32}_{16}S_{16}$ using the Gogny interaction D1 plus the OPE–tensor interaction.[5] We found a relatively low–lying configuration which could be interpreted as two $\bar{K}$–pairs built on an oblate Si–core. This configuration generated a non–vanishing source for the pion field.

If one wants to describe the addition of a $\bar{K}$–pair by the action of a raising operator of a quasi–spin algebra, we have to generalize the B^I– and B^{II}– operators by replacing the magnetic quantum numbers of spin by the ones of the total angular momentum, thereby producing commuting algebras of $B^I_{(m)}$ and $B^{II}_{(m)}$ operators for each given absolute value of the magnetic quantum number m:

$$\mathcal{B}^{I}_{|m|+} = \sum_i a^+_{i\,m-\frac{1}{2}}\, a^+_{i\,-m\,\frac{1}{2}}\ ; \qquad \mathcal{B}^{I}_{|m|-} = \left(\mathcal{B}^{I}_{|m|+}\right)^+ \tag{2.21}$$

$$\mathcal{B}^{I}_{(m)3} := \frac{1}{2}\sum_i \left[a^{+}_{im-\frac{1}{2}} a_{im-\frac{1}{2}} - a_{i-m\frac{1}{2}} a^{+}_{i-m\frac{1}{2}} \right] \tag{2.22}$$

$$\mathcal{B}^{II}_{(m)+} := \sum_i a^{+}_{im\frac{1}{2}} a^{+}_{i-m-\frac{1}{2}} \;;\; \mathcal{B}^{II}_{(m)-} = \left(\mathcal{B}^{II}_{(m)+}\right)^{+} \tag{2.23}$$

$$\mathcal{B}^{II}_{(m)3} := \frac{1}{2}\sum_i \left[a^{+}_{im\frac{1}{2}} a_{im\frac{1}{2}} - a_{i-m-\frac{1}{2}} a^{+}_{i-m-\frac{1}{2}} \right] \tag{2.24}$$

If the generators are summed over all the m-values they satisfy an algebra which is discussed in ref.8, §5.5. We note, however, that the raising operator of this algebra would generate states with a vanishing pion source. If one wants to obtain states producing a non-vanishing pion source one must sum the generators $B^{I}_{(m)\nu}$ and $B^{II}_{(m)\nu}$ over magnetic quantum numbers of one sign only ($\nu = +, -, 3$):

$$\mathcal{B}^{I}_{\nu} := \sum_{m>0} \mathcal{B}^{I}_{(m)\nu} \tag{2.25}$$

$$\mathcal{B}^{II}_{\nu} := \sum_{m>0} \mathcal{B}^{II}_{(m)\nu} \tag{2.26}$$

The generators $B^{I,II}_{(m)\nu}$ as well as $B^{I,II}_{\nu}$ do not commute with the generators of ordinary rotations, whereas the operators (5.25) defined in ref.8 are rotationally invariant quantities. The rotational invariance is not important and perhaps not even desirable in a nucleus where the neutron and proton numbers are far from shell closure and where we thus are faced with deformed nuclei. It is in this region where we may expect a favorable situation for the occurrence of spin–isospin lattice order and a finite pion field source.

We note in passing that the analogous generalization can be obtained for the S^{I}– and S^{II}– algebras.

In very much the same way as the Hamiltonian of the Lipkin model can be formulated as a function of the operators defined in ref.8, §5.5, we may write down the following model Hamiltonian using the generators of the B^{I}– and B^{II}– algebra (equations (2.21) — (2.26)):

$$H_0 = \epsilon\left[\mathcal{B}^{I}_{3} + \mathcal{B}^{II}_{3} + \Omega\right] + G_{np}\left[\mathcal{B}^{I}_{+}\mathcal{B}^{I}_{-} + \mathcal{B}^{II}_{+}\mathcal{B}^{II}_{-}\right] \tag{2.27}$$

G_{np} is an average matrix element of the neutron–proton interaction and ϵ is the energy of a degenerate level. Ω is the summed degeneracy of the subspaces of given m:

$$\Omega = \sum_{m>0} N_{0(m)} \tag{2.28}$$

The operator $B_3^I + B_3^{II} + \Omega$ counts the number of pairs in the degenerate level.

The main defect of the Hamiltonian H_0 is that it contains only an interaction between neutrons and protons, and none between identical particles. It is thus also not invariant with respect to rotations in isospin space. Since we want to deal with nuclei whose neutron and proton numbers are both far from shell closure, the $nn-$ and $pp-$ interactions are as important as the np–interaction. Therefore, a more realistic model Hamiltonian would be:

$$H = H_0 + G_{nn} C_+^{(n)} C_-^{(n)} + G_{pp} C_+^{(p)} C_-^{(p)} \tag{2.29}$$

where $C_\pm^{(n)}$ and $C_\pm^{(p)}$ are defined as:

$$C_+^{(n)} = \sum_{i,m>0} a^+_{i\,m-\frac{1}{2}}\, a^+_{i\,-m-\frac{1}{2}} \; ; \quad C_-^{(n)} = \left(C_+^{(n)}\right)^+ \tag{2.30}$$

$$C_+^{(p)} = \sum_{i,m>0} a^+_{i\,m\frac{1}{2}}\, a^+_{i\,-m\frac{1}{2}} \; ; \quad C_-^{(p)} = \left(C_+^{(p)}\right)^+ \tag{2.30'}$$

The $C_\pm$–operators are the raising and lowering operators of quasi–spin algebras whose third components are given by:

$$C_3^{(n)} = \frac{1}{2}\left(\hat{N} - \Omega\right) ; \quad \hat{N} = \hat{n}_+ + \hat{n}_+ \tag{2.31}$$

$$C_3^{(p)} = \frac{1}{2}\left(\hat{Z} - \Omega\right) ; \quad \hat{Z} = \hat{p}_+ + \hat{p}_+ \tag{2.31'}$$

They are the usual Lipkin algebras used to describe pairing between identical nucleons. For $G_{nn} = G_{pp} = G_{np}$, the Hamiltonian (2.29) is invariant with respect to rotations in isospin space. We did not yet try to diagonalize the Hamiltonian (2.29). This is a non-trivial task since the $B-$ and $C-$ operators do not in general commute with each other. Instead, we used the raising operators $B^{I,II}_{(m)+}$ to generate spin–isospin correlated trial wave functions for subsequent selfconsistent HF-calculations. We used

Gogny's phenomenological density-dependent 2-body force D1[9] to which we added a tensor force $\hat{V}_T^{(\pi)}$ of the OPE-type:

$$\hat{V}_T^{(\pi)}(\vec{r}) = V_T^{(\pi)}\left[1+\frac{3}{\mu_\pi r}+\frac{3}{(\mu_\pi r)^2}\right]\frac{e^{-\mu_\pi r}}{\mu_\pi r}\cdot\hat{O}$$

$$\hat{O} = \sqrt{\frac{5}{32\pi}}\left\{\frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^2}-\vec{\sigma}_1\cdot\vec{\sigma}_2\right\}\underset{\sim}{\tau}_1\cdot\underset{\sim}{\tau}_2 \qquad (2.32)$$

$$\mu_\pi = m_\pi c/\hbar$$

In ref.5 we also investigated the effect of an additional tensor interaction due to ρ-exchange, which we found to be small. In all our HF-calculations we conserved parity and axial symmetry. Parity and the magnetic quantum numbers of the total angular momentum are therefore good quantum numbers. We generated a spin-isospin correlated excited state of ^{32}S by acting twice in the positive parity subspace with the raising operators $B^I_{(m)+}$ on an oblate ^{28}Si core.:

$$|^{32}S\rangle^* = B^I_{(\frac{1}{2})+}\,B^I_{(\frac{1}{2})+}\,|^{28}Si\rangle \qquad (2.33)$$

We minimised the energy of this excited state for various values of the tensor strength $V_T^{(\pi)}$. A convenient measure for the spin-isospin order of the resulting selfconsistent HF-solutions is the expectation value of the spin-isospin density (1.2), whose 3-3-component is proportional to the expectation value of the operator:

$$\hat{P} := B_3^I - B_3^{II} \qquad (2.34)$$

It measures the surplus of pairs of type I over those of type II.

Fig.1 shows the quantity $< \hat{P} >$ as a function of $V_T^{(\pi)}$ for the excited ^{32}S solution.

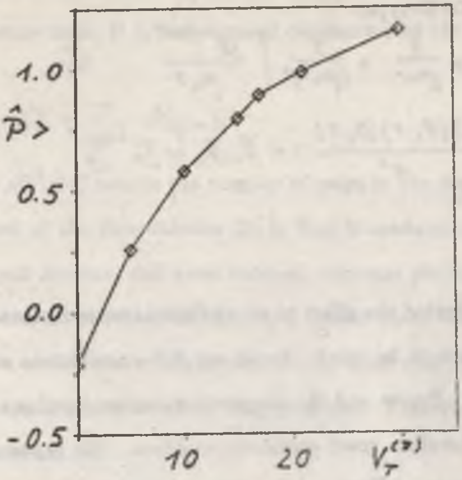

Fig.1: The measure $< \hat{P} >$ of the spin–isospin order in the excited state of ^{32}S as a function of the OPE tensor strength.

It can be seen that the tensor force indeed favors the ordering in $\hat{K}$–pairs. Our HF–calculations did not take nn– and pp– pairing effects into account. Perhaps the Hamiltonian (2.29) could serve as a simple model for a qualitative understanding of the competition between ordinary pairing correlations and the spin–isospin coherence of the $\hat{K}$–type.

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References

1. Migdal A. B.: ZhETF 1971, 61, 2209–2224, JETP 1972, 34, 1184–1191, Rev. Mod. Phys. 1978, 50, 107–172.

2. Blümel R. and Dietrich K.: Nucl. Phys. 1986, A454, 691–719.

3. Weise W. and Brown G. E.: Phys. Rep. 1976, C27, 1–34,
Meyer-ter-Vehn J.: Phys. Rep. 1981, 74, 323–378,
Dickhoff W. H., Faessler A., Meyer-ter-Vehn J. and Müther H.: Phys. Rev. 1981, C23, 1154–1173 and Nucl. Phys. 1981, A368, 445–476.

4. Escudie J.-L., Austin S. M., Boudard A., Bruge G., Chaumeaux A., Farvacque L., Legrand D., Lugol J. C., Mayer B., Belery P., Debevec P. T., Delbar T., Deutsch J., Gregoire G., Prieels R., Cameron J. M., Glashausser C. and Whitten C. A.: Phys. Rev. 1981, C24, 792–795,
Delorme J.: Nucl. Phys. 1982, A374, 541c–555c.

5. Blümel R. and Dietrich K.: Int. Conference and Symposium on Unified Concepts of Many Body Problems, Stony Brook, Sept. 4–6 1986, and preprint 1987, submitted to Nucl. Phys. A.

6. Tamagaki R.: Nucl. Phys. 1979, A328, 352–380,
Palumbo F.: Proc. of the Int. School of Physics Enrico Fermi, Course LXXIX, 408–
Do Dang G.: Phys. Rev. Lett. 1979, 43, 1708–1711.

7. Camiz P., Covello A. and Jean M.: Nuovo Cim. 1965, 36, 663–668 and Nuovo Cim. 1966, 42, 199–237,
Wolter H. H., Faessler A. and Sauer P. U.: Nucl. Phys. 1971, A167, 108–128.

8. Lipkin H. J.: Lie Groups for Pedestrians, North Holland, Amsterdam 1965.

9. Gogny D.: Proc. Int. Conf. on Nucl. Phys., Munich, Aug. 27 – Sept. 1, vol 1, contr. paper 2–12, 48 , and in "Nuclear Selfconsistent Fields", Ripka G. and Porneuf M., ed., North Holland 1975, p. 333– 352.

STRESZCZENIE

Badamy różne algebry spinowe, w których operatory podnoszenia i obniżania zawierają pary neutronów i protonów o ściśle określonych magnetycznych liczbach kwantowych. Pokazujemy, że algebry te mogą być przydatne do opisu pewnych stanów wzbudzonych jądra, scharakteryzowanych określoną siatką w przestrzeni spin-izospin i będących źródłem pola pionów.

Р Е З Ю М Е

Рассматривались разные спиновые алгебры, в которых операторы повышения и снижения включают пары нейтронов и протонов с точно определенными магнитными квантовыми числами. В работе показывается, что эти алгебры могут быть пригодными при описании некоторых возбужденных ядерных состояний, характеризующихся определенной сеткой в спин-изоспинновом пространстве, и являющихся источником пионного поля.