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Exact Diagonalisation of Pairing Interactions for Protons and Neutrons in j-j Coupling. II. Odd Nuclei, $d_{3/2} — f_{7/2}$ Shells

Dokładna diagonalizacja oddziaływania pairing dla protonów i neutronów w sprzężeniu j-j. II. Jądra nieparzyste, powłoki $d_{3/2} — f_{7/2}$

Точная диагонализация парного взаимодействия для протонов и нейтронов в связи $j—j$. II. Нечетные ядра, уровни $d_{3/2}—f_{7/2}$

In this work we follow the notation, the introduction, and references of the paper [1]. The main difference is that we consider the odd nuclei with initial seniority equal to one, and this makes the problem more difficult as compared with seniority zero states.

The first part of the paper presents the results of exact calculations of the energy levels of the Hamiltonian with pairing forces. The second part is devoted to the group-theory discussion of a chosen example.

I. ENERGY LEVELS OF THE PAIRING HAMILTONIAN

For the odd number of nucleons displaced on two levels there are two groups of states which are not mixed by the pairing Hamiltonian [2]. They are the states with unpaired particle on the higher and on the lower level. Numbers of linearly independent states for several numbers of nucleons, with unpaired particles on the higher and lower level, are given in Table 1.

Following the remarks of the paper [1] and the calculated matrix elements of all the operators constructing the pairing Hamiltonian [3], we can calculate matrixes H in all cases under consideration. In order to diminish dimensions of H, we change the basis into the states of definite total isospins T. For example, the matrix 35×35 (nine nucleons

Table 1. Dimensions of the matrix H for several numbers of particles in both cases: an unpaired particle on the higher and on the lower level (7/2 and 3/2)

Total n	3		5		7		9	
	higher	lower	higher	lower	higher	lower	higher	lower
Dim. of H	4	4	11	10	22	18	35	28

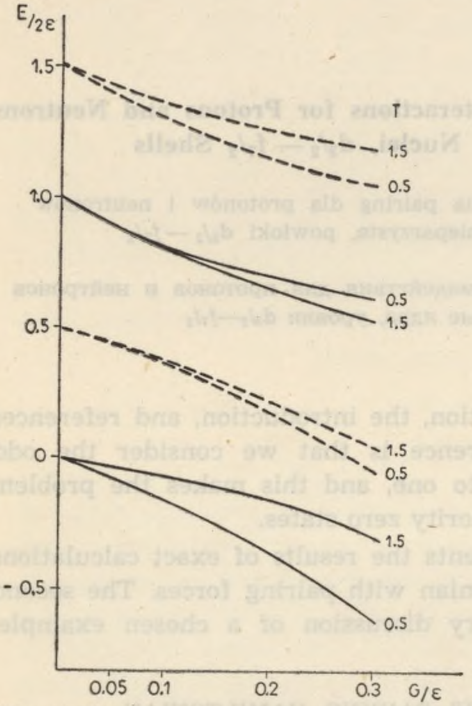

Fig. 1. Energies of the ground and excited states for three nucleons on $j_1=3/2$ and $j_2=7/2$ levels correlated by pairing forces, as function of the strength parameter G. ε is one particle energy of the 7/2 level relatively to the 3/2 level taken as a zero-point energy. Continuous lines are for an unpaired particle on the lower level, and broken lines — for an unpaired particle on the higher level

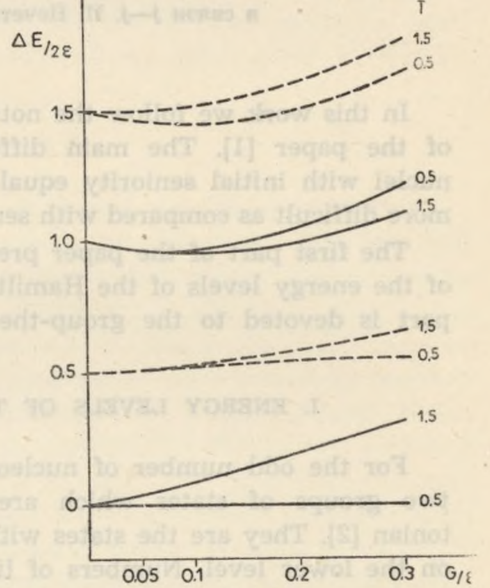

Fig. 2. The same enregies as in Fig. 1 but relatively to the ground state energy taken as $E_0=0$

with unpaired one on the higher, 7/2, level) changes into quasi-diagonal shape with several matrixes of lower dimensions (Table 2).

Transformations have been done with the help of Clebsch-Gordan

Table 2. The problem 35×35 decays into five problems of lower dimensions and with definite isospin T for nine nucleons with unpaired one on the higher, 7/2, level

Total T	0.5	1.5	2.5	3.5	4.5
Dim. of H	9	10	9	5	2

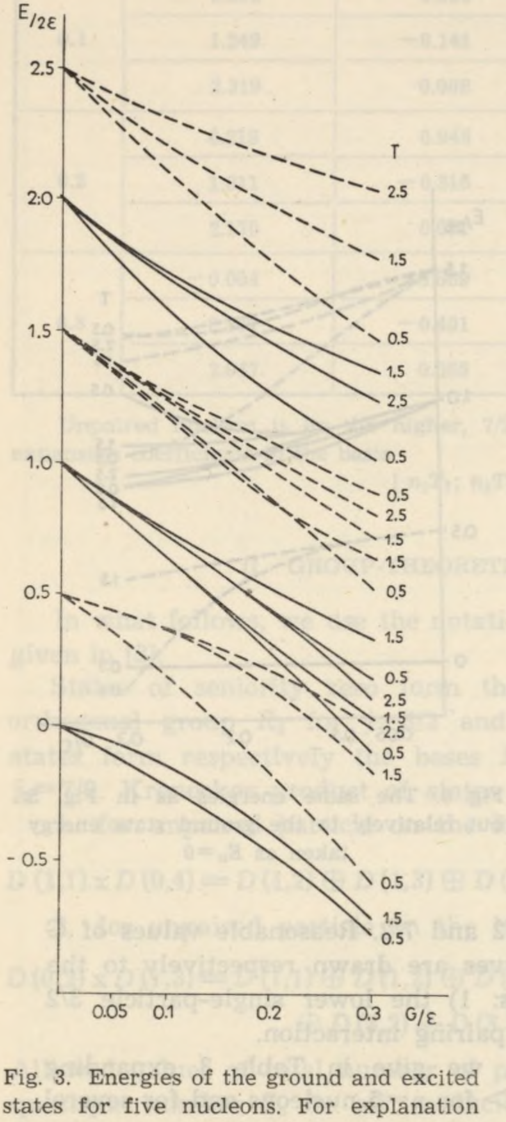

Fig. 3. Energies of the ground and excited states for five nucleons. For explanation see Fig. 1

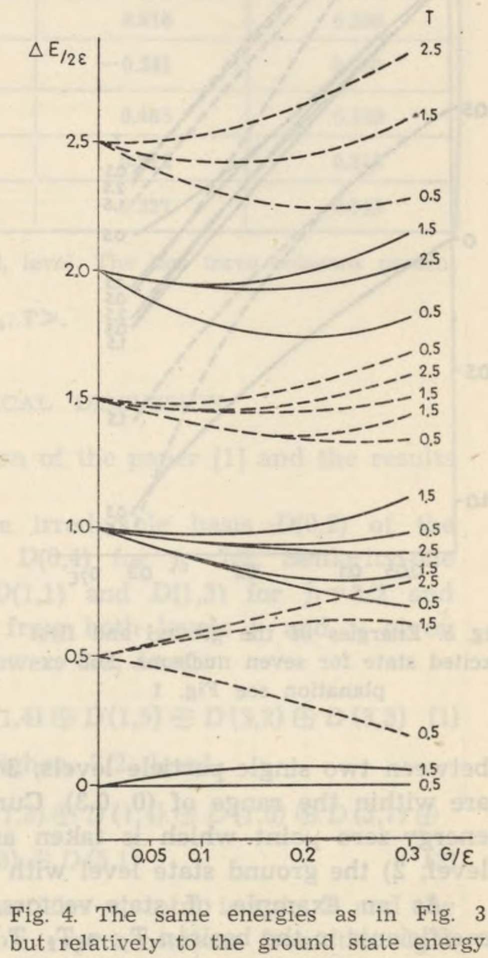

Fig. 4. The same energies as in Fig. 3 but relatively to the ground state energy taken as $E_0=0$

coefficients because the isospin group is the group SU_2 (or R_3). In figures 1—6 we present energies of the ground and excited states as functions of the strength parameter G in units ε, where ε is the energy difference

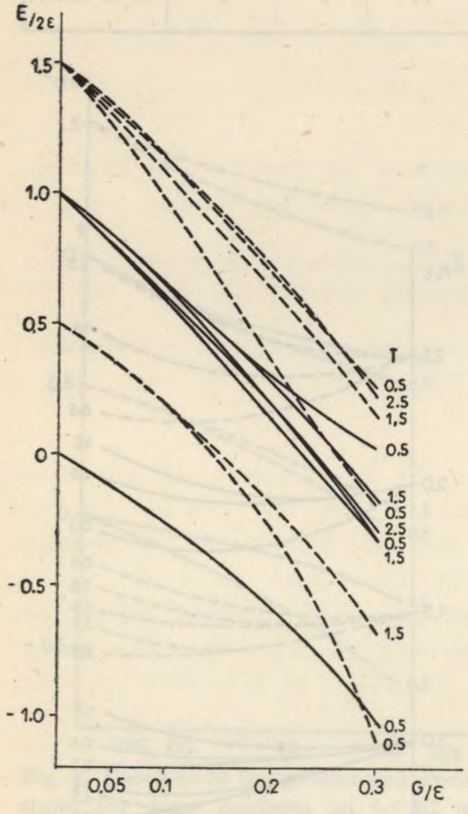

Fig. 5. Energies of the ground and first excited state for seven nucleons. For explanation see Fig. 1

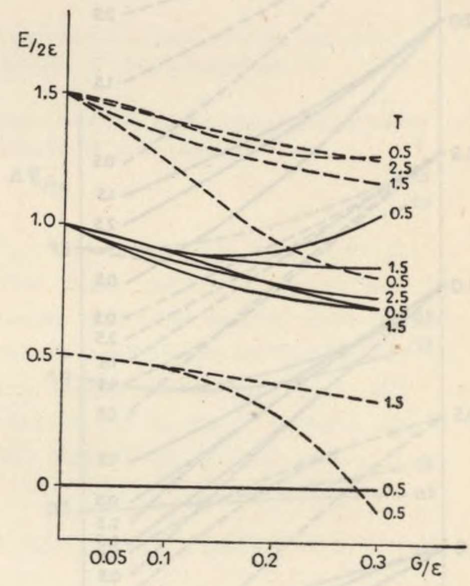

Fig. 6. The same energies as in Fig. 5 but relatively to the ground state energy taken as $E_0 = 0$

between two single-particle levels, 3/2 and 7/2. Reasonable values of G are within the range of (0, 0.3). Curves are drawn respectively to the energy-żero point which is taken as: 1) the lower single-particle 3/2 level; 2) the ground state level with pairing interaction.

As an example of state-vectors, we give in Table 3 expanding coefficients in the basis $|n_1T_1;\, n_2T_2;\, T>$ for $n=5$ nucleons and for several values of G/ε.

Table 3. State-vectors for five nucleons with $T = 5/2$

G/ε	$E/2\varepsilon$	$\|42; 1^1/_2; ^5/_2>$	$\|21; 3^3/_2; ^5/_2>$	$\|00; 5^5/_2; ^5/_1>$
	0.446	0.998	0.066	0.002
0.05	1.374	−0.066	0.995	0.069
	2.405	0.002	−0.069	0.998
	0.382	0.990	0.141	0.010
0.1	1.249	−0.141	0.981	0.132
	2.319	0.008	−0.132	0.991
	0.219	0.948	0.314	0.047
0.2	1.011	−0.316	0.918	0.238
	2.170	0.031	−0.241	0.970
	−0.004	0.869	0.483	0.108
0.3	0.807	−0.491	0.812	0.315
	2.047	0.065	−0.327	0.943

Unpaired nucleon is on the higher, 7/2, level. The last three columns present expansion coefficients in the basis

$$|n_1T_1; n_2T_2; T>.$$

II. GROUP-THEORETICAL DISCUSSION

In what follows, we use the notation of the paper [1] and the results given in [3].

States of seniority zero form the irreducible basis $D(0,2)$ of the orthogonal group R_5 for $j=3/2$ and $D(0,4)$ for $j_2=7/2$. Seniority-one states form respectively the bases $D(1,1)$ and $D(1,3)$ for $j_1=3/2$ and $j_2=7/2$. Kronecker product of states from both levels j_1 and j_2 gives:

1. for unpaired particle on the lower, 3/2, level

$$D(1,1) \times D(0,4) = D(1,2) \oplus D(1,3) \oplus D(1,4) \oplus D(1,5) \oplus D(3,2) \oplus D(3,3) \quad (1)$$

2. for unpaired particle on the higher, 7/2, level

$$D(0,2) \times D(1,3) = D(1,1) \oplus D(1,2) \oplus D(1,3) \oplus D(1,4) \oplus D(1,5) \oplus D(3,1) \oplus$$
$$\oplus D(3,2) + D(3,3) \oplus D(5,1) \quad (2)$$

Allowed values of total number of particles n, total isospin T and the quasispin number S_0 for all irreducible representations from the right hand side of (1) and (2) are given in Table 4.

Table 4. Allowed values of T for irreducible representations $D\ (\lambda_1\lambda_2)$ in Clebsch-Gordan series (1) and (2)

$D\ (\lambda_1, \lambda_2)$	n		$\pm S_0$	T					
(1,1)	1	7	1.5	0.5					
	3	5	0.5	0.5	1.5				
(1,2)	1	11	2.5	0.5					
	3	9	1.5	0.5	1.5				
	5	7	0.5	0.5	1.5	2.5			
(1,3)	1	15	3.5	0.5					
	3	13	2.5	0.5	1.5				
	5	11	1.5	0.5	1.5	2.5			
	7	9	0.5	0.5	1.5	2.5	3.5		
(1,4)	1	19	4.5	0.5					
	3	17	3.5	0.5	1.5				
	5	15	2.5	0.5	1.5	2.5			
	7	13	1.5	0.5	1.5	2.5	3.5		
	9	11	0.5	0.5	1.5	2.5	3.5	4.5	
(1,5)	1	23	5.5	0.5					
	3	21	4.5	0.5	1.5				
	5	19	3.5	0.5	1.5	2.5			
	7	17	2.5	0.5	1.5	2.5	3.5		
	9	15	1.5	0.5	1.5	2.5	3.5	4.5	
	11	13	0.5	0.5	1.5	2.5	3.5	4.5	5.5
(3,1)	3	13	2.5		1.5				
	5	11	1.5	0.5	1.5	2.5			
	7	9	0.5	0.5	1.5 (2×)	2.5			
(3,2)	3	17	3.5		1.5				
	5	15	2.5	0.5	1.5	2.5			
	7	13	1.5	0.5	1.5 (2×)	2.5	3.5		
	9	11	0.5	0.5	1.5 (2×)	2.5 (2×)	3.5		
(3,3)	3	21	4.5		1.5				
	5	19	3.5	0.5	1.5	2.5			
	7	17	2.5	0.5	1.5 (2×)	2.5	3.5		
	9	15	1.5	0.5	1.5 (2×)	2.5 (2×)	3.5	4.5	
	11	13	0.5	0.5	1.5 (2×)	2.5 (2×)	3.5 (2×)	4.5	
(5,1)	5	19	3.5			2.5			
	7	17	2.5		1.5	2.5	3.5		
	9	15	1.5	0.5	1.5	2.5 (2×)	3.5		
	11	13	0.5	0.5	1.5 (2×)	2.5 (2×)	3.5		

For further discussion let us take states with $n=7$ nucleons. They are constructed from states of j_1 and j_2 levels given in Tables 5 and 6 for both cases of unpaired particle. Assigning to each total T its group--theory (R_5) specification we notice that the total S_o for $n=7$ nucleons and for $j_1=3/2$, $j_2=7/2$ is equal to -2.5. Because S_o is an additive

Table 5. Isospin coupling rules for $n = 7$ nucleons with an unpaired one on the lower, $j = 3/2$, level

D (1,1)		D (0,4)		T
n_1	T_1	n_2	T_2	
7	0.5	0	0	0.5
5	0.5 1.5	2	1	0.5 (2×) 1.5 (2×) 2.5
3	0.5 1.5	4	0 2	0.5 (2×) 1.5 (3×) 2.5 (2×) 3.5
1	0.5	6	1 3	0.5 1.5 2.5 3.5

quantum number, we have to choose the states belonging to the irreducible representation of Clebsch-Gordan series (1) and (2) and with $S_o=-2.5$ for both cases of unpaired particle. In such a way we divide all allowed values of T (Tables 5 and 6) into the sets of given irreducible representations of the orthogonal group R_5 (Tables 7 and 8).

Table 6. The same as in Table 5 but with an unpaired nucleon on the higher, $j = 7/2$, level

D (0,2)		D (1,3)		T
n_1	T_1	n_2	T_2	
6	1	1	0.5	0.5 1.5
4	0 2	3	0.5 1.5	0.5 (2×) 1.5 (3×) 2.5 (2×) 3.5
2	1	5	0.5 1.5 2.5	0.5 (2×) 1.5 2.5 (2×) 3.5
0	0	7	0.5 1.5 2.5 3.5	0.5 1.5 2.5 3.5

Similar conclusions as in the work [1] can be drawn here. Starting with seniority-one states we have reached, after taking Kronecker product for two levels j_1 and j_2, the sets of states transforming under the R_5 group like the states of seniority 1, 3, 5, and 7. Seniority number is not an additive guantum number and has to be defined by transformation properties of states under the orthogonal group R_5.

Table 7. Classification of states given in Table 5 according to quantum numbers provided by the orthogonal group R_5

(λ_1, λ_2)	S	t	T			
(1,2)	7	0.5	0.5			
(1,3)	5	0.5	0.5	1.5		
(1,4)	3	0.5	0.5	1.5	2.5	
(1,5)	1	0.5	0.5	1.5	2.5	3.5
(3,2)	5	1.5	0.5	1.5	2.5	
(3,3)	3	1.5	0.5	1.5 (2×)	2.5	3.5

The ground state is the state with unpaired particle on lower, 3/2, level, and with the lowest seniority $s=1$ and lowest total isospin $T=1/2$, as is seen from Fig. 5—6.

Table 8. Classification of states given in Table 6 according to quantum numbers provided by the orthogonal group R_5

(λ_1, λ_2)	S	t	T			
(1,1)	—	—	—			
(1,2)	7	0.5	0.5			
(1,3)	5	0.5	0.5	1.5		
(1,4)	3	0.5	0.5	1.5	2.5	
(1,5)	1	0.5	0.5	1.5	2.5	3.5
(3,1)	7	1.5		1.5		
(3,2)	5	1.5	0.5	1.5	2.5	
(3,3)	3	1.5	0.5	1.5 (2×)	2.5	3.5
(5,1)	5	2.5		1.5	2.5	3.5

Similar considerations can be performed for states with other number of particles.

Numerical calculations were done on the Odra 1013 computer in The Computer Centre of University M. Curie-Skłodowska in Lublin

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STRESZCZENIE

W oparciu o wyliczone wcześniej elementy macierzowe operatorów wchodzących w skład hamiltonianu *pairing*, dokonano diagonalizacji tego hamiltonianu dla przypadku nieparzystej liczby nukleonów na poziomach $d_{3/2}$ oraz $f_{7/2}$. Zagadnienie rozpada się na dwa przypadki niezwiązane siłami *pairing*, a mianowicie na przypadek nieparzystego nukleonu na niższym poziomie i na poziomie wyższym. Otrzymane energie wyrażone zostały w zależności od stałej sprzężenia, całkowitego izospinu oraz liczby cząstek. Dla przypadku 7 cząstek dokonano szczegółowej analizy teriogrupowej, klasyfikując stany o określonym izospinie według nieprzywiedlnych reprezentacji ortogonalnej grupy R_5. Wychodząc z układu cząstek o *seniority* 0 na jednym poziomie, a *seniority* 1 na poziomie drugim, otrzymano poprzez rozłożenie iloczynu Kroneckera dwóch nieprzywiedlnych reprezentacji grupy R_5 stany o *seniority* 1, 3, 5, ... Wskazuje to na nieaddytywny charakter liczby kwantowej *seniority*, która zależy raczej od własności transformacyjnych stanów względem grupy R_5.

РЕЗЮМЕ

Опираясь на раньше вычисленные матричные элементы операторов, входящих в состав гамильтониана для модели оболочек с парными силами, диагонализировали этот гамильтониан для случая нечетного числа протонов и нейтронов на уровнях $d_{3/2}$ и $f_{7/2}$. Эта проблема состоит из двух случаев несвязанных парными силами, а именно: из случая нечетного нуклеона на более низком уровне и на более высоком уровне. Полученные энергии были выражены в зависимости от постоянного сопряжения, постоянного изоспина и числа частиц. Для случая семи частиц сделали подробный теоретически групповой анализ, классифицируя состояния с определенным изоспином по неприводимым представлениям ортогональной группы R_5. Исходя из системы частиц с сеньоритом 0 на одном уровне и сеньоритом 1 на втором уровне, получили, используя разложение произведения Кронекера двух неприводимых представлений, группы R_5, состояния с сеньоритами 1, 3, 5 ... Это указывает, что квантовое число сеньорита неаддитивно и зависит от трансформационных свойств состояний, согласно группе R_5.

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