

Z Katedry Chemii Nieorganicznej Wydziału Farmaceutycznego AM w Lublinie  
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### Some Differences and Analogies in Linear, Radial and Reversed Phase Chromatography

Różnice i analogie w chromatografii liniowej, krążkowej  
i odwróconych faz

Различия и аналогии в хроматографии линейной, радиальной  
и обращенными фазами

Making the assumptions of the plate theory (modified correspondingly for radial chromatography: the cross-sectional areas of the two liquid phases are proportional to their distances from the center of the circle), and assuming that in all three techniques the same organic and water phases are used, at constant ratio of their cross-sectional areas ( $r$ ), we may write the following equations concerning the relative distances travelled by a given substance in linear ( $R_{FL}$ ), radial ( $R_{FC}$ ) and reversed phase chromatography ( $R_{FR}$ ):

$$R_{FC} = \sqrt{R_{FL}}$$

$$R_{FR} = 1 - R_{FL} \quad (1)$$

Therefore, for buffered paper chromatography [1] we have:

$$\text{CC} \quad R_{FC} = \sqrt{\frac{kr}{kr + 1 + 10^{\pm(pH - pK_A)}}}$$

$$\text{LC} \quad R_{FL} = \frac{kr}{kr + 1 + 10^{\pm(pH - pK_A)}} \quad (2)$$

$$\text{RC} \quad R_{FR} = 1 - \frac{kr}{kr + 1 + 10^{\pm(pH - pK_A)}}$$



(upper sign in the power refers to acids, lower — to bases, for which  $pK_A = pK_W - pK_B$ ).

Thus, in all three techniques the relative rate of migration ( $R_F$ ) is a function of four parameters:

1. The partition coefficient  $k$  (ratio of concentrations of unionized substance in organic and water phase, at equilibrium,  $k = c_{org}/c_w$ ).
2. The volume coefficient  $r$  (ratio) of cross-sectional areas of organic and water phase, or their volumes in a sufficiently short section of the system) ( $kr$  — partition number of unionized electrolyte).
3. The ionization constant.
4. The  $pH$  of water phase.

By mathematical analysis of the function (2) it is possible to anticipate the theoretical effect of the four parameters and their variations upon the position and sequence of  $R_F = f(pH)$  curves ( $R_F = R_{FL}$ ,  $R_{FC}$  or  $R_{FR}$ ), the range of changes of  $R_F$ , the  $pH$  of optimal separation etc. The knowledge of these relations may simplify and make easier the choice of optimal conditions of separation and the estimation of its efficiency; moreover, the results of the remaining techniques may be predicted on the basis of the results of either of these techniques; and the value of any of the four parameters of distribution may be calculated when the remaining parameters are known.

#### EFFECT OF PARTITION NUMBER OF UNIONIZED ELECTROLYTE $kr$

The effect of the product  $kr$  upon  $R_F = f(pH)$  curves in the three different chromatographic techniques is illustrated in Fig. 1 ( $K_A = \text{const.}$ ). The partition number influences both the shape and position of  $R_F = f(pH)$  curves.

The height of a curve increases with  $kr$  and is equal to  $\frac{kr}{kr+1}$  (comp. form. 2) in case of linear and reversed phase chromatography ( $R_F$  vs.  $pH$  curves in reversed phase chromatography may be obtained by turning  $R_{FL}$  vs.  $pH$  curves by  $180^\circ$  around a horizontal axis passing through  $R_F = 0.5$ ). In radial chromatography the height of a curve is equal to  $\sqrt{\frac{kr}{kr+1}}$ ; thus  $R_{FC}$  vs.  $pH$  curves are always higher than  $R_{FL}$  vs.  $pH$  curves and the difference is more pronounced at lower values of  $kr$ . At  $kr$  values higher than 100 the height of  $R_F$  vs.  $pH$  curves practically does not increase any more with  $kr$  ( $R_F = 1$ ) and the shape is already invariable: with the increase of  $kr$  the curves are only shifted along the  $pH$  axis, an  $n$ -fold increase in  $kr$  causing a shift by  $\log n$   $pH$  units:



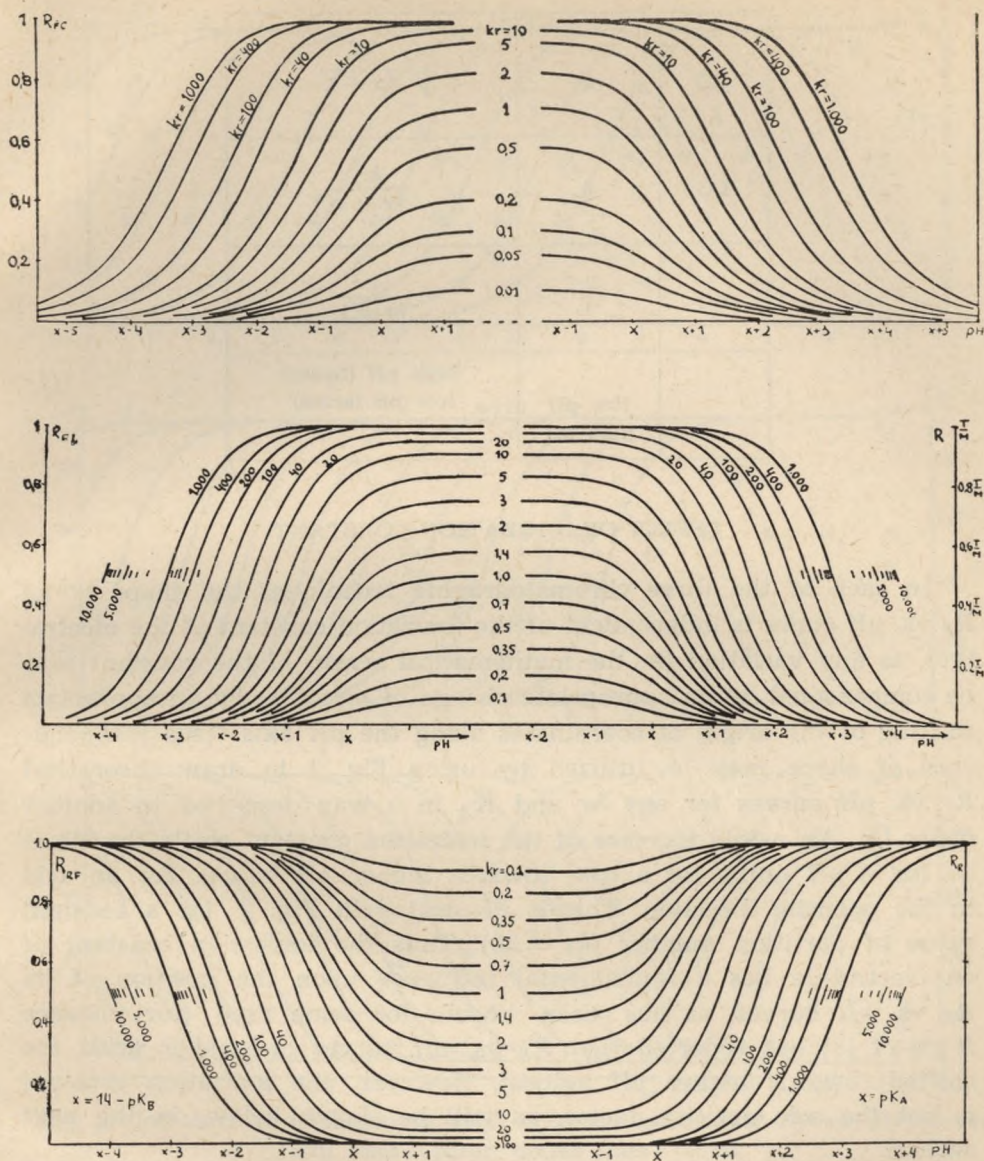


Fig. 1. Influence of partition number upon the shape and position of  $R_F$  vs.  $pH$  curves;  $R_{FC}$  — radial,  $R_{FL}$  — linear (conventional),  $R_{RF}$  — reversed phase chromatography

bases — towards lower  $pH$  values, acids — in the opposite direction (comp. form. 3).

By appropriate variations of  $pH$  one can change the  $R_F$  values within the following ranges:



CC	0	$\sqrt{\frac{kr}{kr+1}}$
LC	0	$\frac{kr}{kr+1}$
RC	1	$\frac{1}{kr+1}$
	low <i>pH</i> high <i>pH</i>	high <i>pH</i> (bases) low <i>pH</i> (acids)

## EFFECT OF IONIZATION CONSTANT

In each of the three chromatographic techniques the shape of an  $R_F$  vs.  $pH$  curve is independent of the ionization constant of the electrolyte, as any variation (in the mathematical sense) of the constant may be compensated by an appropriate change of  $pH$ , i. e., by an appropriate shifting of the origin of coordinates along the  $pH$  axis. This independence of shape may be utilized by using Fig. 1 to draw theoretical  $R_F$  vs.  $pH$  curves for any  $kr$  and  $K_A$  in a way described in another paper [2]. An  $n$ -fold increase of the ionization constant shifts the curve by  $\log n$   $pH$  units; for a base towards higher  $pH$  values, for an acid in the opposite direction. This is illustrated in Fig. 2 for a constant value of partition number ( $kr = 20$ ). Thus the ionization constant of an electrolyte has a fundamental influence upon the position of its  $R_F$  vs.  $pH$  curves, as has been known for some time (for instance Rybař [3] has reported that  $R_F$  vs.  $pH$  curves of weaker acids are shifted towards higher  $pH$  values). However, the ionization constant is not the sole decisive factor, as will be shown below, in the next section.

THE JOINT EFFECT OF IONIZATION CONSTANT AND PARTITION NUMBER UPON THE POSITION AND SEQUENCE OF  $R_F$  VS.  $pH$  CURVES

In case of linear chromatography (conventional or reversed phase) the position of an  $R_F$  vs.  $pH$  curve is most conveniently defined by its inflexion point [4], which lies at its half-height, and, as can be easily demonstrated, is the center of its axial symmetry. From the condition

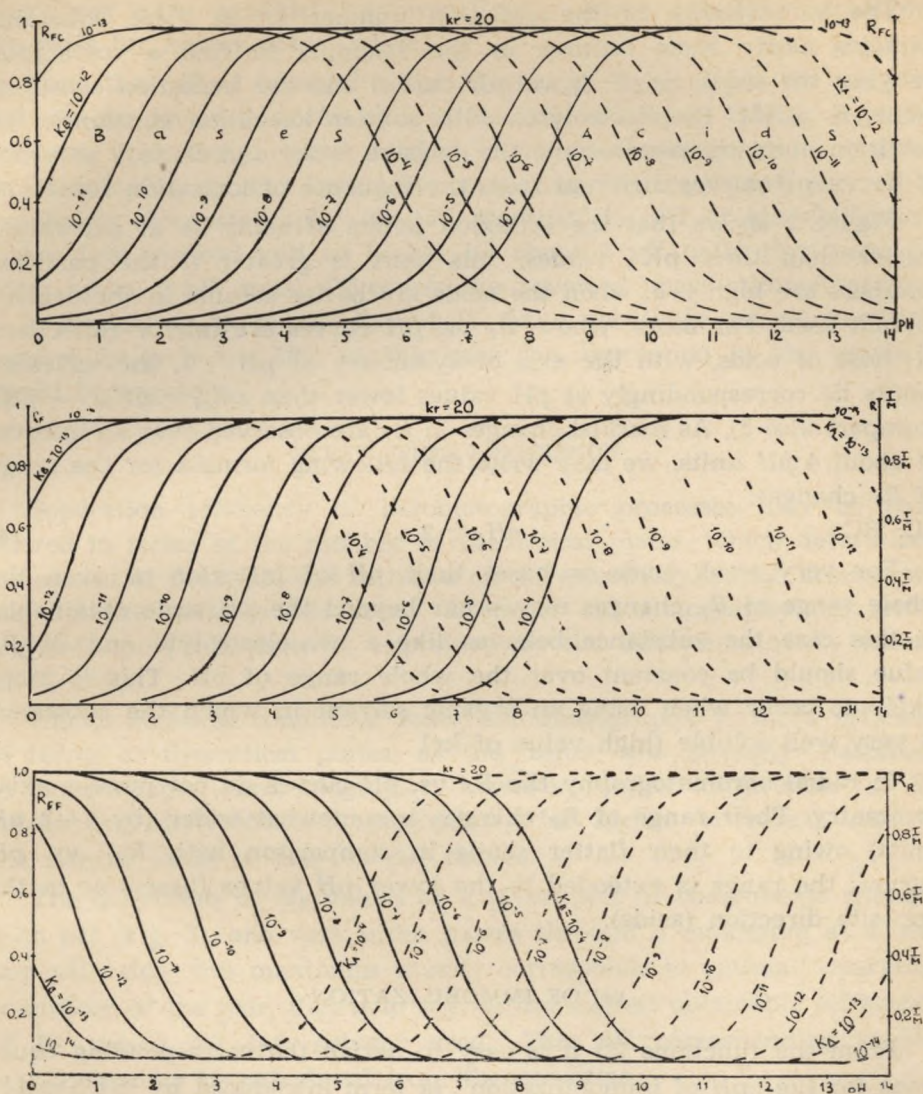


Fig. 2. Influence of ionization constant upon the position of  $R_F$  vs.  $pH$  curves.  
Full lines — bases; dotted lines — acids.

$R_F = \frac{1}{2} \frac{kr}{kr + 1}$  we have for the  $pH$  value ( $pH_i$ ) corresponding to the inflexion point:

$$pH_i = pK_A \pm \log(kr + 1). \quad (3)$$

For high partition numbers ( $kr > 100$ ) the unity may be neglected and then

$$pH_i = pK_A \pm \log kr. \quad (3a)$$



The participation of the partition number  $kr$  in these equations explains easily some failures in the attempts to find a correlation between the sequence of  $R_F$  vs.  $pH$  curves and the ionization constants alone, e. g. [5]: for electrolytes with similar ionization constants, the partition numbers may become the decisive factor and cause a sequence of  $R_F$  vs.  $pH$  curves different from the sequence of ionization constants.

Figure 2 shows that the inflexion points of acids lie at  $pH$  values higher than their  $pK_A$  values; this shift is greater if the partition numbers are high i. e. when the acids are better soluble in the organic solvent used. For bases, whose  $R_F$  vs.  $pH$  curves are mirror reflections of those of acids, with the axis of symmetry at  $pH = 7$ , the inflexion points lie correspondingly at  $pH$  values lower than  $pK_A = pK_W - pK_B$  (compare also 3). As marked changes in  $R_F$  are observed over an interval of about 4  $pH$  units, we may write the following formula for the range of  $R_F$  changes:

$$LC, RC \qquad \qquad \qquad pH_i \pm 2. \qquad (4)$$

For very weak acids or bases their  $pH$  of inflexion or even the whole range of  $R_F$  changes may occur beyond the  $pH$  scale obtainable. In this case the substance behaves like a non-electrolyte and its  $R_F$  value should be constant over the whole range of  $pH$ . This is more likely to occur when using an organic solvent in which the substance is very well soluble (high value of  $kr$ ).

In radial chromatography the  $R_F$  vs.  $pH$  curves do not possess axial symmetry. Their range of  $R_F$  changes is somewhat wider (by 1—2  $pH$  units) owing to their flatter shape in comparison with  $R_{FL}$  vs.  $pH$  curves; the range is extended to the lower  $pH$  values (bases) or in the opposite direction (acids).

#### pH OF IMMOBILIZATION

From the functions (2) it is easy to derive the corresponding equations for the „ $pH$  of immobilization” (a term introduced by Schmall and co-workers [6, 7] in their papers on multi-buffered paper chromatography), that denotes the  $pH$  at which the migration of the electrolyte is stopped. Assuming that at  $pH$  of immobilization the  $R_F$  value of an electrolyte is equal to 0.01 we may derive the following approximate equations:

$$\begin{array}{ll} CC & pH_0 = pK_A \pm \log (10.000 \ kr - 1) = pK_A \pm 4 \pm \log kr \\ LC & pH_0 = pK_A \pm \log (100 \ kr - 1) = pK_A \pm 2 \pm \log kr \\ RC & pH_0 = pK_A \pm \log \left( \frac{kr}{100} - 1 \right) \end{array} \qquad (5)$$



(the upper signs — acids; the lower signs — bases; simplified equations are valid for higher values of  $kr$ ).

Thus the  $pH$  of immobilization is always higher than the  $pK_A$  of the acid (or lower than the  $pK_A$  in the case of bases). It also depends, however, upon the partition number of unionized electrolyte, i. e. upon the developing solvent used. By comparing the equations for radial and linear chromatography we can see that the  $pH$  of immobilization in the first method will be higher by about 2  $pH$  units (for bases — lower by ca. 2  $pH$  units). Of course, for reversed phase chromatography it is impossible to obtain  $R_F = 0.01$  for a substance whose partition number  $kr$  is below 100; in this case the expression in the brackets would be negative.

### SEPARATION EFFICIENCY

Separation efficiency of chromatographic processes may be considered in terms of the number of theoretical plates, which determines the spread of the zone, and in terms of the selectivity, which determines the distance between two zones. In paper chromatography the latter factor is most conveniently expressed by the difference of  $R_F$  values ( $\Delta R_F \geq 0$ ).

Under identical conditions radial chromatography is more efficient in terms of theoretical plates, as the zones are strongly flattened. In linear paper chromatography, however, higher differences in  $R_F$  values are often obtained, particularly for higher  $kr$  values, due to the steeper shape of  $R_{FL}$  vs.  $pH$  curves.

The difference of  $R_F$  values of a given pair of components changes with  $pH$  (Fig. 3), and very often passes through a maximum at a certain  $pH$  value; this maximum usually corresponds to optimal separation conditions of the pair, that is to say, to the highest obtainable difference of  $R_F$  values for the solvent used. This  $pH$  corresponding to the maximum may be calculated from the condition:

$$\frac{d(R'_F - R''_F)}{dpH} = 0. \quad (6)$$

Substituting (1) into (6) the following formulae are obtained:

$$CC \quad pH_{\max} = \log \frac{\frac{k'r + 1}{K'_A} \sqrt[3]{\frac{k''r}{K''_A}} - \frac{k''r + 1}{K''_A} \sqrt[3]{\frac{k'r}{K'_A}}}{\sqrt[3]{\frac{k'r}{K'_A}} - \sqrt[3]{\frac{k''r}{K''_A}}}$$

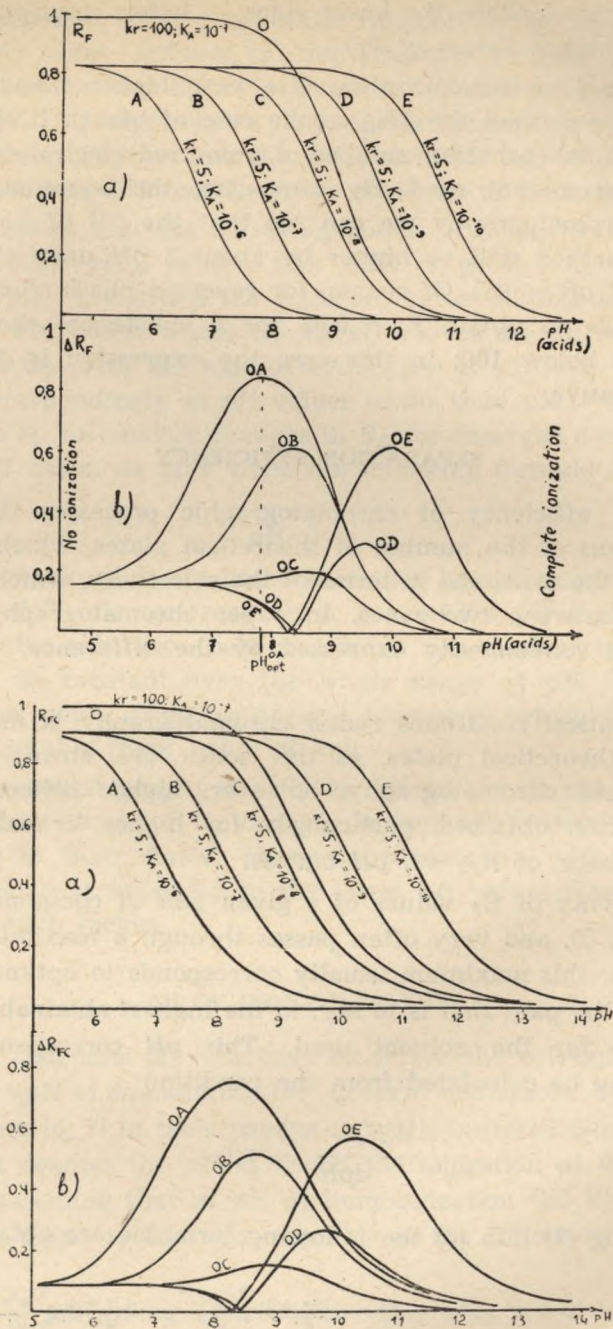


Fig. 3.  $R_F$  vs. pH curves (top) and  $\Delta R_F$  vs. pH curves (bottom) for several pairs of components in linear (upper diagram) and radial chromatography (lower diagram). The maxima correspond to optimal separation conditions:  $pH_{max} = pH_{opt}$ .



$$\text{LC, RC} \quad pH_{\max} = \log \frac{\frac{k'r+1}{K'_A} \sqrt{\frac{k''r}{K''_A}} - \frac{k'r+1}{K''_A} \sqrt{\frac{k'r}{K'_A}}}{\sqrt{\frac{k'r}{K'_A}} - \sqrt{\frac{k''r}{K''_A}}} \quad (7)$$

Of course, for the same parameters of distribution the differences of  $R_F$  values obtained in both conventional and reversed phase linear chromatography will be identical, only the sequence of zones will be reversed. Thus the equation for  $pH_{\max}$  will be the same for both techniques. If, however, one of the parameters of distribution is changed (e.g. the coefficient  $r$ ), then other distribution conditions are obtained and the maximum will be shifted to another  $pH$  value; the extent and direction of the shift may be anticipated on the basis of Fig. 1. The maximal obtainable difference of  $R_F$  values may also change.

On the other hand, in radial chromatography the  $pH$  of optimal separation will be different from that in linear chromatography, even with the same parameters of distribution. It can be seen from Fig. 3 that for higher partition numbers  $kr$  the maxima are lower and longer. The position of  $pH_{\max}$  is somewhat shifted to the lower  $pH$  (bases) or the higher  $pH$  (acids).

However, in certain cases, particularly when the partition numbers of both substances are low (below  $kr = 0.5$ ), it may be possible to obtain higher differences of  $R_F$  values by the radial method. Such a case is illustrated in Fig. 4.

Thus, with low partition numbers (when the substance is better soluble in water than in the developing solvent), radial chromatography

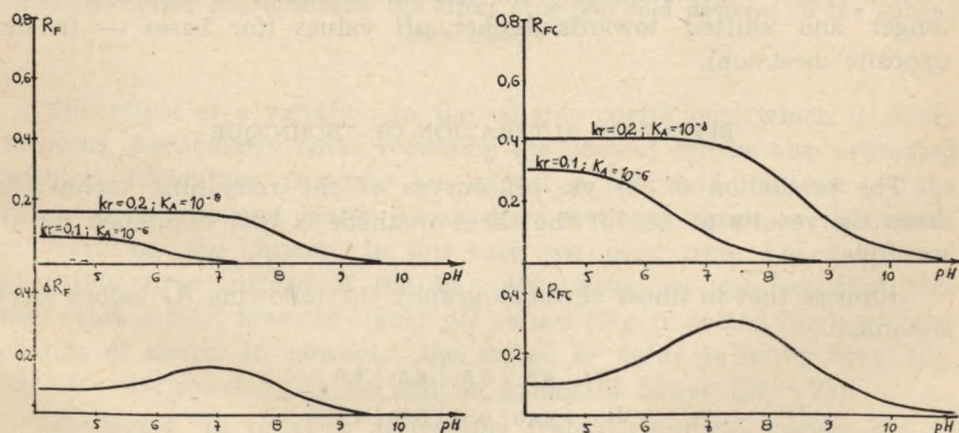


Fig. 4. Case of greater  $R_F$  differences in circular chromatography:  
 $\max \Delta R_{FC} > \max \Delta R_{FL}$



may prove more efficient both in terms of theoretical plates and in terms of differences of  $R_F$  values.

The calculation of  $pH_{\max}$  from form. 7 would be tedious; moreover, in certain cases  $pH_{\max}$  does not correspond to optimal separation conditions, and higher differences in  $R_F$  values may be obtained when the substances are in their unionized forms (an analogous case is also possible in paper electrophoresis, cf. [8]). Furthermore, for multicomponent mixtures different  $pH_{\text{opt}}$  are obtained for each pair of components. Therefore, it would be simpler to plot theoretical  $R_F$  vs.  $pH$  curves from Fig. 1, using the simplified method [2] (on the basis of the distribution parameters  $kr$  and  $K_A$  or on the basis of experimental  $R_F$  values determined at a few values of  $pH$ ), and then to choose the  $pH$  value at which all the substances are best separated.

The case when partition numbers are high ( $kr \geq 100$ ) is less complicated, particularly for linear chromatography (conventional or reversed phase);  $pH_{\max}$  is then identical with  $pH_{\text{opt}}$  and the equations are much simpler; for linear chromatography:

$$\text{LC, RC} \quad pH_{\max} = pH_{\text{opt}} = \frac{1}{2} \log \frac{k'r \cdot k''r}{K'_A \cdot K''_A}$$

Furthermore, in each technique the horizontal distance between two curves is then equal to the logarithm of the coefficient of separation of the pair of components:

$$\beta = \frac{k' K''_A}{k'' K'_A} \geq 1$$

and in radial chromatography the maxima of  $\Delta R_F$  are always lower, longer and shifted towards higher  $pH$  values (for bases — in the opposite direction).

#### EFFECT OF ALTERATION OF TECHNIQUE

The prediction of  $R_F$  vs.  $pH$  curves of the remaining techniques from the results of any of the three methods is best explained by an example.

Suppose that in linear chromatography the following  $R_F$  values were obtained:

$pH$	4,5	5,5	6,5	8,0
$R_{FL}$	0,10	0,52	0,90	1,00

Plotting these values on a sheet of tracing paper, the coordinate scales identical with those in Fig. 1, and making the points fit to



Fig. 1 LC, as described in, [2], we can see that any of the curves with  $kr > 100$  may be the theoretical  $R_{FL}$  vs.  $pH$  curve for these points. For example, we copy the  $kr = 400$  curve, marking also the X point on the  $pH$  axis. The sheet of tracing paper is now transferred over Fig. 1 CC, and with  $pH$  axes and X points overlapping, the  $kr = 400$  curve is copied, giving  $R_{FC}$  vs.  $pH$  relationship for the same substance, under the same distribution conditions, in radial chromatography. The sheet of tracing paper is then transferred over Fig. 1 RC, and with  $pH$  axes and X points overlapping, the  $kr = 400$  curve is copied, giving this time  $R_{FR}$  vs.  $pH$  relationship for reversed phase chromatography under identical distribution conditions (the same parameters  $k$ ,  $r$  and  $K_A$ ). Thus we have obtained  $R_F$  vs.  $pH$  relationship for all three techniques (Fig. 5, full lines).

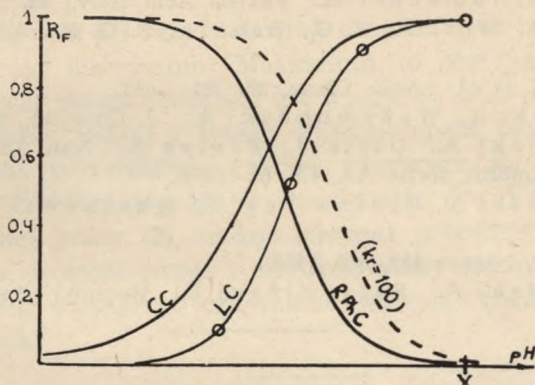


Fig. 5. Prediction of  $R_{FC}$  and  $R_{FR}$  vs.  $pH$  relationships from linear chromatography data. The dotted line illustrates the effect of a four-fold decrease in the volume coefficient  $r$

The effect of a variation in the volume coefficient (which is likely to occur, particularly when reversing the phases) can also be estimated without difficulties. Suppose we expect a four-fold decrease of the ratio of organic and water phase cross-sectional areas as the result of reversing the phases. In this case we copy from Fig. 1 RC the  $kr = 100$  curve instead of the  $kr = 400$  curve; we can see that this will cause a shift towards higher  $pH$  values (Fig. 5, dotted line), without change of shape. If, however, the actual  $kr$  value is lower than 400, say 100, the resulting curve will be somewhat lower ( $kr = 25$ ).

Of course, in practice deflections from theoretical curves are to be expected due to departures from idealizing assumptions of the plate theory: adsorption, formation of  $r$  and  $pH$  gradients, demixion etc.



It seems that the „moist paper method” developed in this laboratory [2, 9—11] may give conditions similar to those of countercurrent extraction processes assumed in the plate theory. Good agreement of experimental and theoretical curves was obtained in the works cited (linear conventional paper chromatography), and the values of the volume coefficient  $r$  were defined and reproducible [12].

## REFERENCES

1. Waksmundzki A., Soczewiński E.: *Roczniki Chem.*, **32**, 863 (1958), **33**, 1423 (1959).
2. Waksmundzki A., Soczewiński E.: *J. Chromat.*, **3**, 252 (1960).
3. Rybař D., Toušek B., Hais I. M.: *Chem. Listy*, **48**, 1532 (1954).
4. Soczewiński E.: *Nature*, **188**, 391 (1960).
5. Büchi J., Schumacher H.: *Pharm. Acta Helv.*, **32**, 273 (1957).
6. Schmall M., Wollish E. G., Schafer E. G. E.: *Anal. Chem.*, **28**, 1373 (1956).
7. Schmall H. et al.: *Anal. Chem.* **29**, 791 (1957).
8. Soczewiński E., Waksmundzki A.: *J. Chromat.*, **5**, 170 (1961).
9. Waksmundzki A., Ościk J., Frelek Z.: *Ann. Univ. Mariae Curie-Skłodowska, Lublin, sectio AA*, **IX**, 83 (1954).
10. Waksmundzki A., Soczewiński E., Aksanowski R.: *Chem. Anal.*, **2**, 459 (1957).
11. Dębska W.: *Nature* **182**, 666 (1958).
12. Waksmundzki A., Soczewiński E.: *Roczniki Chem.*, **35**, 1363 (1961).

## STRESZCZENIE

Zakładając słuszność wzorów (1), wiążących wartości  $R_F$  otrzymane w trzech technikach chromatograficznych: liniowej (konwencjonalnej), krążkowej i odwróconymi fazami, omówiono wpływ stosowanej techniki na wartości  $R_F$  elektrolitów organicznych (buforowana faza polarna) oraz na zdolność rozdzielczą procesu.

1. Wpływ liczby podziału ( $kr$ ) niezdysoncjowanej substancji na przebieg krzywych  $R_F$  vs.  $pH$  zilustrowano na ryc. 1. Krzywe dla chromatografii odwróconych faz otrzymać można przez obrót wykresu dla chromatografii konwencjonalnej (polarna faza nieruchoma) o  $180^\circ$  wokół osi poziomej, leżącej na wysokości  $R_F = 0,5$ . Krzywe dla chromatografii krążkowej są wyższe od krzywych dla chromatografii liniowej (szczególnie przy niskich liczbach podziału  $kr$ ) i bardziej wydłużone.

2. Stała dysocjacji nie wpływa na kształt krzywych we wszystkich trzech technikach, a tylko na ich położenie wzdłuż osi  $pH$ . Dla chromatografii liniowej (LC) i odwróconych faz (RC) położenie krzywych



określone ich punktami przegięcia wyraża się wzorem (3), zaś zakres zmian  $R_F$  wzorem (4). Dla chromatografii krążkowej zakres zmian  $R_F$  jest rozszerzony w kierunku niższych  $pH$  (zasady) lub wyższych  $pH$  (kwasy).

3. Wzory (5) wyrażają wpływ parametrów podziału na  $pH$  potrzebne do zatrzymania elektrolitu organicznego na linii startu.

4. Zdolność rozdzielcza, wyrażona w postaci różnicy wartości  $R_F$ , ze względu na bardziej płaski przebieg krzywych  $F_{FC}$  vs.  $pH$  jest na ogół niższa w chromatografii krążkowej niż liniowej. Jednak przy niskich liczbach podziału, np. poniżej 0,5, możliwe jest uzyskanie wyższych różnic  $R_F$  w chromatografii krążkowej (por. ryc. 4). Z punktu widzenia liczby półek teoretycznych, jak wiadomo, chromatografia krążkowa jest na ogół bardziej skuteczna ze względu na zwięźenie pasm w czasie rozwijania.

Wzór (7) wyraża  $pH$  fazy polarnej, przy których różnica wartości  $R_F$  przechodzi przez maksimum. Maksimum to jest zawsze równoważne optymalnym warunkom rozdziału dla wysokich liczb podziału ( $kr > 100$ ).

5. Z wyników jednej z trzech wymienionych technik chromatograficznych można przewidzieć przebieg krzywych  $R_F$  vs.  $pH$  dla pozostałych technik chromatograficznych, stosując ryc. 1 i metodę graficzną opisaną w innej pracy (2), można również przewidzieć wpływ zmiany współczynnika objętościowego  $r$  spowodowanej zmianą stosowanej techniki, a ponadto wyznaczyć optymalne warunki rozdziału dla poszczególnych technik.

## РЕЗЮМЕ

Предполагая применимость уравнений (1) связывающих величины  $R_f$  получаемые при трёх хроматографических методах: линейном (конвенциональном)  $R_{FL}$ , радиальном  $R_{FC}$  и с обращенными фазами  $R_{FR}$ , рассмотрено влияние применяемой техники на величины  $R_F$  органических электролитов (забуференная полярная фаза) и на разделительную способность процесса.

1. Влияние числа распределения ( $kr$ ) недиссоциированного вещества на ход кривых  $R_F$  vs  $pH$  показано на рис. 1. Кривые для хроматографии обращенными фазами можно получить поворотом графика для конвенциональной хроматографии (полярная неподвижная фаза) на  $180^\circ$  вокруг горизонтальной оси, проходящей через  $R_F = 0,5$ . Кривые для радиальной хроматографии выше чем кривые для линейной хроматографии (особенно при низких числах распределения) и более растянутые.



2. Константа ионизации не влияет на вид кривых при всех трёх способах, а только на их положение вдоль оси  $pH$ . Для линейной хроматографии (LC) и хроматографии обращенными фазами (RC) положение кривых определенное их точками перегиба выражает формула 3, интервал изменений  $R_F$  формула 4. Для радиальной хроматографии интервал изменений  $R_F$  расширен в направлении более низких  $pH$  (основания) или более высоких  $pH$  (кислоты).

3. Формулы 5 устанавливают связь между параметрами распределения и  $pH$  нужном для задержания электролита на линии старта.

4. Разделительная способность выраженная в виде разности величин  $R_F$ , обычно ниже в радиальной хроматографии из-за более плоского хода кривых  $R_{FC}$  vs  $pH$ . Однако для низких чисел распределения, например ниже 0,5, возможны более высокие разности коэффициентов  $R_F$  в радиальной хроматографии (сравни рис. 4). С точки зрения теоретических тарелок, как известно, радиальная хроматография является обычно более эффективной из-за суживания полос во время проявления.

Формула (7) определяет  $pH$  полярной фазы при которой разность коэффициентов  $R_F$  достигает максимума. В случае высоких чисел распределения ( $kr > 100$ ) этот максимум всегда соответствует оптимальным условиям разделения.

5. Из результатов одной из трёх рассматриваемым техник нетрудно теоретически установить ход кривых  $R_F$  vs.  $pH$  для остальных техник, применяя рис. 1 и графический метод описанный в другой работе [2]; возможно также предвидеть влияние изменения коэффициента объемности  $r$  вызванного изменением применяющейся техники, а также определить оптимальные условия разделения для остальных двух методик.