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Estimation of Some Linear Functionals in the Family of Bounded Symmetric Univalent Functions

Oszacowanie pewnych funkcjonałów liniowych w rodzinie funkcji jednolistnych ograniczonych i symetrycznych

Оценки некоторых линейных функционалов в смействе однолистных ограниченных и симметрических функций

Introduction. Denote by S the family of functions

$$F(z) = z + \sum_{n=2}^{\infty} A_{nF} z^n \tag{1}$$

holomorphic and univalent in the disc $E = \{z : |z| < 1\}$, and by S_R the subclass of the family S consisting of functions with real coefficients.

Let $S_R(M)$, $M > 1$, be the subclass of S_R composed of functions bounded by M, i.e. those satysfying the condition

$$|F(z)| \leq M, \ z \in E .$$

It is known that, for each function $F \in S_R(M)$ ([7], [10]),

$$A_{2F} \leq P_{2,M} \text{ if } M > 1 ,$$
$$A_{4F} \leq P_{4,M} \text{ if } M > 11,$$

where $P_{2,M}$, $P_{4,M}$ are, respectively, the second and the fourth coefficients of Taylor expansion (1) of the Pick function $P_M(z)$ given by the equation

$$\frac{P_M(z)}{\left[1 - \frac{P_M(z)}{M}\right]^2} = \frac{z}{(1-z)^2}, \ z \in E , \tag{2}$$

and satysfying the condition $P_M(o) = 0$. It is also known ([5], [6]) that, for each $N = 2, 4, 6, \ldots$, there exists a constant $M_N > 1$ such that, for all $M > M_N$ and each function $F \in S_R(M)$, the estimation

$$A_{NF} \leqslant P_{N,M} \tag{3}$$

takes place, where $P_{N,M}$ is the N-th coefficient of Taylor expansion (1) of the Pick function $P_M(z)$ given by equation (2) and satysfying the condition $P_M(o) = 0$.

Note that an analogous result for any odd N is not valid since, as early as $N = 3$, in the family $S_R(M)$ the sharp estimation

$$A_{3F} \leqslant 1 + 2\lambda^2 - 4\lambda M^{-1} + M^{-2} \quad \text{for } e \leqslant M < +\infty \tag{4}$$

holds, where λ is the greater root of the equation $\lambda \log \lambda = -M^{-1}$; the Pick function does not realize the equality in estimation (4).

In the proof of result (3) one makes essential use of the fact [1] that, for each function $F \in S_R$,

$$A_{nF} \leqslant P_{n,\infty}, \; n = 2, 3, 4, \ldots,$$

where $P_{n,\infty} = n$ is the n-th coefficient in Taylor expansion (1) of the Koebe function

$$\mathcal{K}_o(z) = P_\infty(z) = \frac{z}{(1-z)^2}, \; z \in E. \tag{5}$$

In the present paper we consider a real, linear and continuous functional on the family $S_R(M)$ such that its maximum on S_R is attained for Koebe function (5). By making use of the differential functional equation for extremal functions it, will be proved that, when M is sufficiently large, the maximum of such a functional on the class $S_R(M)$ is attained for the Pick function $P_M(z)$ given by equation (2) and satisfying the condition $P_M(o) = 0$. This result is a generalization of those obtained earlier in papers [3], [12], [13], [5], [6] (see also [4]).

The fundamental theorem. Let K, $K \geqslant 2$, be any positive integer and λ_n, $n = 2, 3, \ldots, K$, real numbers. Consider in the family $S_R(M)$, $M > 1$, a real functional

$$\Phi(F) = \sum_{n=2}^{K} \lambda_n A_{nF} \tag{6}$$

such that

$$\max_{F \in S_R} \Phi(F) = \Phi(\mathcal{K}_o) \tag{7}$$

where $\mathcal{K}_o$ is the Koebe function (5).

The functional Φ is continuous, the family $S_R(M)$ compact in the topology of almost uniform convergence; consequently, for each $M > 1$, in the family $S_R(M)$ there exists at least one function realizing the maximum of the functional Φ . In the sequel, each function F_o, for which $\max_{F \in S_R(M)} \Phi(F) = \Phi(F_o)$, will be shortly called extremal.

From condition (7) and the linearity of the functional Φ it follows that

$$\Phi(s(z, t)) < \Phi(s(z, 1)) = \Phi(\mathcal{K}_o),\ t \in \langle -1, 1),\tag{8}$$

where

$$s(z, t) \equiv \frac{z}{1 - 2tz + z^2}.$$

Making use of the form of the coefficients of the function $s(z, t)$ as well as conditions (6) and (8), we obtain that the parameters λ_n, $n = 2, 3, \ldots$, must satisfy the inequalities:

$$\sum_{n=2}^{K} \left(\frac{\sin n\phi}{\sin \phi} - n\right) \lambda_n < 0,\ e^{i\phi} = t + i\sqrt{1 - t^2},\ -1 < t < 1, \sqrt{1} = 1,\tag{9}$$

$$\sum_{n=2}^{K} [(-1)^{n+1} - 1]\, n\lambda_n < 0,\ t = -1.\tag{10}$$

We shall prove the following

Theorem. *Let K, $K \geqslant 2$, be any positive integer, and λ_n, $n = 2, 3, \ldots, K$, real numbers. Let $\Phi(F) = \sum_{n=2}^{K} \lambda_n A_{nF}$ be a functional defined on the family $S_R(M)$, $M > 1$, such that $\max_{F \in S_R} \Phi(F) = \Phi(\mathcal{K}_o)$ where $\mathcal{K}_o$ is Koebe function (5). Then there exists a constant M_o, $M_o > 1$, such that for all $M > M_o$,*

$$\max_{F \in S_R(M)} \Phi(F) = \Phi(P_M)\tag{11}$$

where P_M is the Pick function defined by equation (2) and satisfying the condition $P_M(o) = 0$. It is the only function realizing equality (11).

Proof. It is known [2] that each function $w = f(z) = \frac{1}{M} F(z)$, where F is an extremal function in the family $S_R(M)$, $M > 1$, satisfies the differential-functional equation:

$$\left(\frac{zw'}{w}\right)^2 \mathcal{M}(w) = \mathfrak{N}(z),\ 0 < |z| < 1,\tag{12}$$

where

$$\mathcal{M}(w)=\frac{A^{(2)}_{2F}}{M}\lambda_2+\frac{A^{(2)}_{3F}}{M}\lambda_3+\ldots+\frac{A^{(2)}_{KF}}{M}\lambda_K\ (w+\frac{1}{w})+\frac{A^{(3)}_{3F}}{M^2}\lambda_3+$$

$$+\frac{A^{(3)}_{4F}}{M^2}\lambda_4+\ldots+\frac{A^{(3)}_{KF}}{M^2}\lambda_K\ (w^2+\frac{1}{w^2})^3+\ldots+\frac{A^{(K-1)}_{K-1,F}}{M^{K-2}}\lambda_{K-1}+$$

$$+\frac{A^{(K-1)}_{K,F}}{M^{K-2}}\lambda_K\quad(w^{K-2}+\frac{1}{w^{K-2}})+\frac{A^{(K)}_{KF}}{M^{K-1}}\lambda_K\,(w^{K-1}+\frac{1}{w^{K-1}})-\mathcal{P},$$

$$\begin{aligned}\mathcal{N}(z)=(A_{2F}\lambda_2+2A_{3F}\lambda_3+3A_{4F}\lambda_4+\ldots+(K-1)A_{KF}\lambda_K)+(\lambda_2+2A_{2F}\lambda_3+\\+3A_{3F}\lambda_4+\ldots+(K-1)A_{K-1,F}\lambda_K)(z+1/z)+(\lambda_3+2A_{2F}\lambda_4+\\+3A_{3F}\lambda_5+\ldots+(K-2)A_{K-2,F}\lambda_K)(z^2+1/z^2)+\ldots+(\lambda_{K-1}+\\+2A_{2F}\lambda_K)(z^{K-2}+1/z^{K-2})+\lambda_K(z^{K-1}+1/z^{K-1})-\quad,\end{aligned}\tag{13}$$

$$\mathcal{P}=2\min_{0\le x\le\pi}\ \frac{A^{(2)}_{2F}}{M}\lambda_2+\frac{A^{(2)}_{3F}}{M}\lambda_3+\ldots+\frac{A^{(2)}_{KF}}{M}\lambda_K)\cos x+\frac{A^{(3)}_{3F}}{M^2}\lambda_3+$$

$$+\frac{A^{(3)}_{4F}}{M^2}\lambda_4+\ldots+\frac{A^{(3)}_{KF}}{M^2}\lambda_K)\cos 2x+\ldots+\frac{A^{(K-1)}_{K-1,F}}{M^{K-2}}\lambda_{K-1}+$$

$$+\frac{A^{(K-1)}_{K,F}}{M^{K-2}}\lambda_K)\cos(K-2)x+\frac{A^{(K)}_{KF}}{M^{K-1}}\lambda_K\cos(K-1)x\ ,$$

$$F^m(z)=\sum_{n=m}^{\infty}A^{(m)}_{nF}\ z^n,\quad m=2,3,\ldots,\ n=m,m+1,\ldots\ .$$

The functions $\mathcal{M}(w)$ and $\mathcal{N}(z)$ take non-negative real values on the circles $|w|=1$ and $|z|=1$, respectively. Each of these functions has on the respective circle at least one zero of even multiplicity. Let us still notice that if $\mathcal{M}(w_o)=0$, then $\mathcal{M}(\bar{w}_o)=0$, $\mathcal{M}(1/w_o)=0$, $\mathcal{M}(1/\bar{w}_o)=0$, and if $\mathcal{N}(z_o)=0$, then also $\mathcal{N}(\bar{z}_o)=0$, $\mathcal{N}(1/z_o)=0$ and $\mathcal{N}(1/\bar{z}_o)=0$.

From condition (7) it follows that, for any $\epsilon>0$, there exists a constant $M'>1$ such that, for all $M>M'$ and each $z\in\Delta$,

$$|z^{K-1}(\mathcal{N}(z)-\mathcal{N}_o(z))|<\epsilon\tag{14}$$

where Δ is an arbitrary compact set containing in its inside all zeros of the function $\mathcal{W}_o(z)$, while $\mathcal{W}(z)$ is given by formula (13) and $\mathcal{W}_o(z)$ is defined as follows:

$$\mathcal{W}_o(z) = [2\lambda_2 + 2\cdot 3\lambda_3 + 3\cdot 4\lambda_4 + \dots + (K-1)K\lambda_K] + \lambda_2 (z + 1/z) + \\ + \lambda_3 [2^2 (z+1/z) + (z^2 + 1/z^2)] + \dots + \lambda_{K-1} [(K-2)^2 (z+1/z) + (K-3)^2 (z^2 + \\ + 1/z^2) + \dots + 2^2 (z^{K-3} + 1/z^{K-3}) + (z^{K-2} + 1/z^{K-2})] + \lambda_K [(K-1)^2 (z + \\ + 1/z) + (K-2)^2 (z^2 + 1/z^2) + \dots + 2^2 (z^{K-2} + 1/z^{K-2}) + (z^{K-1} + 1/z^{K-1})]. \tag{15}$$

We shall determine the zeros of the function $\mathcal{W}_o(z)$ on the circle $|z|=1$. Without loss of generality, let us assume that K is even (in the case where K is odd, the proof runs analogously). Since

$$\sum_{m=2}^{N} (N-m+1)^2 z^{-m+1} = 1/z^N \sum_{m=2}^{N} (N-m+1)^2 z^{N-m+1} =$$

$$= 1/z^N \sum_{n=1}^{N-1} n^2 z^n = 1/z^N [((\sum_{n=1}^{N-1} z^n)' z)' z] = 1/z^N [((\frac{z^N - z}{z-1})' z)' z] =$$

$$= \frac{1}{(z-1)^3} [(N-1)^2 z^2 - (2N^2 - 2N - 1) z + N^2 - z^{-N+2} - z^{-N+1}],$$

therefore, proceeding in an analogous way with all the addends of $\mathcal{W}_o(z)$, after some transformations we get:

$$\mathcal{W}_o(z) = \frac{(z+1)^2}{(z-1)^2} L_o(z) \tag{16}$$

where

$$L_o(z) = \lambda_2 [(z+1/z) - 2] + \lambda_3 [(z^2 + 1/z^2) - 2] + \lambda_4 [(z^3 + 1/z^3) + (z + 1/z) - 4] + \\ + \lambda_5 [(z^4 + 1/z^4) + (z^2 + 1/z^2) - 4] + \dots + \lambda_{K-1} [(z^{K-2} + 1/z^{K-2}) + \\ + (z^{K-4} + 1/z^{K-4}) + \dots + (z^2 + 1/z^2) - (K-2)] + \lambda_K [(z^{K-1} + 1/z^{K-1}) + \\ + (z^{K-3} + 1/z^{K-3}) + \dots + (z + 1/z) - K]. \tag{17}$$

Adopting in (17) $z = e^{i\phi}$, $0 < \phi \leqslant \Pi$, after transformations and making use of inequalities (9) and (10), we obtain that the only zero of the function $L_o(z)$ of the circle $|z| = 1$ is the point $z = 1$ which, in view of (15), is not a zero of $\mathcal{N}_o(z)$.

So, finally, from formula (16) it follows that the function $\mathcal{N}_o(z)$ has on the circle $|z| = 1$ one double zero $z = -1$ and $K - 2$ zeros both inside and outside this circle.

Let us surround all zeros of the function $\mathcal{N}_o(z)$ with sufficiently small disjoint disc. From the Hurwitz theorem as well as condition (14) we deduce that there exists an $M'' > M'$ such that, for all $M > M''$, zeros of the function $\mathcal{N}(z)$ given by formula (13) lie, respectively, in chosen neighbourhoods of zeros of the function $\mathcal{N}_o(z)$, with that in each of these neighbourhoods the number of zeros of both those functions, considering miltiplicities, is the same.

It is known [2] that the function $\mathcal{N}(z)$ has on the circle $|z| = 1$ at least one zero of even multiplicity. Let $\bar{z} \neq -1$, $|\bar{z}| = 1$, be one of these zeros. Then, for $M > M''$, it lies in the vicinity of the double zero $z = -1$ of the function $\mathcal{N}_o(z)$. Since $\mathcal{N}(z)$ is a non--negative function on the circle $|z| = 1$, the multiplicity of such a zero is at least 2; besides, in the same neighbourhood there must lie a zero $\tilde{z}$ of multiplicity at least 2, which contradicts the fact that the function $\mathcal{N}(z)$ must have exactly two zeros there considering mutiplicities. Consequently, $\tilde{z} = -1$ is the only zero of the function $\mathcal{N}(z)$ on the circle $|z| = 1$.

So, from the form of $\mathcal{N}(z)$ it results that, for $M > M''$, this function can be represented as follows:

$$\mathcal{N}(z) = \frac{(z+1)^2}{z^{K-1}} L(z) \tag{18}$$

where $L(z)$ is some polynomial of degree $2K - 4$, and $L(z) \neq 0$ for $|z| = 1$. From the properties of the function $\mathcal{N}(z)$, given before, we know that if $L(z_o) = 0$, then also $L(\bar{z}_o) = 0$, $L(1/z_o) = 0$ and $L(1/\bar{z}_o) = 0$.

From equation (12) we infer that the images $\tilde{w} = f(\tilde{z})$ of zeros $\tilde{z}$, $|\tilde{z}| < 1$, of the function $\mathcal{N}(z)$ are zeros of the function $\mathcal{M}(w)$ since $f'(\tilde{z}) \neq 0$, whereas from the very form of the function $\mathcal{M}(w)$ it follows that also the points $\bar{\tilde{w}}$, $1/\tilde{w}$, $1/\bar{\tilde{w}}$ are its zeros. Moreover, it is well known [2] that the function $\mathcal{M}(w)$ has on the circle $|w| = 1$ at least one double zero w_o. From the above properties of the function $\mathcal{M}(w)$ we deduce that, for $M > M''$,

$$\mathcal{M}(w) = \frac{(w - w_o)^2}{w^{K-1}} \hat{L}(w) \tag{19}$$

where $w_o = -1$ or $w_o = 1$, and $\hat{L}(w)$ is some polynomial of degree $2K - 4$, and $\hat{L}(w) \neq 0$ for $|w| = 1$.

We have thus demonstrated that, for $M > M''$, each function $w = f(z) = 1/M\,F(z)$, where F is an extremal function, satisfies the equation (12) where $\mathcal{M}(w)$ and $\mathcal{N}(z)$ are given by formulae (18) and (19), respectively.

Using now the Royden theorem [8], the theory of Γ-structures [9] as well as the fact that, for the classes $S_R(M)$, the image $f(E)$ of the disc E under the mapping $w = f(z) = 1/M\, F(z)$ is symmetrical with respect to the real axis, one proves that, for $M > M''$, each function $w = f(z) = 1/M\, F(z)$, where F is an extremal function, maps the disc $|z| < 1$ onto the disc $|w| < 1$ lacking a segment on the real axis with a) one end at the point $w_o = -1$ and the other at some point of the negative real half-axis between -1 and 0 or b) one end at the point $w_o = 1$ and the other at some point of the positive half-axis between 0 and 1. Consequently, from the property of the Pick function P_M (e.g. [6]) and from the Riemann theorem it follows that the only such function is in case a) the function $p_M(z) = 1/M\, P_M\,(z)$, while in case b) the function $-p_M(-z) = -1/M\, P_M\,(-z) =$

$= z + \sum_{n=2}^{\infty} (-1)^{n-1} P_{n,M} z^n$ where P_M is a Pick function.

One knows (e.g. [6]) that $\lim_{M \to \infty} P_{n,M} = n$, $n = 2, 3, \ldots$. From this and inequality (10) it follows that there exists an $M_o > M''$ such that, for all $M > M_o$, the inequality

$$\sum_{n=2}^{K} \lambda_n P_{n,M} \geqslant \sum_{n=2}^{K} (-1)^{n+1} \lambda_n P_{n,M}$$

is satisfied. So, finally, the only extremal function in the family $S_R(M)$ for $M > M_o$ is the Pick function P_M given by equation (2) and satisfying the condition $P_M(o) = 0$.

Remark. Proceeding in the way similar to that given above, one can prove that if the functional Φ of the form (6) is such that

$$\max_{F \in S_R} \Phi(F) = \Phi(\tilde{\mathcal{K}}_o), \quad \tilde{\mathcal{K}}_o(z) = -\mathcal{K}_o(-z),$$

where $\mathcal{K}_o$ is Koebe function (5), then there exists an $\tilde{M}_o > 1$ such that for all $M > \tilde{M}_o$,

$$\max_{F \in S_R(M)} \Phi(F) = \Phi(\tilde{P}_M), \quad \tilde{P}_M(z) = -P_M(-z),$$

where P_M is the Pick function defined by equation (2) and satisfying the condition $P_M(o) = 0$.

In virtue of the Toeplitz theorem on the general form of a linear functional ([11], p. 36), the estimation of the functional $\Phi(F) = \sum_{n=2}^{\infty} \lambda_n A_{nF}$ remains an open problem; however, the method applied in this paper allows one to consider functionals depending on a finite number of coefficients (see [2]).

REFERENCES

[1] Dieudonné J., *Sur les fonctions univalentes*, Compt. rend. Acad. Sci. 192 (1931), 1148–1150.
[2] Dziubiński I., *L'Equation des fonctions extremales dans la famille des fonctions univalentes symetriques et bornées*, Łódzkie Towarzystwo Naukowe, Sec. III, Nr 65, 1960.
[3] Jakubowski Z. J., *Maximum funkcjonału $A_3 + \alpha A_2$ w rodzinie funkcji jednolistnych o współczynnikach rzeczywistych*, Zeszyty naukowe UŁ, Ser. II, Zeszyt 20 (1966), 43–61.
[4] Jakubowski Z. J., *Sur les coefficients des fonctions univalentes dans la cercle unite*, Ann. Polon. Math., XIX, (1967).
[5] Jakubowski Z. J., Zielińska A., Zyskowska K., *Sharp Estimation of Even Coefficients of Bounded Symmetric Univalent Functions*, Abstract of short communications and poster sessions, International Congress of Mathematicians, Helsinki, 1978, p. 118.
[6] Jakubowski Z. J., Zielińska A., Zyskowska K., *Sharp Estimation of Even Coefficients of Bounded Symmetric Univalent Functions*, Ann. Polon. Math., to appear.
[7] Pick G., *Über die Konforme Abbildung eines Kreises auf ein schlichtes und zugleich beschränktes Gebiet*, Sitrgsber. Kaiserl. Akad. Wiss. Wien., Abt. II a, 126 (1917), 247–263.
[8] Royden H. L., *The Coefficient Problem for Bounded Schlicht Functions*, Proc. Nat. Acad. Sci., Vol. 35 (1949), 657–662.
[9] Schaeffer A. C., Spencer D. C., *Coefficient Regions for Schlicht Functions*, Amer. Math. Soc., Colloqium Publications, Vol. XXXV (1950).
[10] Schiffer M., Tammi O., *The Fourth Coefficient of Bounded Real Univalent Functions*, Ann., Acad. Sci. Fennicae, Ser. AI, No 354 (1965), 1–34.
[11] Schober G., *Univalent Functions*, Lecture Notes in Mathematics, 1975.
[12] Zielińska A., Zyskowska K., *Estimation of the Sixth Coefficients in the Class of Univalent Bounded Functions with Real Coefficients*, Ann. Polon. Math., to appear.
[13] Zielińska A., Zyskowska K., *On Estimation of the Eighth Coefficient of Bounded Univalent Functions with Real Coefficients*, Demonstratio Mathematica, Vol. XII, No 1 (1979), 231–246.

STRESZCZENIE

Niech $S_R(M)$, $M > 1$, będzie rodziną funkcji

$$F(z) = z + \sum_{n=2}^{\infty} A_{nF} z^n$$

holomorficznych i jednolistnych w kole jednostkowym E, mających współczynniki rzeczywiste i takich, że $|F(z)| < M$ dla $z \in E$.

Niech $K \geq 2$ będzie liczbą całkowitą oraz λ_n, $n = 2, 3, \ldots, K$, niech będą liczbami rzeczywistymi. W pracy rozważa się funkcjonały rzeczywiste postaci

$$\Phi(F) = \sum_{n=2}^{K} \lambda_n A_{nF}$$

w rodzinie $S_R(M)$ takie, że

$$\max_{F \in S_R} \Phi(F) = \Phi(\mathcal{K}_0), \text{ gdzie } S_R = S_R(\infty) \text{ oraz } \mathcal{K}_0(z) = z(1-z)^{-2}.$$

Dowodzi się, że istnieje stała $M_0 > 1$ taka, że dla wszystkich $M > M_0$

$$\max_{F \in S_R(M)} \Phi(F) = \Phi(P_M),$$

gdzie P_M jest funkcją Picka określoną wzorem (2).

РЕЗЮМЕ

Пусть $S_R(M)$, $M > 1$ семейство одинлистных функций

$$F(z) = z + \sum_{n=2}^{\infty} A_{nF} z^n$$

в единоличном круге E, имеющих действительные коеффиценты, таких что $|F(z)| < M$ для $z \in E$.

Пусть $K \geq 2$ целое число, λ_n, $n = 2, 3, \ldots, K$, действительные числа. В этой работе рассматривается действительные функционалы вида

$$\Phi(F) = \sum_{n=2}^{K} \lambda_n A_{nF}$$

в семи $S_R(M)$, для которых

$$\max_{F \in S_R} \Phi(F) = \Phi(\mathcal{K}_0), \quad \text{где} \quad S_R = S_R(\infty) \quad \text{и} \quad \mathcal{K}_0(z) = z(1-z)^{-2}.$$

Выказано, что существует постронная $M_0 > 1$ такая, что для всех $M > M_0$,

$$\max_{F \in S_R(M)} \Phi(F) = \Phi(P_M),$$

где P_M функция Пика определенная формулой (2).

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