ANNALES UNIVERSITATIS MARIAE CURIE-SKŁODOWSKA
LUBLIN–POLONIA
VOL. XXXV, 6 SECTIO A 1981

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On a Structure of a Linearized Tangent Bundle of Second Order

O strukturze uliniowionej wiązki stycznej drugiego rzędu

О структуре линеаризованого касательного расслоения второго порядка

The purpose of this paper is to investigate a structure of a linearized tangent bundle of second order ${}^2M \to M$ on an n-dimensional manifold M.

Introducing a linear connection Γ on a manifold M (i.e. in the tangent bundle $TM \to M$) allows to endow its tangent bundle of second order with a vector bundle structure ([1], [2]).

A vector bundle structure in the tangent bundle of second order ${}^2M$ allows to produce a horizontal subbundle ${}^2M^H$ and a vertical subbundle ${}^2M^V$.

Equivalence classes of sections of a horizontal subbundle ${}^2M^H$ determine geodesics on the manifold M and geodesics flows on TM. A vector bundle structure of ${}^2M$ allows to introduce horizontal, vertical and complete lifts for section of the tangent bundle $TM \to M$ into the tangent bundle of second order ${}^2M \to M$. These lifts are closely related with classical lifts of a section of the tangent bundle $TM \to M$ into the bundle $TTM \to TM$, [3].

1. Let M be an n-dimensional C^∞ Hausdorff manifold. We consider a tangent bundle over the manifold M:

$$ {}^1_0\pi : TM \to M \,, \tag{1.1}$$

and analogously a tangent bundle over the manifold TM:

$$ T_\pi : TTM \to TM \,. \tag{1.2}$$

We consider also a tangent bundle over the manifold TM:

$$ {}^1_0\pi_* : TTM \to TM \,, \tag{1.3}$$

with the tangent map ${}^1_0\pi_*$ as projection. The following diagram is commutative:

$$\begin{array}{ccc} TTM & \xrightarrow{{}^1_0\pi_*} & TM, \\ T_\pi \downarrow & & \downarrow {}^1_0\pi \\ TM & \xrightarrow{{}^1_0\pi} & M. \end{array} \tag{1.4}$$

A total space of a tangent bundle of second order over M may be defined, [1], as a submanifold ${}^2M$ of the manifold TTM by formula:

$$ {}^2M = \left\{ A \in TTM : T_\pi A = {}^1_0\pi_* A \right\} . \tag{1.5}$$

Then, one may form a fibre bundle:

$$ {}^2_0\pi : {}^2M \longrightarrow M, \text{ where } {}^2_0\pi = {}^1_0\pi \circ {}^1_0\pi_* |_{{}^2M} . \tag{1.6}$$

The fibre bundle ${}^2_0\pi : {}^2M \to M$ is called a tangent bundle of second order over a manifold M. The local chart (U, x^i) in a neighbourhood $U \subset M$ of $p \in U$ on M induces local chart $({}^1_0\pi^{-1}(U), x^{0i}, x^{1i})$ in neighbourhood $X_p \in {}^1_0\pi^{-1}(U) \subset TM$ such that: $x^{0i} = x^i, x^{1i} = X_p \cdot x^i$. Analogously these charts induce the local chart $(T_\pi^{-1} \circ {}^1_0\pi^{-1}(U), x^{0i}, x^{1i}, \widetilde{x}^{1i}, x^{2i})$ on TTM. For the submanifold ${}^2M \subset TTM$, from (1.5), we have: $x^{1i} = \widetilde{x}^{1i}$. Thus, a local chart (U, x^i) on M induces a local chart $({}^2_0\pi^{-1}(U), x^{0i}, x^{1i}, x^{2i})$ on total space ${}^2M$ of a tangent bundle of second order.

A total space ${}^2M \subset TTM$ may be identified with a set $J^2_0(R, M)$ of second order jets of a map: $f : R \to M$ at 0. Then, for induced coordinates in ${}^2M$ we have:

$$ x^{0i} = f^i(0),\ x^{1i} = \frac{df^i}{dt}(0),\ x^{2i} = \frac{d^2 f^i}{dt^2}(0), \text{ where: } f^i = x^i \circ f . \tag{1.7}$$

A tangent bundle of second order with respect to induced coordinates (x^{0i}, x^{1i}, x^{2i}) is not a vector bundle.

Let Γ be a linear connection on a manifold M. We consider a connection Γ in a bundle ${}^1_0\pi : TM \to M$ given by a left splitting of exact sequence of vector bundles over TM and requining to be linear on fibres of a bundle: ${}^1_0\pi_* : TTM \to TM$,

$$ 0 \longrightarrow VTM \underset{\Gamma}{\overset{i}{\rightleftarrows}} TTM \longrightarrow TM \times_M TM \longrightarrow 0 . \tag{1.8}$$

A connection map D for the connection $\Gamma : TTM \to VTM$ is called a map $D : TTM \to TM$ given by formula:

$$ D = p_2 \circ i_{VTM} \circ \Gamma , \tag{1.9}$$

where: $i_{VTM} : VTM \to TM \times_M TM$ is canonical isomorphism of a vertical subbundle

VTM over TM into Whitney sum of bundles TM and TM over M, $p_2 : TM \underset{M}{\times} TM \to TM$ is a projection on the second component. In local induced coordinates in TTM, we have:

$$D : (x^{0i}, x^{1i}; y^{0i}, y^{1i}) \xrightarrow{\Gamma} (x^{0i}, x^{1i}; 0, y^{1i} + \Gamma^i_{jk} x^{1k} y^{0j}) \xrightarrow{i_{VTM}} (x^{0i}, x^{1i}; y^{1i} + \Gamma^i_{jk} x^{1k} y^{0j}) \xrightarrow[p_2]{} (x^{0i}, y^{1i} + \Gamma^i_{jk} x^{1k} y^{0j}) .$$

If is given a linear connection Γ on a manifold M then P. Dombrowski [2] has defined a vector bundle structure for the bundle: $\tau = {}^1_0\pi \cdot T_\pi : TTM \to M$ by the diffemorphism:

$$T_\pi \times {}^1_0\pi_* \times D : TTM \longrightarrow TM \underset{M}{\times} TM \underset{M}{\times} TM , \tag{1.10}$$

into Whitney sum of bundles TM over M.

A vector structure in fibres of the bundle $\tau : TTM \longrightarrow M$ defines the following operations $\oplus$, $\circledcirc$.

For any $A, B \in TTM$, $\lambda \in \mathbf{R}$ such that $\tau(A) = \tau(B)$, we have:

$$\begin{aligned} &T_\pi(A \oplus B) = T_\pi A + T_\pi B , && T_\pi(\lambda \circledcirc A) = \lambda \cdot T_\pi A , \\ &{}^1_0\pi_*(A \oplus B) = {}^1_0\pi_* A + {}^1_0\pi_* B , && {}^1_0\pi_*(\lambda \circledcirc A) = \lambda \cdot {}^1_0\pi_* A , \\ &D(A \oplus B) = D(A) + D(B) , && D(\lambda \circledcirc A) = \lambda \cdot D(A) . \end{aligned} \tag{1.11}$$

As shown in [1] a vector structure of the bundle $\tau : TTM \longrightarrow M$ defined by diffeomorphism: $T_\pi \times {}^1_0\pi_* \times D$ induces a vector structure for the tangent bundle of second order ${}^2_0\pi : {}^2M \longrightarrow M$ by means of the diffeomorphism:

$${}^1_0\pi_* \times D : {}^2M \longrightarrow TM \underset{M}{\times} TM . \tag{1.12}$$

A restriction of the operations: $\oplus$, $\circledcirc$ to the submanifold ${}^2M$ defines a linear structure in fibres of ${}^2M$. It follows from the equality $T_\pi = {}^1_0\pi_*$ for the submanifold ${}^2M \subset TTM$ and from (1.10).

For a given linear connection Γ in the bundle ${}^1_0\pi : TM \longrightarrow M$ a local adopted basis in fibres of the bundle $T_\pi : TTM \longrightarrow TM$ forms the following system of $2n$-vectors:

$$X^H_{i\,|(x^{0i}, x^{1i})} = \frac{\partial}{\partial x^{0i}} - \Gamma^k_{ij} x^{1j} \frac{\partial}{\partial x^{1k}} , \quad X^V_{i\,|(x^{0i}, x^{1i})} = \frac{\partial}{\partial x^{1i}} . \tag{1.13}$$

The vectors: $\dfrac{\partial}{\partial x^{0i}}$, $\dfrac{\partial}{\partial x^{1i}}$ denote natural basis in chart $({}^1_0\pi^{-1}(U), x^{0i}, x^{1i})$. The vectors of adopted basis $X^H_{i\,|(x^0, x^1)}$, $X^V_{i\,|(x^0, x^1)}$ in $T_{(x^0, x^1)} TM$ do not belong to a tangent bundle of second order. But they allows to describe a local basis in fibres of the bundle: ${}^2_0\pi : {}^2M \longrightarrow M$. For any $A \in {}^2M$ we have:

$$A = x^{1i}\frac{\partial}{\partial x^{0i}} + x^{2i}\frac{\partial}{\partial x^{1i}} =$$

$$= x^{1i}\left[\frac{\partial}{\partial x^{0i}} - \Gamma^k_{ij}x^{1j}\frac{\partial}{\partial x^{1k}}\right] + (x^{2i} + \Gamma^i_{jk}\, x^{1j}x^{1k})\frac{\partial}{\partial x^{1i}} = \tag{1.14}$$

$$= x^{1i}\, X^H_{i\,|(x^0,\, x^1)} + (x^{2i} + \Gamma^i_{jk}\, x^{1j}x^{1k})\cdot X^V_{i\,|(x^0,\, x^1)}\,.$$

Using the operations $\oplus$, $\odot$ and putting:

$$z^{0i} = x^{0i},\quad z^{1i} = x^{1i},\quad z^{2i} = x^{2i} + \Gamma^i_{jk}x^{1j}x^{1k}, \tag{1.15}$$

we can write:

$$A = z^{1i}\, X^H_{i\,|(z^{0j},\, z^{1j})} + z^{2i}\, X^V_{i\,|(z^{0j},\, z^{1j})} = z^{1i}\, X^V_{i\,|(z^{0j},\, z^{1k}\delta^j_k)} \oplus z^{2i}\, X^V_{i\,|(z^0,\, 0)} =$$

$$= z^{1i} \odot X^H_{i\,|(z^{0j},\, \delta^j_i)} \oplus z^{2i} \odot X^V_{i\,|(z^{0j},\, 0)}\,. \tag{1.16}$$

Thus, a local basis in fibres of the bundle: ${}^2_0\pi : {}^2M \to M$ forms a system of $2n$-sections: E^0_{1i}, E^0_{2i} defined in a local chart (U, x^{0i}) by the following formulas:

$$E^0_{1i\,|(x^0)} = X^H_{i\,|(x^{0j},\, \delta^j_i)} = \frac{\partial}{\partial x^{0i}} - \Gamma^k_{ij}\,\delta^j_i\frac{\partial}{\partial x^{1k}}\,{}_{|(x^{0j},\, \delta^j_i)}\,,$$

$$E^0_{2i\,|(x^0)} = X^V_{i\,|(x^{0j},\, 0)} = \frac{\partial}{\partial x^{1i}}\,{}_{|(x^{0j},\, 0)}\,. \tag{1.17}$$

Local coordinates in the vector bundle ${}^2M$ with respect to a local basis E^0_{1i}, E^0_{2i} have the form (1.15). If local charts: (U, x^{0i}), $(U', x^{0i'})$ describe basis: (E^0_{1i}, E^0_{2i}), $(E^0_{1i'}, E^0_{2i'})$ then changing charts: $x^{0i} = x^{0i}(x^{0i'})$ on $U \cap U'$ we obtain:

$$E^0_{1i'} = A^i_{i'}\, E^0_{1i},\quad E^0_{2i'} = A^i_{i'}\, E^0_{2i}\,,$$

$$z^{1i} = A^i_{i'}\, z^{1i'}\,,\quad z^{2i} = A^i_{i'}\, z^{2i'}\,, \tag{1.18}$$

where $A^i_{i'} = \dfrac{\partial x^{0i}}{\partial x^{0i'}}$. For a given linear connection Γ in the bundle ${}^1_0\pi : TM \to M$ we can describe in natural way a horizontal subbundle ${}^2M^H$ and vertical subbundle ${}^2M^V$ for the tangent bundle of second order ${}^2M$ as follows:

$$
\begin{aligned}
{}^2M^H &= \left\{ A \in TTM : T_\pi A = {}^1_0\pi_* A,\ DA = 0 \right\} , \\
{}^2M^V &= \left\{ A \in TTM : T_\pi A = {}^1_0\pi_* A = 0 \right\} .
\end{aligned} \tag{1.19}
$$

Thus we have:

Theorem 1. ([1], [2]) *Let M be C^∞ manifold and Γ be a linear connection in the bundle ${}^1_0\pi : TM \to M$ with a connection map D. Then the tangent bundle of second order ${}^2_0\pi : {}^2M \to M$ has a vector bundle structure defined by the diffeomorphism:*

$$
{}^1_0\pi_* \times D : {}^2M \longrightarrow TM \underset{M}{\times} TM , \tag{1.12}
$$

into Whitney sum of the bundles TM and TM over M.

The tangent bundle of second order is Whitney sum of the horizontal subbundle ${}^2M^H$ and the vertical subbundle ${}^2M^V$: ${}^2M = {}^2M^H \underset{M}{\times} {}^2M^V$. Local basis of sections of the bundle ${}^2M$ consist of sections E^0_{1i}, E^0_{2i} spanning ${}^2M^H$, ${}^2M^V$ respectively and being defined locally by the formula:

$$
\begin{aligned}
E^0_{1i\,|\,(x^0)} &= X^H_{i\,|\,(x^{0j},\,\delta^j_i)} = \frac{\partial}{\partial x^{0i}} - \Gamma^k_{ij}\,\delta^j_i \frac{\partial}{\partial x^{1k}}\,{}_{|\,(x^{0j},\,\delta^j_i)} , \\
E^0_{2i\,|\,(x^0)} &= X^V_{i\,|\,(x^{0j},\,0)} = \frac{\partial}{\partial x^{1i}} .
\end{aligned} \tag{1.17}
$$

A local chart of type of vector bundle on ${}^2M$ corresponding to a vector structure defined by sections E^0_{1i}, E^0_{2i} has the form: $({}^2_0\pi^{-1}(U), z^{0i}, z^{1i}, z^{2i})$

$$
z^{0i} = x^{0i},\ z^{1i} = x^{1i},\ z^{2i} = x^{2i} + \Gamma^i_{jk}\, x^{1j} x^{1k}, \tag{1.15}
$$

where (x^{0i}, x^{1i}, x^{2i}) denotes induced coordinates in ${}^2M$.

2. A total space of tangent bundle of second order ${}^2M$ is a submanifold of TTM. We consider the natural injection:

$$
\begin{aligned}
&i : {}^2M \longrightarrow TTM \\
&i : z^{1i} E^0_{1i} + z^{2i} E^0_{2i\,|\,(z^0)} \longrightarrow x^{1i} \frac{\partial}{\partial x^{0i}} + x^{2i} \frac{\partial}{\partial x^{1i}}\,{}_{|\,(x^0,\, x^1)} .
\end{aligned} \tag{2.1}
$$

We consider the horizontal subbundle: ${}^2_0\pi : {}^2M^H \to M$ of the bundle ${}^2M$. We define an equivalence relation in the set of sections horizontal subbundle ${}^2M^H$:

$$
A \sim B \Longleftrightarrow \bigvee_{\lambda \in \mathbf{R}^*} B = \lambda \odot A, \quad \mathbf{R}^* = \mathbf{R} \setminus \{0\}. \tag{2.2}
$$

For sections: A, B: $U \to {}^2M^H$ we have:

$$A = A^{1i} \circledcirc E^0_{1i\,|\,(x^0)} = A^{1i} \circledcirc X^H_{i\,|\,(x^{0j},\,\delta^j_i)} = A^{1i}\, X^H_{i\,|\,(x^{0j},\,A^{1j})}\,,$$
$$B = \lambda \circledcirc A = (\lambda A^{1i}) \circledcirc E^0_{1i} = \lambda A^{1i}\, X^H_{i\,|\,(x^{0j},\,\lambda A^{1j})}\,. \tag{2.3}$$

Using the natural injection i, (2.1), for sections of bundles ^2M and TTM we have:

$$i : A = A^{1i} E^0_{1i} \longrightarrow iA = A^{1i} \frac{\partial}{\partial x^{0i}} - \Gamma^i_{jk} A^{1j} A^{1k} \frac{\partial}{\partial x^{1i}}\;{}_{|\,(x^{0j},\,A^{1j})}.$$

The section $A = A^{1i} E^0_{1i}$ has coordinates of type vector bundles in ^2M of the form: $(x^{0i};\, A^{1i},\, 0)$ and the induced coordinates iA in TTM are of the form: $(x^{0i},\, A^{1i};\, A^{1i},\, -\Gamma^i_{jk} A^{1j} A^{1k})$. A curve $c : t \to c(t)$ on M is called an integral curve of a section $A \in {}^2M$, if its canonical lift to TM: $C = (c, \dot{c}) : t \to C(t) = (c(t), \dot{c}(t))$ ($\dot{c}(t)$ being tangent to $c(t)$) is an integral curve of the section $iA \in TTM$: $\dot{C}(t) = (iA)(C(t))$. Then in a local chart (U, x^{0i}) we have:

$$C = (c, \dot{c}) : t \longrightarrow \left((x^{0i} \cdot c)(t),\ \frac{d(x^{0i} \cdot c)}{dt}(t) \right),$$
$$\frac{d(x^{0i} \cdot c)}{dt}\Big|_t = A^{1i}(c(t)),$$
$$\frac{d^2 (x^{0i} \cdot c)}{dt^2}\Big|_t = -\Gamma^i_{jk}(c(t))\, A^{1j}(c(t))\, A^{1k}(c(t)). \tag{2.4}$$

Thus for an integral curve of section: $A : U \to {}^2M$ in virtue (2.4) we get following equation:

$$\frac{d^2 (x^{0i} \cdot c)}{dt^2}\Big|_t + \Gamma^i_{jk}(c(t)) \frac{d(x^{0j} \cdot c)}{dt}\Big|_t \frac{d(x^{0k} \cdot c)}{dt}\Big|_t = 0. \tag{2.5}$$

Similarly an integral curve of a section $B = \lambda \circledcirc A$ equivalent to A has the same equation (2.5).

For the equivalence class $[A]$ of a section $A \in {}^2M$ with respect to the relation $\sim$, the class of the sections $iA \in T(TM)$ is called a geodesic flow of the connection Γ, [2].

Thus we get:

Theorem 2. *Let M be a manifold with a given linear connection Γ and ^2M be a linearized tangent bundle of second order by connection Γ. Equivalence class $[A]$ of section $A \in {}^2M^H$ horizontal subbundle $^2M^H \to M$ with respect relation $\sim$:*

$$A \sim B \longleftrightarrow B = \lambda \odot A, \ \lambda \in R^*;$$

describe locally a set of geodesic (2.5) *on manifold* M *with respect connection* Γ, *as integral curve of the section* A. *If* $A = A^{1i} E^0_{1i}$ *is a section and* $A_p \in {}^2M_p$ *for fixed* $p \in M$, *then geodesic* $c : t \to c(t)$ *for the connection* Γ *through* p *i.e.* $c(0) = p$, $\dot{c}(0) = {}^2_0\pi_* A_p$ *is integral curve of section* A *through* $p = c(0)$.

3. Let A be a section of tangent bundle ${}^1_0\pi : TM \to M$. Horizontal lift of section A of tangent bundle ${}^1_0\pi : TM \to M$ into tangent bundle of second order ${}^2_0\pi : {}^2M \to M$ is called a section ${}^2A^H \in {}^2M^H$ such that:

$$^1_0\pi_* (i\,({}^2A^H)) = A, \ D\,(i\,({}^2A^H)) = 0. \tag{3.1}$$

Vertical lift of section A of tangent bundle ${}^1_0\pi : TM \to M$ into bundle ${}^2_0\pi : {}^2M \to M$ is called a section ${}^2A^V \in {}^2M^V$ such that:

$$^1_0\pi_* (i\,({}^2A^V)) = 0, \ D\,(i\,({}^2A^V)) = A\,. \tag{3.2}$$

Complete lift of section A of bundle ${}^1_0\pi : TM \to M$ into bundle ${}^2_0\pi : {}^2M \to M$ is called a section ${}^2A^C \in {}^2M$ such that:

$$^1_0\pi_* (i\,({}^2A^C)) = \mathrm{A}, \ \mathrm{D}\,(i\,({}^2A^C)) = \nabla_A A\,. \tag{3.3}$$

Locally, for a section $A = A^i \dfrac{\partial}{\partial x^{0i}}$ of a tangent bundle ${}^1_0\pi : TM \to M$ its horizontal, vertical and complete lifts into the tangent bundle of second order ${}^2_0\pi : {}^2M \to M$ are of the form respectively:

$$^2A^H = A^i E^0_{1i}\,,$$

$$^2A^V = A^i E^0_{2i}\,, \tag{3.4}$$

$$^2A^C = A^i E^0_{1i} + (\nabla_A A)^i E^0_{2i}\,.$$

Next, we have:

Theorem 3. *Let* A *be a section of the tangent bundle* ${}^1_0\pi : TM \to M$ *and* A^H, A^C *denote its horizontal, complete lifts into the bundle* $T_\pi : TTM \to TM$, *respectively. Then its horizontal and complete lifts into the tangent bundle of second order are defined by a composition:*

$$^2A^H = A^H \cdot A, \ {}^2A^C = A^C \cdot A \tag{3.5}$$

Proof. The value of the section ${}^2A^H$ at a point x is

$$^2A^H_x = (A^H \cdot A)\,(x) = A^H\,(A\,(x)) = A^H_{A(x)}\,.$$

Its induced coordinates in TTM are of the form:

$$(x^i, A^i; A^i, -\Gamma^i_{jk} A^j A^k)\,.$$

On the other hand the vector bundle coordinates in ${}^2M$ are the form $(x^i; A^i, 0)$. Thus we get: $A^H \cdot A = {}^2A^H \in {}^2M^H$. The complete lift: ${}^2A^C_x = (A^C \cdot A)(x) = A^C(A(x)) = A^C_{A(x)}$ has induced coordinates in TTM of the form: $(x^i, A^i; A^i, \partial_k A^i A^k)$, and the vector bundle coordinates of the form: $(x^i; A^i, (\nabla_A A)^i)$. Thus we get: ${}^2A^C = A^C \cdot A$.

Remark: *Let A be a section of the tangent bundle ${}^1_0\pi : TM \to M$ and γ be a geodesic on a manifold M with a given connection Γ such that: $\dot{\gamma} = A(\gamma)$. Then complete and horizontal lifts of into the bundle ${}^2_0\pi : {}^2M \to M$ coincide: ${}^2A^C = {}^2A^H$.*

Proof: In virtue of (3.4) for $A(\gamma) = \dot{\gamma}$ we get: ${}^2A^C = {}^2A^H$.

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STRESZCZENIE

W pracy badamy strukturę uliniowionej wiązki stycznej drugiego rzędu ${}^2_0\pi : {}^2M \to M$ rozmaitości M.

Wprowadzenie koneksji liniowej Γ na rozmaitości M (t.j. w wiązce stycznej $TM \to M$) pozwala wyposażyć jej wiązkę styczną drugiego rzędu ${}^2_0\pi : {}^2M \to M$ w strukturę wiązki wektorowej ([1], [2]). Struktura ta w wiązce ${}^2M \to M$ pozwala utworzyć jej subwiązkę horyzontalną ${}^2M^H$ oraz subwiązkę wertykalną ${}^2M^V$.

Klasy równoważności przekrojów subwiązki horyzontalnej ${}^2M^H$ wyznaczają geodezyjne na rozmaitości M.

Ponadto, struktura wiązki wektorowej ${}^2_0\pi : {}^2M \to M$ pozwala wprowadzić podniesienia horyzontalne, wertykalne i zupełne przekrojów wiązki stycznej $TM \to M$ do wiązki stycznej drugiego rzędu ${}^2M \to M$. Podniesienia te są związane z klasycznymi podniesieniami przekrojów wiązki $TM \to M$ do wiązki $T(TM) \to TM$, ([3]).

РЕЗЮМЕ

В работе исследуется структура линеаризованного касательного расслоения второго порядка ${}^2_0$: ${}^2M \to M$ на дифференцируемом многообразии M.

Введение линейной связности Γ на многообразии M (т.е. в касательном расслоении ${}^1_0\pi : TM \to M$) дает возможность определить в касательном расслоении второго порядка ${}^2_0\pi : {}^2M \to M$ структуру векторного расслоения ([1], [2]).

Эта структура в расслоении ${}^2_0\pi : {}^2M \to M$ позволяет образовать горизонтальные подрасслоения ${}^2M$ и вертикальные подрасслоения ${}^2M^H$. Классы эквивалентности сечений горизонтального подрасслоения ${}^2M^H$ определяют геодезические на многообразии M Структура векторного расслоения ${}^2_0\pi : {}^2M \to M$ позволяет определить горизонтальный, вертикальный и полный лифт сечений касательного расслоения $TM \to M$ до касательного расслоения второго порядка ${}^2_0\pi : M \to M$. Эти лифты связаны с классическими лифтами сечений расслоения $TM \to M$ до расслоения $T(TM) \to TM$, [3].