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On a Determinant and Spectrum of some Matrix, I

O wyznaczniku i widmie pewnej macierzy, I

Об определителе и спектре некоторой матрицы, I

This paper considers the problem of computing of determinant and spectrum of the matrix:

$$(1) \qquad \mathcal{A}_{2m\times 2m} = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

where A, B, C, D denote real, upper (right) triangular $m\times m$ matrices. It is well known ([1]) that the spectral radius condition $\rho(G)<1$ (G denotes a real $m\times m$ matrix) guarantees the convergence of the iterative process of the form

$$(2) \qquad x_{n+1} = Gx_n, \quad x_{n+1}, x_n \in R^{2m}, \quad n = 0,1,\dots, \quad x_0 \text{ - given.}$$

The criterions for the convergence and divergence of the process (2), generated by the matrix of the form (1), are presented below; it seems to us that these criterions will be useful in some numerical calculations.

Let us mention that the process (2) can be also considered as an initial value problem for a first order ordinary difference equation ([4]). Oldenburger has found ([3]) the connection between the spectral radius of G and the stability of the trivial solution of the equation (2). Some corollaries for stability of the trivial solution of the equation (2) are also presented in this paper.

LEMMA 1. *Matrix of the form* (1) *is reducible.*

In order to prove this fact it is sufficient - taking into account the theorem given in [5] (p. 50) - to point out such index subset $J \subset \{1,2,\ldots,2m\}$ that

(3) $\mathcal{A}_{2m\times 2m}[i,j]$ for any pair (i,j), $(i\in J,\ j\notin J)$.

It is easily seen that for the set $J = \{m,2m\}$ the condition (3) is satisfied. So, Lemma 1 is proved.

THEOREM 1. *For the matrix* (1) *the following formula holds*

$$\det \mathcal{A}_{2m\times 2m} = \prod_{i=1}^{m} \det \begin{bmatrix} a_{i,i} & b_{i,i} \\ c_{i,i} & d_{i,i} \end{bmatrix}$$

P r o o f by induction. For $m = 2$ we have:

$$\mathcal{A}_{4\times 4} = \begin{bmatrix} a_{1,1} & a_{1,2} & b_{1,1} & b_{1,2} \\ 0 & a_{2,2} & 0 & b_{2,2} \\ c_{1,1} & c_{1,2} & d_{1,1} & d_{1,2} \\ 0 & c_{2,2} & 0 & d_{2,2} \end{bmatrix}$$

Since matrix $\mathcal{A}_{4\times4}$ is reducible then there exists a transformation P (by "orthogonality") such that matrix $P\mathcal{A}_{4\times4}P^T$ has the form

$$(4)\qquad \begin{bmatrix} B_{1,1} & B_{1,2} \\ \Theta & B_{2,2} \end{bmatrix}$$

where $B_{1,1}$, $B_{2,2}$ denote square matrices and Θ-nullmatrix. Let us consider the matrix $P = [e_1, e_3, e_2, e_4]$, where e_i $(i = \overline{1(1)4})$ denote unity vectors of real 4-dimensional space R^4. Matrix P is non-singular and has the inverse of the form

$$P^{-1} = P^T = \begin{bmatrix} e_1^T \\ e_3^T \\ e_2^T \\ e_4^T \end{bmatrix}$$

Since the transformation by "orthogonality" does not change the value of determinant we have

$$\det \mathcal{A}_{4\times4} = \det P\mathcal{A}_{4\times4}P^T =$$

$$= \det\left\{\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_{1,1} & a_{1,2} & b_{1,1} & b_{1,2} \\ 0 & a_{2,2} & 0 & b_{2,2} \\ c_{1,1} & c_{1,2} & d_{1,1} & d_{1,2} \\ 0 & c_{2,2} & 0 & d_{2,2} \end{bmatrix}\right)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right\} =$$

$$= \det \begin{bmatrix} a_{1,1} & a_{1,2} & b_{1,1} & b_{1,2} \\ c_{1,1} & c_{1,2} & d_{1,1} & d_{1,2} \\ 0 & a_{2,2} & 0 & b_{2,2} \\ 0 & c_{2,2} & 0 & d_{2,2} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \det \begin{bmatrix} a_{1,1} & b_{1,1} & a_{1,2} & b_{1,2} \\ c_{1,1} & d_{1,1} & c_{1,2} & d_{1,2} \\ 0 & 0 & a_{2,2} & b_{2,2} \\ 0 & 0 & c_{2,2} & d_{2,2} \end{bmatrix}$$

Hence, by [2] (p. 52) we obtain:

$$\det \mathcal{A}_{4\times 4} = \det \begin{bmatrix} a_{1,1} & b_{1,1} \\ c_{1,1} & d_{1,1} \end{bmatrix} \det \begin{bmatrix} a_{2,2} & b_{2,2} \\ c_{2,2} & d_{2,2} \end{bmatrix} = \prod_{i=1}^{2} \det \begin{bmatrix} a_{i,i} & b_{i,i} \\ c_{i,i} & d_{i,i} \end{bmatrix} .$$

So, Theorem 1 for $m = 2$ is proved.

Let us now assume that

$$(5) \qquad \det \mathcal{A}_{2m\times 2m} = \prod_{i=1}^{m} \det \begin{bmatrix} a_{i,i} & b_{i,i} \\ c_{i,i} & d_{i,i} \end{bmatrix} ;$$

we are going now to show that

$$\det \mathcal{A}_{2(m+1)\times 2(m+1)} = \prod_{i=1}^{m+1} \det \begin{bmatrix} a_{i,i} & b_{i,i} \\ c_{i,i} & d_{i,i} \end{bmatrix} ,$$

where
$$\mathcal{A}_{2(m+1)\times 2(m+1)} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,m+1} & b_{1,1} & \cdots & b_{1,m+1} \\ & \ddots & \vdots & & \ddots & \vdots \\ 0 & & a_{m+1,m+1} & 0 & & b_{m+1,m+1} \\ c_{1,1} & \cdots & c_{1,m+1} & d_{1,1} & \cdots & d_{1,m+1} \\ & \ddots & \vdots & & \ddots & \vdots \\ 0 & & c_{m+1,m+1} & 0 & & d_{m+1,m+1} \end{bmatrix}$$

By Lemma 1, there exists a transformation $\widetilde{P}$ (by "orthogonality") such that $\widetilde{P}\mathcal{A}_{2(m+1)\times 2(m+1)}\widetilde{P}^T$ has the form (4); let us consider the matrix $\widetilde{P} = [\tilde{e}_1, \tilde{e}_2, \ldots, \tilde{e}_m, \tilde{e}_{2m+1}, \tilde{e}_{m+2}, \ldots$ $\ldots, \tilde{e}_{2m-1}, \tilde{e}_{2m}, \tilde{e}_{m+1}, \tilde{e}_{2m+2}]$, where $\tilde{e}_i$ $(i = \overline{1(1)2m + 2})$ are unity vectors of real $2m+2$-dimensional space R^{2m+2} so,

$$\widetilde{P}\hat{\mathcal{A}}_{2(m+1)\times 2(m+1)}\widetilde{P}^T =$$

$$\left[\begin{array}{cccccccc}
a_{1,1} & \cdots & a_{1,m} & a_{1,m+1} & b_{1,1} & \cdots & b_{1,m} & b_{1,m+1} \\
 & \ddots & \vdots & \vdots & & \ddots & \vdots & \vdots \\
0 & & a_{m,m} & a_{m,m+1} & 0 & & b_{m,m} & b_{m,m+1} \\
 & & c_{m,m} & c_{m,m+1} & & & d_{m,m} & d_{m,m+1} \\
c_{1,1} & c_{1,m-1} & c_{1,m} & c_{1,m+1} & d_{1,1} & d_{1,m-1} & d_{1,m} & d_{1,m+1} \\
 & \ddots & \vdots & \vdots & & \ddots & \vdots & \vdots \\
 & c_{m-1,m-1} & c_{m-1,m} & c_{m-1,m+1} & & d_{m-1,m-1} & d_{m-1,m} & d_{m-1,m+1} \\
0 & & 0 & a_{m+1,m+1} & 0 & & 0 & b_{m+1,m+1} \\
 & & 0 & c_{m+1,m+1} & & & 0 & d_{m+1,m+1}
\end{array}\right]\widetilde{P}^T =$$

$$
= \begin{bmatrix}
a_{1,1} & \cdots & a_{1,m-1} & a_{1,m} & b_{1,m} & b_{1,1} & \cdots & b_{1,m-1} & a_{1,m+1} & b_{1,m+1} \\
 & \ddots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots \\
0 & & a_{m-1,m-1} & a_{m-1,m} & b_{m-1,m} & 0 & & b_{m-1,m-1} & a_{m-1,m+1} & b_{m-1,m+1} \\
 & & 0 & a_{m,n} & b_{m,m} & & & 0 & a_{m,m+1} & b_{m,m+1} \\
 & & 0 & c_{m,m} & d_{m,m} & & & 0 & c_{m,m+1} & d_{m,m+1} \\
c_{1,1} & & c_{1,m-1} & c_{1,m} & d_{1,m} & d_{1,1} & & d_{1,m-1} & c_{1,m+1} & d_{1,m+1} \\
 & \ddots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots \\
 & & c_{m-1,m-1} & c_{m-1,m} & d_{m-1,m} & 0 & & d_{m-1,m-1} & c_{m-1,m+1} & d_{m-1,m+1} \\
0 & & & & & & & 0 & a_{m+1,m+1} & b_{m+1,m+1} \\
 & & & 0 & & & & 0 & c_{m+1,m+1} & d_{m+1,m+1}
\end{bmatrix}
$$

or in a much closer form

$$
(6) \qquad \tilde{P}\mathcal{A}_{2(m+1)\times 2(m+1)}\tilde{P}^T = \begin{bmatrix} \tilde{\mathcal{A}}_{2m\times 2m} & \tilde{\tilde{\mathcal{A}}}_{2m\times 2} \\ \Theta_{2\times 2m} & R_{2\times 2} \end{bmatrix}
$$

where $R_{2\times 2} = \begin{bmatrix} a_{m+1,m+1} & b_{m+1,m+1} \\ c_{m+1,m+1} & d_{m+1,m+1} \end{bmatrix}$; so, the matrix (6) has the form (4). Thus, by [2] (p. 52) we get

$$
(7) \quad \det \tilde{P}\mathcal{A}_{2(m+1)\times 2(m+1)}\tilde{P}^T = \det\tilde{\mathcal{A}}_{2m\times 2m}\det\begin{bmatrix} a_{m+1,m+1} & b_{m+1,m+1} \\ c_{m+1,m+1} & d_{m+1,m+1} \end{bmatrix}.
$$

Evidently:

$$\det \tilde{P} \mathcal{A}_{2(m+1)\times 2(m+1)} \tilde{P}^T = \det \mathcal{A}_{2(m+1)\times 2(m+1)} ,$$

so with respect to (7) we obtain

$$\det \mathcal{A}_{2(m+1)\times 2(m+1)} = \det \tilde{\mathcal{A}}_{2m\times 2m} \det \begin{bmatrix} a_{m+1,m+1} & b_{m+1,m+1} \\ c_{m+1,m+1} & d_{m+1,m+1} \end{bmatrix}. \tag{8}$$

Let us now use the transformation $\tilde{\tilde{P}}$ (by "orthogonality"), $\tilde{\tilde{P}} = [\tilde{\tilde{e}}_1, \tilde{\tilde{e}}_2, \dots, \tilde{\tilde{e}}_m, \tilde{\tilde{e}}_{m+2}, \tilde{\tilde{e}}_{m+3}, \dots, \tilde{\tilde{e}}_{2m}, \tilde{\tilde{e}}_{m+1}]$, where $\tilde{\tilde{e}}_i$ $(i = 1(1)2m)$ are unity vectors of real 2m-dimensional space R^{2m}. Because the matrix $\tilde{\mathcal{A}}_{2m\times 2m}$ has the form:

$$\begin{bmatrix}
a_{1,1} & \cdots\, a_{1,m-2} & a_{1,m-1} & a_{1,m} & b_{1,m} & b_{1,1} & \cdots\, b_{1,m-2} & b_{1,m-1} \\
 & \ddots & \vdots & \vdots & \vdots & & \ddots & \vdots \\
 & 0 & a_{m-1,m-1} & a_{m-1,m} & b_{m-1,m} & 0 & & b_{m-1,m-1} \\
 & 0 & & a_{m,m} & b_{m,m} & 0 & & 0 \\
0 & 0 & & c_{m,m} & d_{m,m} & 0 & & 0 \\
c_{1,1} & \cdots\, c_{1,m-2} & c_{1,m-1} & c_{1,m} & d_{1,m} & d_{1,1} & \cdots\, d_{1,m-2} & d_{1,m-1} \\
 & \ddots & \vdots & \vdots & \vdots & & \ddots & \vdots \\
0 & c_{m-2,m-2} & c_{m-2,m-1} & c_{m-2,m} & d_{m-2,m} & 0 & d_{m-2,m-2} & d_{m-2,m-1} \\
0 & & c_{m-1,m-1} & c_{m-1,m} & d_{m-1,m} & 0 & & d_{m-1,m-1}
\end{bmatrix}$$

thus

$$\tilde{\tilde{P}}\,\mathcal{A}_{2m\times 2m}\tilde{\tilde{P}}^{T} =$$

$$\left[\begin{array}{cccccccc}
a_{1,1} & \cdots\; a_{1,m-2} & a_{1,m-1} & a_{1,m} & b_{1,m} & b_{1,1} & \cdots\; b_{1,m-2} & b_{1,m-1} \\
0 & \ddots & \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\
 & & a_{m-1,m-1} & a_{m-1,m} & b_{m-1,m} & & & b_{m-1,m-1} \\
 & & 0 & a_{m,m} & b_{m,m} & 0 & 0 & 0 \\
c_{1,1} & \cdots\; c_{1,m-2} & c_{1,m-1} & c_{1,m} & d_{1,m} & d_{1,1} & \cdots\; d_{1,m-2} & d_{1,m-1} \\
0 & \ddots & \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\
 & & c_{m-1,m-1} & c_{m-1,m} & d_{m-1,m} & & 0 & d_{m-1,m-1} \\
0 & & & c_{m,m} & d_{m,m} & & 0 & 0
\end{array}\right] P^{T} =$$

$$\left(\begin{array}{cccccc}
a_{1,1} & \cdots\; a_{1,m-1} & a_{1,m} & b_{1,1} & \cdots\; b_{1,m-1} & b_{1,m} \\
 & \ddots\;\vdots & \vdots & & \ddots\;\vdots & \vdots \\
 & a_{m-1,m-1} & a_{m-1,m} & & b_{m-1,m-1} & b_{m-1,m} \\
 & \ddots & a_{m,m} & & & b_{m,m} \\
c_{1,1} & \cdots\; c_{1,m-1} & c_{1,m} & d_{1,1} & \cdots\; d_{1,m-1} & d_{1,m} \\
 & \ddots\;\vdots & \vdots & & \ddots\;\vdots & \vdots \\
 & c_{m-1,m-1} & c_{m-1,m} & & d_{m-1,m-1} & d_{m-1,m} \\
 & & c_{m,m}. & & & d_{m,m}
\end{array}\right) = \mathcal{A}_{2m\times 2m}$$

Hence

$$\det \tilde{\tilde{P}}\,\tilde{\mathcal{A}}_{2m\times 2m}\tilde{\tilde{P}}^{T} = \det \mathcal{A}_{2m\times 2m}$$

but

$$\det \tilde{\tilde{P}} \tilde{\mathcal{A}}_{2m\times 2m} \tilde{\tilde{P}}^T = \det \tilde{\mathcal{A}}_{2m\times 2m} ;$$

so

(9) $$\det \tilde{\mathcal{A}}_{2m\times 2m} = \det \mathcal{A}_{2m\times 2m} .$$

Taking into account (5), (8) and (9) we get

$$\det \mathcal{A}_{2(m+1)\times 2(m+1)} = \det \tilde{\mathcal{A}}_{2m\times 2m} \det \begin{bmatrix} a_{m+1,m+1} & b_{m+1,m+1} \\ c_{m+1,m+1} & d_{m+1,m+1} \end{bmatrix} =$$

$$= \det \mathcal{A}_{2m\times 2m} \det \begin{bmatrix} a_{m+1,m+1} & b_{m+1,m+1} \\ c_{m+1,m+1} & d_{m+1,m+1} \end{bmatrix} = \prod_{i=1}^{m+1} \det \begin{bmatrix} a_{i,i} & b_{i,i} \\ c_{i,i} & d_{i,i} \end{bmatrix}$$

and the proof of our Theorem is completed.

THEOREM 2. The spectrum of the matrix (1) is equal to the union of the spectra of the matrices of the form $\begin{bmatrix} a_{i,i} & b_{i,i} \\ c_{i,i} & d_{i,i} \end{bmatrix}$ $i = \overline{1(1)m}$,

i.e. $$\sigma(\mathcal{A}_{2m\times 2m}) = \bigcup_{i=1}^{m} \sigma\left(\begin{bmatrix} a_{i,i} & b_{i,i} \\ c_{i,i} & d_{i,i} \end{bmatrix}\right).$$

P r o o f. Because the eigenvalues of the matrix $\mathcal{A}_{2m\times 2m}$ are roots of the equation

$$\det \left[\mathcal{A}_{2m\times 2m} - \lambda I_{2m\times 2m}\right] = 0$$

and matrix $\mathcal{A}_{2m\times 2m} - \lambda I_{2m\times 2m}$ has the form (1); so, by previous Theorem, we have

$$\det\left[\mathcal{A}_{2m\times 2m} - \lambda I_{2m\times 2m}\right] = \prod_{i=1}^{m} \det\begin{bmatrix} a_{i,i}-\lambda & b_{i,i} \\ c_{i,i} & d_{i,i}-\lambda \end{bmatrix} =$$

$$= \prod_{i=1}^{m} \det\left(\begin{bmatrix} a_{i,i} & b_{i,i} \\ c_{i,i} & d_{i,i} \end{bmatrix} - I_{2\times 2}\right)$$

That proves the Theorem.

From Theorem 2 and Kakeya theorem ([6]) it follows:

COROLLARY 1. If there exists such number l $(1\leqslant l\leqslant m)$ that

$$0<1<-(a_{l,l}+d_{l,l})<\det\begin{bmatrix} a_{l,l} & b_{l,l} \\ c_{l,l} & d_{l,l} \end{bmatrix},$$

then matrix $\mathcal{A}_{2m\times 2m}$ has an eigenvalue with modulus greater than one.

COROLLARY 2. If for any l $(l=\overline{1(1)m})$ the inequalities

$$1>-(a_{l,l}+d_{l,l})>\det\begin{bmatrix} a_{l,l} & b_{l,l} \\ c_{l,l} & d_{l,l} \end{bmatrix}>0$$

are satisfied, then the spectral radius of $\mathcal{A}_{2m\times 2m}$ is less than one.

REMARK 1. Corollary 1 gives a simple criterion for the divergence of the iterative proces (2) generated by the matrix $\mathcal{A}_{2m\times 2m}$.

By Oldenburger's theorem and the form of spectrum of $\mathcal{A}_{2m\times 2m}$ we immediately obtain

THEOREM 3. The trivial solution of difference equation (2) (with matrix $\mathcal{A}_{2m\times 2m}$) is asymptotically stable iff for any i $(i = \overline{1(1)m})$ all roots of equations

$$\lambda^2 - (a_{i,i} + d_{i,i})\lambda + a_{i,i}b_{i,i} - c_{i,i}d_{i,i} = 0 \tag{10}$$

have moduli less than one.

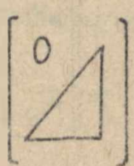
From Faddiejewa's theorem ([1]) pp. 100-101) and from the form of spectrum of $\mathcal{A}_{2m\times 2m}$ we obtain:

THEOREM 4. Iterative process (2), with matrix $\mathcal{A}_{2m\times 2m}$, is convergent iff for any i $(i = \overline{1(1)m})$ all roots of equations (10) have moduli less than one.

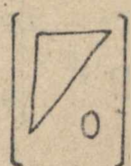
REMARK 2. Because the transformation by "orthogonality" does not change the value of determinant and the spectrum of matrix, thus Theorems 1, 2, 3, 4 and Corollaries 1, 2 are valid for every matrix G which is similar to $\mathcal{A}_{2m\times 2m}$. In particular for the matrices of the forms:

$$\begin{bmatrix} M & N \\ P & Q \end{bmatrix}, \quad \begin{bmatrix} M & R \\ S & D \end{bmatrix}, \quad \begin{bmatrix} M & B \\ C & Q \end{bmatrix}, \quad \begin{bmatrix} A & R \\ S & Q \end{bmatrix}, \quad \begin{bmatrix} A & S \\ R & Q \end{bmatrix}, \tag{11-15}$$

where M, N, P, Q denote lower (left) and A, B, C, D upper (right) real, triangular $m\times m$ matrices and R, S-real, $m\times m$ matrices of the forms:



and



respectively; the transformation P (by "orthogonality") has the form

$$(11') \quad \begin{bmatrix} P & \Theta_{m\times m} \\ \Theta_{m\times m} & P \end{bmatrix} \left(\tilde{P}^T = \begin{bmatrix} P^T & \Theta_{m\times m} \\ \Theta_{m\times m} & P \end{bmatrix} \right),$$

$$(12') \quad \begin{bmatrix} P & \Theta_{m\times m} \\ \Theta_{m\times m} & I_{m\times m} \end{bmatrix} \left(\tilde{P}^T = \begin{bmatrix} P^T & \Theta_{m\times m} \\ \Theta_{m\times m} & I_{m\times m} \end{bmatrix} \right),$$

$$(13') \quad \begin{bmatrix} P & \Theta_{m\times m} \\ \Theta_{m\times m} & P^T \end{bmatrix} \left(\tilde{P}^T = \begin{bmatrix} P^T & \Theta_{m\times m} \\ \Theta_{m\times m} & P \end{bmatrix} \right),$$

$$(14') \quad \begin{bmatrix} I_{m\times m} & \Theta_{m\times m} \\ \Theta_{m\times m} & P^T \end{bmatrix} \left(\tilde{P}^T = \begin{bmatrix} I_{m\times m} & \Theta_{m\times m} \\ \Theta_{m\times m} & P \end{bmatrix} \right),$$

$$(15') \quad \begin{bmatrix} I_{m\times m} & \Theta_{m\times m} \\ \Theta_{m\times m} & P \end{bmatrix} \left(P^T = \begin{bmatrix} I_{m\times m} & \Theta_{m\times m} \\ \Theta_{m\times m} & P^T \end{bmatrix} \right)$$

for the matrices: (11), (12), (13), (14) and (15), respectively ($P = [e_m, \ldots, e_1]$ and $\Theta_{m\times m}$ is the nullmatrix).

For example:

$$\tilde{P}_{(11')} \mathcal{A}_{2m\times 2m} \tilde{P}_{(11')}^T = \begin{bmatrix} P & \Theta_{m\times m} \\ \Theta_{m\times m} & P \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P^T & \Theta_{m\times m} \\ \Theta_{m\times m} & P^T \end{bmatrix} = \begin{bmatrix} M & N \\ P & Q \end{bmatrix}$$

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STRESZCZENIE

W pracy podano nowe kryteria zbieżności względnie rozbieżności procesu iteracyjnego postaci $x_{n+1} = Gx_n$, generowanego przez macierz G.

Резюме

В работе представлены новые критерия сходимости или расходимости итерационного процесса типа $x_{n+1} = Gx_n$, порождаемого матрицей G .

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