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JAN STANKIEWICZ

**Quasibordination and Quasimajorization of Analytic Functions**

Quasipodporządkowanie i quasimajoryzacja funkcji analitycznych

Квазиподчинение и квазимажорация аналитических функций

Let  $B$  be the class of analytic functions in  $K_R$ ,  $K_R = \{z: |z| < R\}$  and bounded  $|\varphi(z)| \leq 1$  for  $z \in K_R$ .

Let  $\Omega$  denote the class of analytic functions in  $K_R$ , such that  $|\omega(z)| \leq |z|$  for  $z \in K_R$ .

Let  $f(z)$ ,  $F(z)$  be two functions which are single-valued and analytic in the disc  $K_R$ .

**Definition 1.** The function  $f(z)$  is said to be subordinate to  $F(z)$  in  $K_R$  if there exists a function  $\omega(z) \in \Omega$ , for which

$$f(z) = F(\omega(z)), \quad z \in K_R.$$

In this case we write

$$f(z) \rightarrow F(z) \text{ in } K_R.$$

**Definition 2.** The function  $f(z)$  is said to be majorized by  $F(z)$  in  $K_R$ , if there exists a function  $\varphi(z) \in B$ , such that

$$f(z) = \varphi(z)F(z) \text{ for } z \in K_R.$$

We write then

$$f(z) \ll F(z) \text{ in } K_R.$$

Note that  $f(z) \ll F(z)$  in  $K_R$ , if and only if, for every  $z \in K_R$  we have  $|f(z)| \leq |F(z)|$ .

Several theorems exist in the literature that relate  $f(z)$  and  $F(z)$  when  $f(z) \rightarrow F(z)$  which have their counterparts that relate  $g(z)$  and  $G(z)$  when  $g(z) \ll G(z)$ . In order to establish some semblance of unification for parallel results for subordination and majorization, M. S. Robertson [6] introduced the concept of quasibordination.

**Definition 3.** [6]. Let  $f(z)$  and  $F(z)$  be analytic in  $K_R$ . Let  $\varphi(z)$  be analytic and bounded for  $z \in K_R$ ,  $|\varphi(z)| \leq 1$ , such that  $f(z)/\varphi(z)$  is regular and subordinate to  $F(z)$  in  $K_R$ . Then  $f(z)$  is said to be quasisubordinate to  $F(z)$  w.r.t. to  $\varphi(z)$  in  $K_R$ .

If  $f(z)$  is quasisubordinate to  $F(z)$  w.r.t. to  $\varphi(z)$  we shall often say simply that  $f(z)$  is quasisubordinate to  $F(z)$  in  $K_R$  and write

$$f(z) \rightarrow_q F(z) \text{ in } K_R.$$

From this definition we have that  $f(z) \rightarrow_q F(z)$  in  $K_R$ , if and only if there exists the function  $\varphi(z) \in B$  and  $\omega(z) \in \Omega$ , such that

$$f(z) = \varphi(z)F(\omega(z)) \text{ for } z \in K_R.$$

The concept of quasisubordination can be also defined as follows:

**Definition 4.** Let  $f(z)$ ,  $F(z)$  be analytic in  $K_R$ . If there exists an analytic function  $g(z)$ , such that  $f(z) \ll g(z)$  in  $K_R$  and  $g(z) \rightarrow F(z)$  in  $K_R$ , then  $f(z)$  is said to be quasisubordinate to  $F(z)$  relative to  $g(z)$  in  $K_R$  and we shall often say simply that  $f(z)$  is quasisubordinate to  $F(z)$  in  $K_R$  and write  $f(z) \rightarrow_q F(z)$  in  $K_R$ .

These two definitions 3 and 4 are equivalent. If  $f(z)$  is quasisubordinate to  $F(z)$  relative to  $\varphi(z)$ , then in Definition 4 we can put  $g(z) = f(z)/\varphi(z)$ . Now if  $f(z)$  is quasisubordinate to  $F(z)$  relative to  $g(z)$ , then we can put  $\varphi(z) = f(z)/g(z)$  in Definition 3.

Thus we have

$$f(z) \rightarrow_q F(z) \text{ in } K_R \Leftrightarrow \exists_{g(z)} (f(z) \ll g(z) \text{ in } K_R) \wedge (g(z) \rightarrow F(z) \text{ in } K_R).$$

Using the Definition 4 we can obtain immediately the generalizations on quasisubordination of all these theorems, which have the same conclusions and the assumption subordination  $f(z) \rightarrow F(z)$  is replaced by that of majorization  $f(z) \ll F(z)$ . In particular we have

**Theorem 1,** [6]. If  $f(z) \rightarrow_q F(z)$  in  $K_R$ , then for every  $\lambda > 0$  and  $r \in (0, 1)$  we have

$$\int_0^{2\pi} |f(re^{i\theta})|^{\lambda} d\theta \leq \int_0^{2\pi} |F(re^{i\theta})|^{\lambda} d\theta.$$

**Theorem 2,** [6]. If  $f(z) \rightarrow_q F(z)$  in  $K_R$  and

$$f(z) = \sum_{k=0}^{\infty} a_k z^k, \quad F(z) = \sum_{k=0}^{\infty} A_k z^k$$

for  $z \in K_R$  then for  $n = 0, 1, 2, \dots$  and  $r \in (0, R)$

$$\sum_{k=0}^n |a_k|^2 r^{2k} \leq \sum_{k=0}^n |A_k|^2 r^{2k}.$$

If the series  $\sum_{k=0}^{\infty} |A_k|^2 r^{2k}$  is convergent for  $r \leq R$  then

$$\sum_{k=0}^{\infty} |a_k|^2 r^{2k} \leq \sum_{k=0}^{\infty} |A_k|^2 r^{2k}, \quad 0 < r \leq R.$$

**Theorem 3.** If  $f(z) \rightarrow_q F(z)$  in  $K_1$  then

$$\sum_{k=0}^n |a_k|^2 \leq \sum_{k=0}^n |A_k|^2, \quad n = 0, 1, 2, \dots,$$

and if the series  $\sum_{k=0}^{\infty} |A_k|^2$  is convergent then

$$\sum_{k=0}^{\infty} |a_k|^2 \leq \sum_{k=0}^{\infty} |A_k|^2.$$

The following theorem is a generalization of Theorem 3 in the paper [3] p. 211.

**Theorem 4.** Let  $\lambda_k$ ,  $k = 1, 2, \dots$ , be a sequence of nonnegative real numbers, such that  $\lambda_k \geq \lambda_{k+1} \geq 0$  for  $k = 1, 2, \dots$ .

If  $f(z) \rightarrow_q F(z)$  in  $K_1$  then

$$\sum_{k=1}^n \lambda_k |a_k|^2 \leq \sum_{k=1}^n \lambda_k |A_k|^2, \quad n = 1, 2, \dots$$

and if the series  $\sum_{k=1}^{\infty} \lambda_k |A_k|^2$  is convergent, then

$$\sum_{k=1}^{\infty} \lambda_k |a_k|^2 \leq \sum_{k=1}^{\infty} \lambda_k |A_k|^2.$$

Let  $A$  be a class of functions analytic in the unit disc  $K_1$  and normalized by the conditions

$$f(0) = 0, \quad f'(0) \geq 0.$$

Now we can introduce the concept of so-called quasibordination in a normalized way.

**Definition 5.** We say that a function  $f(z) \in A$  is quasibordinate in a normalized way to  $F(z) \in A$ , if there exists a function  $g(z) \in A$  such that  $f(z) \ll g(z)$  in  $K_1$  and  $g(z) \rightarrow F(z)$  in  $K_1$ . We write then

$$f(z) \rightarrow_{qu} F(z).$$

For this kind of subordination we can prove the following theorems.

**Theorem 5.** Let  $S_a^*$  denote the class of  $a$ -starlike normalized functions in  $K_1$  that is  $F(0) = F'(0) - 1 = 0$  and  $\operatorname{Re}\{zF'(z)/F(z)\} > a$  for  $z \in K_1$ . If  $F(z) \in S_{1/2}^*$  and  $f(z) \rightarrow_{qu} F(z)$  then for  $|z| = r < 1$  we have

$$|f(z)| \leq T(r, S_{1/2}^*) |F(z)|$$

where

$$T(r, S_{1/2}^*) = \max \left\{ 1, \frac{r}{1-r} \right\}.$$

**Theorem 6.** Let  $S^c$  denote the class of convex normalized functions in  $K_1$  that is  $F(0) = 0 = F'(0) - 1$  and  $\operatorname{Re}\{1 + zF''(z)/F'(z)\} > 0$  in  $K_1$ .

If  $F(z) \in S^c$  and  $f(z) \rightarrow_{qu} F(z)$  then for  $|z| = r < 1$  we have

$$|f(z)| \leq T(r, S^c) |F(z)|$$

where

$$T(r, S^c) = T(r, S_{1/2}^*) = \max \left\{ 1, \frac{r}{1-r} \right\}.$$

**Theorem 7.** If  $F(z) \in S_0^*$  and  $f(z) \rightarrow_{qu} F(z)$ , then for  $|z| = r < 1$ , we have

$$|f(z)| \leq T(r, S_0^*) |F(z)|$$

where

$$T(r, S_0^*) = \max \left\{ 1, \frac{r}{(1-r)^2} \right\}.$$

The results of the theorems, 5, 6 and 7 are the best possible in this sense that we could not replace the functions  $T(r, S_{1/2}^*) = T(r, S^c)$  and  $T(r, S_0^*)$  by any smaller functions of  $r$  respectively.

The Theorems 5, 6 and 7 follow immediately from one general theorem. In order to formulate it we must introduce first some notations.

Let  $H_z$  denote so-called the Rogosinski's domain bounded by an arc of circumference  $|\zeta| = |z|^2$  and two arcs of circumferences going through the point  $\zeta = z$  and tangent to the circumference  $|\zeta| = |z|^2$  at the points  $\zeta_1 = iz|z|$ ,  $\zeta_2 = -iz|z|$  respectively.

Let  $U$  be an arbitrary fixed subclass of the class  $S$  of normalized ( $f(0) = 0$ ,  $f'(0) = 1$ ) and univalent functions in  $K_1$ , with the following property:  $f(z) \in U \Rightarrow e^{-i\theta} f(e^{i\theta} z) \in U$  for all real  $\theta$ .

Let us put

$$Q(z, U) = \left\{ w : w = \frac{F(\zeta)}{F(z)}, \zeta \in H_z, F(z) \in U \right\}.$$

**Theorem 8.** If  $f(z) \rightarrow_{qu} F(z)$  and  $F(z) \in U$  then for  $r \in (0, 1)$  we have

$$\sup_{|z|=r} \left| \frac{f(z)}{F(z)} \right| \leq T(r, U),$$

where

$$T(r, U) = \sup \{ |w| : w \in Q(r, U) \}.$$

**Proof of Theorem 8.**  $f(z) \rightarrow_{qu} F(z)$  implies that there exists a function  $g(z)$  such that  $f(z) \ll g(z)$ , that is  $|f(z)/g(z)| < 1$  for  $z \in K_1$  and  $g(z) \rightarrow F(z)$  in  $K_1$ . Now by the results of paper [2] (Theorem 1 and Corollary 1) we have that  $|g(z)/F(z)| \leq T(r, U)$  for  $|z| = r < 1$ . Therefore

$$\left| \frac{f(z)}{F(z)} \right| = \left| \frac{f(z)}{g(z)} \right| \cdot \left| \frac{g(z)}{F(z)} \right| \leq T(r, U).$$

Because the above mentioned results of paper [2] are best possible and subordination  $f(z) \rightarrow F(z)$  in  $K_1$  implies quasibordination  $f(z) \rightarrow_q F(z)$  in  $K_1$ , therefore the result given in Theorem 8 is the best possible, too.

**Proof of Theorems 5, 6 and 7.** It is enough to use the Theorem 8 and the functions  $T(r, S_{1/2}^*)$ ,  $T(r, S^c)$  and  $T(r, S_0^*)$  determined in [2].

If in the definitions 4 and 5 we change the role of subordination and majorization then we obtain one new concept.

**Definition 6.** Let  $f(z)$ ,  $F(z)$  be analytic in  $K_R$ . If there exists an analytic function  $h(z)$  such that  $f(z) \rightarrow h(z)$  in  $K_R$  and  $h(z) \ll F(z)$  in  $K_R$ , then  $f(z)$  is said to be quasimajorized by  $F(z)$  relative to  $h(z)$  in  $K_R$  and we shall often say simply that  $f(z)$  is quasimajorized by  $F(z)$  in  $K_R$  and write  $f(z) \ll_q F(z)$  in  $K_R$ .

It is easy to see that  $f(z) \ll_q F(z)$  in  $K_R$  if and only if there exist the functions  $\varphi_1(z) \in B$  and  $\omega_1(z) \in \Omega$  such that

$$f(z) = \varphi_1(\omega_1(z)) \cdot F(\omega_1(z)), \text{ for } z \in K_R.$$

**Definition 7.** We shall say that a function  $f(z) \in A$  is quasimajorized in a normalized way by a function  $F(z) \in A$ , if there exists a function  $h(z) \in A$  such that  $f(z) \rightarrow h(z)$  in  $K_1$  and  $h(z) \ll F(z)$  in  $K_1$ . We shall write then

$$f(z) \ll_{qu} F(z).$$

Quasimajorization and quasibordination are connected by the following relations.

**Lemma 1.** If  $f(z) \ll_q F(z)$  in  $K_R$  then  $f(z) \rightarrow_q F(z)$  in  $K_R$ .

**Lemma 2.** If  $f(z) \ll_{qu} F(z)$  then  $f(z) \rightarrow_{qu} F(z)$ .

**Proof.** In order to prove it it is enough to put  $\varphi(z) = \varphi_1(\omega_1(z))$  and  $\omega(z) = \omega_1(z)$ . Furthermore if  $f, h, F$  belong to  $A$  then  $\varphi_1(0) \geq 0$ ,  $\omega_1'(0) \geq 0$  and therefore  $\varphi(0) \geq 0$ ,  $\omega'(0) \geq 0$ . This implies that  $h(z) = f(z)/\varphi(z) \in A$  and the both lemmas are proved.

**Remark.** By the lemmas 1 and 2 it follows that the theorems 1-8 of this paper are valid if we replace in them quasibordination by quasimajorization.

**Problem.** Lemma 1 says that  $f(z) \ll_q F(z) \Rightarrow f(z) \rightarrow_q F(z)$ . We can change the direction of this implication that is, the concepts of quasibordination and quasimajorization are equivalent.

**Remark.** If  $F(z) = z$  is identity function then  $f(z) \ll_{qu} F(z)$  if and only if  $f(z) \rightarrow_{qu} F(z)$ .

**Proof of the remark.** Necessity follows by Lemma 2. Sufficiency. In this case  $f(z) \rightarrow_{qu} z$  and we have

$$f(z) = \varphi(z)\omega(z) = \varphi_1(\omega_1(z))\omega_1(z)$$

where  $\omega_1(z) = \varphi(z)\omega(z)$ ,  $\varphi_1(z) \equiv 1$ . Therefore  $f(z) \ll_{qu} F(z)$ .

Thus we see that for identity these two concepts are equivalent. In a general case the problem is open.

We can generalize also some such theorems on quasimajorization of which we could not generalize on quasibordination.

**Theorem 9.** If  $f(z) \ll_{qu} F(z)$  and  $F(z) \in S_0^*$  then for every  $R \in (0, 1)$  we have

$$f(K_{r(R)}) \subset F(K_R).$$

where

$$r(R) = r(R, S_0^*) = \min \left\{ R, \frac{\sqrt{R}}{1 + \sqrt{R} + R} \right\}.$$

**Theorem 10.** If  $f(z) \ll_{qu} F(z)$  and  $F(z) \in S_{1/2}^*$  then for every  $R \in (0, 1)$  we have

$$f(K_{r(R)}) \subset F(K_R),$$

where

$$r(R) = r(R, S_{1/2}^*) = \min \left\{ R, \frac{\sqrt{5R^2 + 4R} - R}{2(1 + R)} \right\}.$$

**Corollary.** *Theorem 10 is valid if the hypothesis  $F(z) \in S_{1/}^*$  is replaced by  $F(z) \in S^c$  since  $S_{1/2}^* \supseteq S^c$ .*

The theorems 9 and 10 follow from some general theorem which is a generalization of Theorem 2.2 of paper [4]. Now we formulate

**Theorem 11.** *If  $f(z) \ll_{qu} F(z)$  and  $F(z) \in U$  then for every  $R \in (0, 1)$  we have*

$$f(K_{r(R)}) \subset F(K_R)$$

where

$$r(R) = r(R, U) = \sup_{r \leq R} \{r : O_r \cap D(R, r, U) = \emptyset\}$$

and

$$O_r = \{w : w = \varphi(z), |z| \leq r, \varphi(z) \in B, \varphi(0) \geq 0\},$$

$$D(R, r, U) = \left\{ w : w = \frac{F(z)}{F(\zeta)}, |z| = R, |\zeta| = r, F \in U \right\}.$$

**Proof of Theorem 11.** By the hypothesis  $f(z) \ll_{qu} F(z)$  we have

1°  $f(z) \rightarrow h(z)$  in  $K_1$ ,  $h(z) \in A$ ,

2°  $h(z) \ll F(z)$  in  $K_1$ .

The functions  $h(z), F(z)$  satisfy the hypotheses of Theorem 2.2 [4] and therefore

$$h(K_{r(R)}) \subset F(K_R).$$

From 1° we have that

$$f(K_\varrho) \subset h(K_\varrho)$$

for every  $\varrho \in (0, 1)$ .

Thus we obtain

$$f(K_{r(R)}) \subset F(K_R)$$

and the proof is complete. In an analogous way: Theorem 9 follows from Theorem 2 of paper [5] p. 924 and Theorem 10 follows from Theorem 2 of paper [1] p. 7.

Let us put in the Theorems 11, 10 and 9  $R \rightarrow 1$ . Then we have the following corollaries.

### Corollaries.

Suppose  $f(z) \ll_{qu} F(z)$ .

1. If  $F(z) \in U$ , then

$$f(K_{r_1}) \subset F(K_1)$$

where

$$r_1 = \lim_{R \rightarrow 1} r(R).$$

2. If  $F(z) \in S_{1/2}^*$  then  $f(K_{1/2}) \subset F(K_1)$ .
3. If  $F(z) \in S_0^*$  then  $f(K_{1/3}) \subset F(K_1)$ .

The Theorems 1-11 are not all which we can obtain. There are many other theorems which can be extended on quasibordination and quasi-majorization. Some of these generalizations will be studied in the next paper.

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#### STRESZCZENIE

Od dawna znane są pojęcia podporządkowania  $f \rightarrow F$  i majoryzacji (modułowej)  $f \ll F$ . M. S. Robertson w 1970 r. uogólnił te dwa pojęcia wprowadzając nowe pojęcie quasipodporządkowania  $f \rightarrow_q F$ , które w szczególnych przypadkach daje podporządkowania lub majoryzację. W tej pracy podana jest definicja quasipodporządkowania w postaci

$$f \rightarrow_q F \Leftrightarrow \bigvee_g \{(f \ll g) \wedge (g \rightarrow F)\}.$$

Definicja ta pozwala na otrzymanie niemal bez dowodów wielu twierdzeń dotyczących funkcji quasipodporządkowanych.

Wprowadzone jest tu również pojęcie quasimajoryzacji  $f \ll_q F$  poprzez zamianę roli majoryzacji i podporządkowania w definicji quasipodporządkowania.

$$f \ll_q F \Leftrightarrow \bigvee_g \{(f \rightarrow h) \wedge (h \ll F)\}.$$

Okazuje się, że  $f \ll_q F \Rightarrow f \rightarrow_q F$ , problem odwrotny pozostaje otwarty. Ponadto podanych jest kilka przykładowych twierdzeń wiążących quasipodporządkowanie ze wzajemnym wzrostem funkcji  $f$  i  $F$  oraz quasimajoryzację też ze wzrostem funkcji i dodatkowo z zawieraniem się obszarów  $f(K_r)$ ,  $F(K_R)$  gdzie  $K_r = \{z: |z| < r\}$ .

## РЕЗЮМЕ

Уже давно известно понятие подчинения  $f \prec F$  и мажорации (модульной)  $f \ll F$ . Робертсон М. С. в 1970 г. обобщил эти два понятия, вводя новое понятие квазиподчинения  $f \rightarrow_q F$ , которое в особых случаях создает подчинение или мажорации. В данной работе представлена дефиниция квазиподчинения в виде:

$$f \rightarrow_q F \Leftrightarrow \bigvee_g \{(f \ll g) \wedge (g \prec F)\}$$

Эта дефиниция дает возможность получить почти без доказательств многое теоремы, относящихся к квазиподчиненным функциям.

Введено здесь понятие квазимажорации  $f \ll_q F$ , заменяя роль мажорации и подчинения в дефиницию квазиподчинения.

$$f \prec_q F \Leftrightarrow \bigvee_n \{(f \prec h) \wedge (h \ll F)\}$$

Оказывается, что  $f \ll_q F \Rightarrow f \rightarrow_q F$  оборотная проблема остаётся открытой. Также было представлено несколько примерных теорем, связанных квазиподчинение с обоюдным ростом функций  $f$  и  $F$ , а также квазимажорации с ростом функции и дополнительно с возмещающими областями  $f(K_r), F(K_R)$ , где  $K_r = \{z: |z| < r\}$ .

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