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On an Estimation of the Fraction Defective in Curtailed Sampling under an Inflation

O estymacji frakcji wadliwych elementów w uciętych próbach z populacji o rozkładach ze zniekształceniem

Об оценивании доли дефектных объектов в усеченных выборках из „раздутых" совокупностей

1. Introduction. In a quality inspection the different criteria of the acceptance or of the rejection of a lot are applied. In [2] A. G. Phattak and M. N. Bhatt have considered the following sampling plans.

Plan 1. Inspect a random sample of n units from the lot. Accept the lot if there are fewer than k defectives. Reject the lot if there are k or more defectives.

Plan 2. Inspect randomly selected units of the lot one at a time until either k defectives have been observed or until n units have been inspected. Reject the lot if k defectives are observed. Accept the lot if n units are inspected, provided that the number of defectives observed is less than k.

Plan 3. Inspected randomly selected units of the lot one at a time until either k defectives have been observed or $n-k+1$ nondefectives. Reject the lot if there are k defectives.

It is supposed that in all these plans k and n are predetermined numbers and k is much less than n.

To receive the probability distributions related with the above plans we define the following discrete random variable:

X — number of defectives in n inspected articles.

Y — number of articles inspected when the k-th defective is found.

Z — number of articles inspected when the $(n-k+1)$-th nondefective is found.

I — number of defective found when sampling is curtailed by the finding of the $(n-k+1)$-th nondefective.

$$S = \begin{cases} 0, 1, \ldots, k-1 & \text{when a lot is accepted,} \\ k, k+1, \ldots, n & \text{when a lot is rejected.} \end{cases}$$

The probability distributions of the random variable S are respectively as follows:

Plan 1

$$\text{a) } f_1(s) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & s = x = 0, 1, 2, \ldots, k-1, \\ \binom{n}{x} p^x q^{n-x}, & s = x = k,\ k+1, \ldots, n. \end{cases}$$

Plan 2

$$\text{b) } f_2(s) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & s = x = 0, 1, 2, \ldots, k-1, \\ \binom{y-1}{k-1} p^k q^{y-k}, & s = y = k,\ k+1, \ldots, n. \end{cases}$$

Plan 3

$$\text{c) } f_3(s) = \begin{cases} \binom{n-k+i}{n-k} p^i q^{n-k+1}, & s = i = 0, 1, \ldots, k-1, \\ \binom{y-1}{k-1} p^k q^{y-k}, & s = y = k,\ k+1, \ldots, n, \end{cases}$$

where p is the probability of selecting a defective in a single trial. Further on, it is assumed that the trials are stochastically independent.

In [2] it has been obtained the maximum likelihood estimate and the asymptotic variance of p in the above plans.

In this paper there are considered the similar problems of the rejection or acceptance of a lot in the case when the decision random variable is distributed according to:

Plan 1

$$g_1(s) = \begin{cases} \beta + \alpha q^n, & s = 0, \\ \alpha \binom{n}{s} p^s q^{n-s}, & s = x = 1, 2, \ldots, k-1, \\ \alpha \binom{n}{s} p^s q^{n-s}, & s = x = k,\ k+1, \ldots, n. \end{cases}$$

Plan 2

$$g_2(s) = \begin{cases} \beta + aq^n, & s = 0, \\ a\binom{n}{s} p^s q^{n-s}, & s = x = 1, 2, \ldots, k-1, \\ a\binom{s-1}{k-1} p^k q^{s-k}, & s = y = k, k+1, \ldots, n. \end{cases}$$

Plan 3

$$g_3(s) = \begin{cases} \beta + aq^{n-k+1}, & s = 0, \\ a\binom{n-k+s}{n-k} p^s q^{n-k+1}, & s = i = 1, 2, \ldots, k-1, \\ a\binom{s-1}{k-1} p^k q^{s-k}, & s = y = k, k+1, \ldots, n, \end{cases}$$

where $0 < a \leqslant 1$, $a + \beta = 1$. The parameter a can be interpreted as the fraction of population which has one of the distributions a), b) or c).

Let us observe, that $\sum_{s=0}^{k-1} g_j(s)$, $j = 1, 2, 3$ is the probability of acceptance of a lot, $\sum_{s=k}^{n} g_j(s)$, $j = 1, 2, 3$ is the probability of a rejection of a lot, and

$$\sum_{s=1}^{k-1} g_j(s) + \sum_{s=k}^{n} g_j(s) = 1, \quad j = 1, 2, 3.$$

The considered models of inspection of a random sample can appear, for instance, in such cases, when the part of articles is double controlled. The first control is realized by a producer who sends defectives to be repaired. The second control is realized by a customer. The number of defectives received by a customer is in fact less than the number of produced defectives. The inspected population is the mixture of two groups of elements, where one of them has the distribution a), b) or c) and the distribution of second part of population is degenerated at the point $s = 0$.

2. Estimates of the parameters of the distributions $g_1(s)$, $g_2(s)$ and $g_3(s)$. Let T be a number of lots which have undergone inspection under one of the described plans. Let us introduce the following notations:

$a_{x,j}$ — number of accepted lots under plan j-th in which the number of defectives in the sample was $x (j = 1, 2)$.

$a_{i,3}$ — number of accepted lots under plan 3 in which the number of defectives in the sample was i.

$a_{z,3}$ — number of accepted lots under Plan 3 in which the number of articles in the sample was z.

$r_{x,1}$ – number of rejected lots under Plan 1 in which the number of defectives in the sample was x.

$r_{y,j}$ – number of rejected lots under Plan j in which the number of articles in the sample was $y (j = 2, 3)$.

Note that in Plan 3, when a lot is accepted, the experimenter can observe either I or Z, because $i = z-(n-k+1)$.

Let $T_{a,j}$ $(j = 1, 2, 3)$ denote a total number of accepted lots and $T_{r,j}$ – a number of rejected lots $(j = 1, 2, 3)$ in the respective plans. We have:

Plan 1

$$T_{a,1} = \sum_{x=0}^{k-1} a_{x,1}, \quad T_{r,1} = \sum_{x=k}^{n} r_{x,1},$$

Plan 2

$$T_{a,2} = \sum_{x=0}^{k-1} a_{x,2}, \quad T_{r,2} = \sum_{y=k}^{n} r_{y,2},$$

Plan 3

$$T_{a,3} = \begin{cases} \sum_{i=0}^{k-1} a_{i,3} & \text{when the random variable } I \text{ is recorded,} \\ \sum_{z=n-k+1}^{n} a_{z,3} & \text{when the random variable } Z \text{ is recorded,} \end{cases}$$

$$T_{r,3} = \sum_{y=k}^{n} r_{y,3}.$$

Let us denote

$$D_{a,j} = \sum_{x=0}^{k-1} x a_{x,j}, \; j = 1, 2, \quad D_{a,3} = \sum_{i=0}^{k-1} i a_{i,3},$$

$$D_{r,1} = \sum_{x=k}^{n} x r_{x,1}, \quad D_{r,j} = \sum_{y=k}^{n} y r_{y,j}, \; j = 2, 3.$$

Note that $D_{a,j}$, $j = 1, 2, 3$ is the total number of observed defectives in accepted lots under Plan j $(j = 1, 2, 3)$, $D_{r,1}$ is the total number of observed defectives in rejected lots under Plan 1, and $D_{r,j}$ $(j = 2, 3)$ is the total number of inspected articles in rejected lots under Plan j $(j = 2, 3)$.

The likelihood functions in respective plans are as follows:

Plan 1

$$L_1 = (\beta + \alpha q^n)^{a_{0,1}} \prod_{x=1}^{k-1} \left[\alpha \binom{n}{x} p^x q^{n-x}\right]^{a_{x,1}} \prod_{x=k}^{n} \left[\alpha \binom{n}{x} p^x q^{n-x}\right]^{r_{x,1}},$$

Plan 2

$$L_2 = (\beta + aq^n)^{a_{0,2}} \prod_{x=1}^{k-1} \left[a \binom{n}{x} p^x q^{n-x} \right]^{a_{x,2}} \prod_{y=k}^{n} \left[a \binom{y-1}{k-1} p^k q^{y-k} \right]^{r_{y,2}},$$

Plan 3

$$L_3 = (\beta + aq^{n-k+1})^{a_{0,3}} \prod_{i=1}^{k-1} \left[a \binom{n-k+i}{n-k} p^i q^{n-k+1} \right]^{a_{i,3}} \times$$

$$\times \prod_{y=k}^{n} \left[a \binom{y-1}{k-1} p^k q^{y-k} \right]^{r_{y,3}}.$$

Therefore,

(1) $$\operatorname{Log} L_1 = a_{0,1} \log(\beta + aq^n) + \sum_{x=1}^{k-1} a_{x,1} \left[\log a + \log \binom{n}{x} + x \log p + \right.$$

$$\left. + (n-x) \log q \right] + \sum_{x=k}^{n} r_{x,1} \left[\log a + \log \binom{n}{x} + x \log p + (n-x) \log q \right],$$

(2) $$\log L_2 = a_{0,2} \log(\beta + aq^n) + \sum_{x=1}^{k-1} a_{x,2} \left[\log a + \log \binom{n}{x} + x \log p + \right.$$

$$\left. + (n-x) \log q \right] + \sum_{y=k}^{n} r_{y,2} \left[\log a + \log \binom{y-1}{k-1} + k \log p + (y-k) \log q \right],$$

(3) $$\log L_3 = a_{0,3} \log(\beta + aq^{n-k+1}) + \sum_{i=1}^{k-1} a_{i,3} \left[\log a + \log \binom{n-k+i}{n-k} + \right.$$

$$\left. + i \log p + (n-k+1) \log q \right] + \sum_{y=k}^{n} r_{y,3} \left[\log a + \log \binom{y-1}{k-1} + k \log p + \right.$$

$$\left. + (y-k) \log q \right].$$

Differentiating the equalities (1), (2) and (3) with respect to p and a and equating it to zero, after easy calculations, we obtain

Plan 1

(4) $$\hat{p} = \frac{(D_{a,1} + D_{r,1})(1 - \hat{q}^n)}{n(T_{a,1} + T_{r,1} - a_{0,1})},$$

(5) $$\hat{a} = \frac{T_{a,1} + T_{r,1} - a_{0,1}}{(T_{a,1} + T_{r,1})(1 - \hat{q}^n)},$$

Plan 2

(6) $$\hat{p} = \frac{(D_{a,2} + kT_{r,2})(1 - \hat{q}^n)}{n(T_{a,2} - a_{0,2}) + D_{r,2} - \hat{q}^n (D_{r,2} - nT_{r,2})},$$

(7) $$\hat{a} = \frac{T_{a,2} + T_{r,2} - a_{0,2}}{(T_{a,2} + T_{r,2})(1 - \hat{q}^n)},$$

Plan 3

(8) $$\hat{p} = \frac{(D_{a,3}+kT_{r,3})(1-\hat{q}^{n-k+1})}{[D_{a,3}+(n-k+1)T_{a,3}+D_{r,3}](1-\hat{q}^{n-k+1})+(n-k+1)[(T_{a,3}+T_{r,3})\hat{q}^{n-k+1}-a_{0,3}]},$$

(9) $$\hat{a} = \frac{T_{a,3}+T_{r,3}-a_{0,3}}{(T_{r,3}+T_{a,3})(1-\hat{q}^{n-k+1})}.$$

The maximum likelihood estimates obtained from (4) — (9) are not linear in p and a. Thus, there is some trouble with their calculate. In order to get the pilot estimates $\bar{p}$ and $\bar{a}$ of p and a respectively, we can use the method of linearization considered in [4].

Pilot estimates $\bar{p}$ and $\bar{a}$ of p and a concerning Plan 1 and 2 can be obtained from the following equations

(10) $$\begin{cases} f_0 = \beta + aq^n, \\ f_1 = anpq^{n-1}, \end{cases}$$

where f_0 and f_1 are observed relative frequencies for 0 defectives and 1 defectives respectively in sample of size n, and in the case of Plan 3 from

(11) $$\begin{cases} f_0 = \beta + aq^{n-k+1}, \\ f_1 = a(n-k+1)pq^{n-k+1}, \end{cases}$$

where f_0 is observed relative frequency for $n-k+1$ nondefective and f_1 is observed relative frequency for 1 defective and $n-k+1$ nondefective. The right-hand sides of (10) and (11) are the probabilities of the mentioned events.

Eliminating parameter a from (10) and (11), we have

(10′) $$1-q^n = \frac{np(1-f_0)}{qf_1+np(1-f_0)},$$

(11′) $$1-q^{n-k+1} = \frac{p(1-f_0)(n-k+1)}{f_1+p(n-k+1)(1-f_0)}.$$

Putting (10′) into (4), (5), (6) and (7) and next putting (11′) into (8) and (9), we obtain linear equations for estimators of p and a respectively. Thus, we have:

Plan 1

$$\bar{p} = \frac{(D_{a,1}+D_{r,1})(1-f_0)-f_1(T_{a,1}+T_{r,1}-a_{0,1})}{(T_{a,1}+T_{r,1}-a_{0,1})[n(1-f_0)-f_1]},$$

$$\bar{a} = \frac{(D_{a,1}+D_{r,1})(T_{a,1}+T_{r,1}-a_{0,1})[n(1-f_0)-f_1]}{n(T_{a,1}+T_{r,1})[(D_{a,1}+D_{r,1})(1-f_0)-f_1(T_{a,1}+T_{r,1}-a_{0,1})]},$$

Plan 2

$$\bar{p} = \frac{(1-f_0)(D_{a,2}+kT_{r,2})-f_1(T_{a,2}+T_{r,2}-a_{0,2})}{(1-f_0)(D_{r,2}+nT_{a,2}-na_{0,2})-f_1(T_{a,2}+T_{r,2}-a_{0,2})},$$

$$\bar{a} = \frac{(T_{a,2}+T_{r,2}-a_{0,2})[n(1-f_0)(D_{a,2}+kT_{r,2})-f_1(D_{a,2}+kT_{r,2}-D_{r,2}+nT_{r,2})]}{n(T_{a,2}+T_{r,2})[(1-f_0)(D_{a,2}+kT_{r,2})-f_1(T_{a,2}+T_{r,2}-a_{0,2})]},$$

Plan 3

$$\bar{p} = \frac{(1-f_0)(D_{a,3}+kT_{r,3})-f_1(T_{a,3}+T_{r,3}-a_{0,3})}{(1-f_0)[D_{a,3}+D_{r,3}+(n-k+1)(T_{a,3}-a_{0,3})]},$$

$$\bar{a} =$$

$$\frac{(T_{a,3}+T_{r,3}-a_{0,3})\{f_1[D_{a,3}+D_{r,3}-(n-k+1)T_{r,3}]+(n-k+1)(D_{a,3}+kT_{r,3})(1-f_0)\}}{(T_{a,3}+T_{r,3})(n-k+1)[(1-f_0)(D_{a,3}+kT_{r,3})-f_1(T_{a,3}+T_{r,3}-a_{0,3})]}.$$

3. Variance and covariance of the estimators. The asymptotic variance and covariance of the estimators $\hat{p}$ and $\hat{a}$ are given by the following matrix M

$$M = \begin{bmatrix} -E\dfrac{\partial^2 \log L}{\partial a^2} & -E\dfrac{\partial^2 \log L}{\partial a \partial p} \\ -E\dfrac{\partial^2 \log L}{\partial a \partial p} & -E\dfrac{\partial^2 \log L}{\partial p^2} \end{bmatrix}^{-1} = \tag{12}$$

$$= \begin{bmatrix} \sigma^2(\hat{a}) & \mathrm{Cov}(\hat{a},\hat{p}) \\ \mathrm{Cov}(\hat{a},\hat{p}) & \sigma^2(\hat{p}) \end{bmatrix}.$$

The second derivatives of the likelihood functions are of the form:

Plan 1

$$\frac{\partial^2 \log L_1}{\partial p^2} = \frac{p-q}{p^2q^2}(D_{a,1}+D_{r,1}) - \frac{n}{q^2}(T_{a,1}+T_{r,1}) + \frac{\beta n a_{0,1}[\beta+a(n+1)q^n]}{q^2(\beta+aq^n)^2},$$

$$\frac{\partial^2 \log L_1}{\partial a^2} = -\frac{1}{a^2}(T_{a,1}+T_{r,1}) + \frac{a_{0,1}(1-2a+2aq^n)}{a^2(\beta+aq^n)^2}$$

and

$$\frac{\partial^2 \log L_1}{\partial a \partial p} = -\frac{na_{0,1}q^{n-1}}{(\beta+aq^n)^2}.$$

Plan 2

$$\frac{\partial^2 \log L_2}{\partial p^2} = \frac{p-q}{p^2q^2}[D_{a,2}+kT_{r,2}-p(nT_{a,2}+D_{r,2})] - \frac{1}{pq}(nT_{a,2}+D_{r,2}) + \beta n a_{0,2}\frac{\beta+a(n+1)q^n}{q^2(\beta+aq^n)^2},$$

$$\frac{\partial^2 \log L_2}{\partial a^2} = -\frac{1}{a^2}(T_{a,2}+T_{r,2}) + \frac{a_{0,2}(1-2a+2aq^n)}{a^2(\beta+aq^n)^2}$$

and

$$\frac{\partial^2 \log L_2}{\partial a \partial p} = -\frac{n a_{0,2} q^{n-1}}{(\beta + a q^n)^2}.$$

Plan 3

$$\frac{\partial^2 \log L_3}{\partial p^2} = \frac{p-q}{p^2 q^2}(D_{a,3} + kT_{r,3}) - \frac{1}{q^2}[D_{a,3} + (n-k+1)T_{a,3} + D_{r,3}] +$$

$$+ \beta(n-k+1) a_{0,3} \frac{\beta + a(n-k+2) q^{n-k+1}}{q^2(\beta + a q^{n-k+1})^2},$$

$$\frac{\partial^2 \log L_3}{\partial a^2} = -\frac{1}{a^2}(T_{a,3} + T_{r,3}) + a_{0,3} \frac{1 - 2a + 2a q^{n-k+1}}{a^2(\beta + a q^{n-k+1})^2},$$

and

$$\frac{\partial^2 \log L_3}{\partial a \partial p} = -\frac{(n-k+1) a_{0,3} q^{n-k}}{(\beta + a q^{n-k+1})^2}.$$

Let us observe that

$$E\frac{a_{0,j}}{T} = \begin{cases} \beta + a q^n, & j = 1, 2, \\ \beta + a q^{n-k+1}, & j = 3, \end{cases}$$

$$E\frac{D_{a,1} + D_{r,1}}{T} = anp,$$

$$E\frac{D_{a,2}}{T} = anpB(p, n-1, k-2),$$

where $B(p, u, w) = \sum_{x=0}^{w} \binom{u}{x} p^x q^{u-x}$

and $B(p, u+1, w) = pB(p, u, w-1) + qB(p, u, w)$,

$$E\frac{D_{a,2} + kT_{r,2}}{T} = aJ_1,$$

where

$$J_1 = np\ B(p, n-1, k-2) + k[1 - B(p, n, k-1)],$$

$$pE\frac{nT_{a,2} + D_{r,2}}{T} = \beta np + aJ_2,$$

where

$$J_2 = np\ B(p, n, k-1) + k[1 - B(p, n+1, k)],$$

$$E\frac{D_{a,3} + kT_{r,3}}{T} = aJ_3,$$

where

$$J_3 = (n-k+1)\,B(p, n+1, k-1)/q - (n-k+1)\,B(p, n, k-1) + + k[1 - B(p, n, k-1)],$$

and

$$pE\,\frac{D_{a,3} + (n-k+1)T_{a,3} + D_{r,3}}{T} = \beta(n-k+1)p + aJ_4,$$

where

$$J_4 = (n-k+1)pB(p, n+1, k-1)/q + k[1 - B(p, n+1, k)].$$

Taking into account the above relations, we get:

Plan 1

$$\sigma^2(\hat{p}) = \frac{T(1-q^n)}{a(\beta + aq^n)}\,/\Delta_1,$$

$$\sigma^2(\hat{a}) = \frac{anT}{pq}\left[1 - \frac{\beta npq^{n-1}}{\beta + aq^n}\right]/\Delta_1,$$

$$\mathrm{cov}(\hat{a}, \hat{p}) = -\frac{nTq^{n-1}}{\beta + aq^n}\,/\Delta_1,$$

where

$$\Delta_1 = \frac{nT^2(1 - q^n - npq^{n-1})}{pq(\beta + aq^n)}.$$

Plan 2

$$\sigma^2(\hat{p}) = \frac{T(1-q^n)}{a(\beta + aq^n)}\,/\Delta_2,$$

$$\sigma^2(\hat{a}) = aT\left[\frac{J_2}{p^2q} - \frac{\beta n^2 q^{n-2}}{\beta + aq^n}\right]/\Delta_2,$$

$$\mathrm{Cov}(\hat{a}, \hat{p}) = -\frac{nTq^{n-1}}{\beta + aq^n}\,/\Delta_2,$$

where

$$\Delta_2 = \frac{T^2}{p^2q(\beta + aq^n)}\left[(1-q^n)J_2 - n^2p^2q^{n-1}\right].$$

Plan 3

$$\sigma^2(\hat{p}) = \frac{T(1-q^{n-k+1})}{a(\beta + aq^{n-k+1})}\,/\Delta_3,$$

$$\sigma^2(\hat{a}) = aT\left[\frac{J_3}{p^2q} - \frac{\beta(n-k+1)^2q^{n-k+1}}{\beta + aq^{n-k+1}}\right]/\Delta_3,$$

$$\mathrm{Cov}(\hat{a}, \hat{p}) = -\frac{T(n-k+1)q^{n-k}}{\beta + aq^{n-k+1}}\,/\Delta_3,$$

where

$$\Delta_3 = \frac{T^2}{p^2 q(\beta + aq^{n-k+1})} [(1-q^{n-k+1})J_3 - (n-k+1)^2 p^2 q^{n-k}].$$

After making use of the linearization, we have:

Plan 1

$$\sigma^2(\bar{p}) = pq(1-f_0)/(anM_1),$$

$$\sigma^2(\bar{a}) = a(af_0 - \beta f_1)/M_1,$$

$$\mathrm{Cov}(\bar{a}, \bar{p}) = -p(f_0 - \beta)/M_1,$$

where

$$M_1 = T(1-f_0-f_1).$$

Plan 2

$$\sigma^2(\bar{p}) = p^2 q(1-f_0)/(aTM_2),$$

$$\sigma^2(\bar{a}) = \frac{a}{T}[\beta + J_2(f_0 - \beta)/M_2],$$

$$\mathrm{Cov}(\bar{a}, \bar{p}) = -pqf_1/(TM_2),$$

where

$$M_2 = (1-f_0)J_2 - npf_1.$$

Plan 3

$$\sigma^2(\bar{p}) = p^2 q^2(1-f_0)/(aTM_3),$$

$$\sigma^2(\bar{a}) = \frac{a}{T}[\beta + q(f_0 - \beta)J_3/M_3],$$

$$\mathrm{Cov}(\bar{a}, \bar{p}) = -pqf_1/(TM_3),$$

where

$$M_3 = q(1-f_0)J_3 - p(n-k+1)f_1.$$

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STRESZCZENIE

W pracy rozważa się problem kontroli jakości partii towaru, w przypadku gdy partia ta jest niejednorodna i obserwowana zmienna losowa (liczba elementów wadliwych w próbce) ma rozkład ze zniekształceniem w punkcie zero z parametrami p i a. Autor uzyskuje wyrażenia (uproszczone przez linearyzację) dla estymatorów największej wiarygodności parametrów p i a dla trzech planów pobierania próby, a ponadto znajduje asymptotyczne wariancje i kowariancje tych estymatorów.

РЕЗЮМЕ

В работе рассматривается проблема контроля качества партии товара, в случае когда эта партия неоднородна и наблюдавшаяся случайная величина (число дефектных объектов в выборке) имеет „раздутое" распределение в точке нуль с параметрами p и a. Автор получает выражения (упрощенные линейным преобразованием) для оценок максимального правдоподобия параметров p и a при трех выборочных планах контроля и кроме того находит асимптотические дисперсии и ковариации этих оценок.