ANNALES
UNIVERSITATIS MARIAE CURIE-SKLODOWSKA
LUBLIN-POLONIA
VOL. LI.2, 18 SECTIO A 1997

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Generalized Vector Variational Inequalities for Multifunctions

ABSTRACT. In this paper, we consider a generalized vector variational inequality and a generalized vector quasivariational inequality for multifunctions, and give some existence theorems of solutions for our inequalities.

1. Introduction and preliminaries. Recently, Giannessi [8] firstly introduced a vector variational inequality for vector-valued functions in a finite-dimensional Euclidean space. Since then, many authors (Chen et al. [2-7], Lee et al. [9-10], Lee et al. [11-16], Siddiqi et al. [19] and Yang. [22-24]) have intensively studied several kinds of vector variational inequalities for vector-valued functions or multifunctions in abstract spaces. In paricular, Chen et al. [7] and Lee et al. [16] obtained some existence theorems of solutions of vector quasivariational inequalities for vector-valued functions in abstract spaces. Lee et al. [9] and Lee et al. [13-14] obtained existence theorems of solutions for vector variational inequalities for multifunctions with vector values in abstract spaces.

The first and second authors were supported by KOSEF Grant 961-0102-009-2.
1991 *Mathematics Subject Classification.* 47H10.
Key words and phrases. Multifunctions, generalized vector variational inequality, generalized vector quasivariational inequality, Fan-Browder fixed point theorem, Fan-Glicksberg-Kakutani fixed point theorem.

In this paper, we will consider the following generalized vector variational inequality and generalized vector quasivariational inequalities for multifunctions. Our generalized vector variational inequality is a more general form than that of many authors [2, 5, 11, 14, 22-23]. Also, our generalized vector quasivariational inequality is a more general form than that of Chen et al. [7] and Lee et al. [16]

Let X be a nonempty convex subset of a Hausdorff topological vector space, Y be a Hausdorff topological vector space and Z be a Hausdorff topological vector space with a convex cone C such that $intC \neq \phi$, where int denotes the interior. Let $S : X \to 2^X$, $T : X \to 2^Y$, and $G : X \times N \times X \to 2^Z$ be multifunctions, where N is a subset of Y containing $T(X)$.

Consider the following generalized vector variational inequality (GVVI) and generalized vector quasivariational inequality (GVQVI) for multifunctions;

(GVVI) · Find $\overline{x} \in X$ and $\overline{y} \in T(\overline{x})$ such that for any $x \in X$ and any $z \in G(\overline{x}, \overline{y}, x)$, $z \notin -intC$.

(GVQVI) Find $\overline{x} \in S(\overline{x})$ and $\overline{y} \in T(\overline{x})$ such that for any $x \in S(\overline{x})$ and any $z \in G(\overline{x}, \overline{y}, x)$, $z \notin -intC$.

In this paper, we give an existence theorem of solutions for (GVVI), using Fan-Browder fixed point theorem, and an existence theorem of solutions for (GVQVI), using Fan-Glicksberg-Kakutani fixed point theorem.

Now we give some definitions and preliminary results needed in the later sections.

Definition 1.1 [17,21]. Let X be a convex subset of a Hausdorff topological vector space and Z be a Hausdorff topological vector space with a convex cone C . Let $f : X \to Z$ be a vector-valued function.

(1) f is said to C-convex if for any $x_1, x_2 \in X$ and $\lambda \in [0, 1]$,

$$f(\lambda x_1 + (1 - \lambda)x_2) \in \lambda f(x_1) + (1 - \lambda)f(x_2) - C.$$

(2) f is said to be quasi C-convex if for any $z \in Z$, the set $B(z) := \{x \in X | z - f(x) \in C\}$ is convex.

Now we extend the above definition to the multifunction case.

Definition 1.2 [18] Let X be a convex subset of a Hausdorff topological vector space and Z be a Hausdorff topological vector space with a convex cone C. Let $F : X \to 2^Z$ be a multifunction.

(1) F is said to be C-convex if for every $x_1, x_2 \in X$, $\lambda \in [0,1]$, $y_1 \in F(x_1)$ and $y_2 \in F(x_2)$, there exists $y_3 \in F(\lambda x_1 + (1-\lambda)x_2)$ such that

$$\lambda y_1 + (1-\lambda)y_2 - y_3 \in C.$$

(2) F is said to be quasi C-convex if for any $z \in Z$, the set $B(z) = \{x \in X :$ there is a $y \in F(x)$ such that $z - y \in C\}$ is convex.

Remark 1.1. If F is C-convex, then F is quasi C-convex.

Definition 1.3. Let F be a multifunction from a Hausdorff topological space X to a Hausdorff topological space Y.

(1) F is said to be upper semi-continuous (shortly, u.s.c.) at $x_0 \in X$ if for every open set U in Y containing $F(x_0)$, there is a neighborhood V of x_0 in X such that $F(x) \subset U$ for all $x \in V$.

(2) F is said to be lower semi-continuous (shortly, l.s.c.) at $x_0 \in X$ if for every open set U in Y with $F(x_0) \cap U \neq \phi$, there is a neighborhood V of x_0 in X such that $F(x) \cap U \neq \phi$ for all $x \in V$. This definition is equivalent to: for any net $\{x_\alpha\}$ in X converging to x_0 in X and any $y_0 \in F(x_0)$, there exists a net $\{y_\alpha\}$ in Y such that $y_\alpha \in F(x_\alpha)$ for any α converging to y_0.

(3) F is said to be continuous at $x_0 \in X$ if F is u.s.c. and l.s.c. at $x_0 \in X$. Also, F is called continuous on X if it is continuous at every point $x \in X$.

(4) F is said to be closed if the graph of F, $graph(F) := \{(x,y) \in X \times Y : y \in F(x)\}$, is closed.

Lemma 1.1. *Let X be a convex subset of a Hausdorff topological vector space and $F : X \to 2^{\mathbb{R}}$ be a multifunction, where $\mathbb{R}$ is the real number system.*

(1) *If F is quasi $\mathbb{R}^+$-convex, then for any $\lambda \in \mathbb{R}$, the set $\{x \in X : \exists s \in F(x)$ such that $s < \lambda\}$ is convex, where $\mathbb{R}^+ = \{\alpha \in \mathbb{R} : \alpha \geq 0\}$.*

(2) *If F is compact-valued, the converse holds.*

Proof. (1) is clear.

We will prove that (2) holds. Let $\lambda \in \mathbb{R}$. Then we have

$$\{x \in X : \exists s \in F(x) \text{ such that } s \leq \lambda\} \subset \bigcap_{n=1}^{\infty} \Big\{x \in X : \exists s \in F(x) \text{ such that } s < \lambda + \frac{1}{n}\Big\}.$$

Suppose that

$$x \in \bigcap_{n=1}^{\infty} \Big\{x \in X : \exists s \in F(x) \text{ such that } s < \lambda + \frac{1}{n}\Big\} \tag{1.1}$$

and

$$x \notin \{x \in X : \exists s \in F(x) \text{ such that } s \leq \lambda\}. \tag{1.2}$$

Then from (1.1), for each $n \in \mathbb{N}$, there exists a sequence $\{s_n\}$ in $F(x)$ such that $s_n < \lambda + \frac{1}{n}$. Since $F(x)$ is compact, there exists a subsequence $\{s_{n_i}\}$ of $\{s_n\}$ such that s_{n_i} converges to $s_* \in F(x)$. Hence $s_* \leq \lambda$ and $s_* \in F(x)$. This is a contradiction to the fact that for any $s \in F(x)$, $s > \lambda$ from (1.2). Therefore we have

$$\{x \in X : \exists s \in F(x) \text{ such that } s \leq \lambda\}$$
$$= \bigcap_{n=1}^{\infty} \Big\{ x \in X : \exists s \in F(x) \text{ such that } s < \lambda + \frac{1}{n} \Big\}.$$

By assumption, $\{x \in X : \exists s \in F(x) \text{ such that } s \leq \lambda\}$ is convex.

Lemma 1.2. *Let X be a convex subset of a Hausdorff topological vector space and Z be a Hausdorff topological vector space with a convex cone C such that $intC \neq \phi$. Let $F : X \to 2^Z$ be a multifunction.*

(1) *For any fixed $e \in intC$ and any fixed $a \in Z$,*

$$\xi(y) := \min\{t \in \mathbb{R} : y \in a + te - \bar{C}\}$$

is a continuous and strictly monotonically increasing function from Z to $\mathbb{R}$, that is, $\xi(a) > \xi(b)$ if $a - b \in intC$, where $\bar{C}$ is the closure of C.

(2) *If F is quasi $\bar{C}$-convex, then the composite multifunction $\xi F : X \to 2^{\mathbb{R}}$ is quasi $\mathbb{R}^+$-convex.*

Proof. (1) By the argument similar to the proof of Theorem 1.6 in [17, pp 83-85], we can obtain the conclusion of (1).

(2) By the argument similar to the proof of Proposition 2.3 in [18], we can obtain the conclusion of (2).

Lemma 1.3 [1]. *Let X and Z be two Hausdorff topological spaces, and $F : X \to 2^Z$ be a multifunction.*

(1) *If F is u.s.c. and compact-valued, then F is closed.*

(2) *If Z is compact and F is closed, then F is u.s.c. and compact-valued.*

(3) *If X is compact, and F is u.s.c. and compact-valued, then $F(X)$ is compact, where $F(X) = \bigcup_{x \in X} F(x)$.*

We can easily prove the following lemma:

Lemma 1.4. *Let ξ be a continuous function from a Hausdorff topological space Z to $\mathbb{R}$ and F be a multifunction from a Hausdorff topological space X to 2^Z.*
(1) *If F is u.s.c., then the composite multifunction $\xi F : X \to 2^{\mathbb{R}}$ is u.s.c..*
(2) *If F is l.s.c., then the composite multifunction $\xi F : X \to 2^{\mathbb{R}}$ is l.s.c..*

Lemma 1.5 [20]. *Let X, Y and Z be Hausdorff topological spaces, A and B nonempty subsets of X and Y, respectively. Suppose that*
(1) *the set A is compact;*
(2) *a multifunction $F : A \times B \to 2^Z$ is u.s.c.; and*
(3) *a multifunction $y \to F(x, y)$ is l.s.c..*
Then a multifunction $T : B \to 2^Z$ defined by $T(y) = F(A, y)$, is continuous, where $F(A, y) = \bigcup_{x\in A} F(x, y)$.

Theorem 1.1 (Fan-Browder fixed point theorem). *Let X be a compact convex subset of a Hausdorff topological vector space and $T : X \to 2^X$ a multifunction such that*
(1) *for each $x \in X$, $T(x)$ is a nonempty convex set; and*
(2) *for each $y \in X$, $T^{-1}(y) := \{x \in X \mid y \in T(x)\}$ is open.*
Then there exists an $\bar{x} \in X$ such that $\bar{x} \in T(\bar{x})$.

Theorem 1.2 (Fan-Glicksberg-Kakutani fixed point theorem). *Let X be a compact convex subset of a locally convex Hausdorff topological vector space and $T : X \to 2^X$ a multifunction such that*
(1) *for each $x \in X$, $T(x)$ is a nonempty convex and closed set ; and*
(2) *T is u.s.c..*
Then there exists an $\bar{x} \in X$ such that $\bar{x} \in T(\bar{x})$.

2. Existence Theorem of Solutions for (GVVI). Now we give an existence theorem for the generalized vector variational inequality (GVVI) for multifunctions, using the Yannelis-Prabhakar selection theorem and the Fan-Browder fixed point theorem.

Theorem 2.1. *Let X be a compact convex subset of a Hausdorff topological vector space, Y be a Hausdorff topological vector space and Z be a Hausdorff topological vector space with a convex cone C such that $intC \neq \phi$. Suppose that*
(1) *$T : X \to 2^Y$ is a multifunction with convex values and $T^{-1} : Y \to 2^X$ is open-valued; and*

(2) $G : X \times Y \times X \to 2^Z$ *is a multifunction satisfying*

(i) $G(x, y, x) \subset C$ *for any* $x \in X$ *and* $y \in T(x)$;

(ii) *the multifunction* G *is u.s.c. and compact-valued, and the multifunction* $(x, y) \mapsto G(x, y, u)$ *is l.s.c.; and*

(iii) *the multifunction* $u \mapsto G(x, y, u)$ *is quasi* $\bar{C}$*-convex.*

Then (GVVI) is solvable.

Proof. By Lemma 1.2, there exists a continuous and strictly monotonically increasing function ξ from Z to $\mathbb{R}$. By Yannelis-Prabhakar selection theorem in [25], there is a continuous function $f : X \to Y$ such that $f(x) \in T(x)$ for all $x \in X$. Define a multifunction $F_n : X \to 2^X$, $n = 1, 2, \cdots$, by $F_n(x) = \{z \in X : \exists s \in \xi G(x, f(x), z); s < \min \xi G(x, f(x), X) + \frac{1}{n}\}$, for any $x \in X$.

By Lemma 1.4, ξG is u.s.c. and compact-valued, and by Lemma 1.3 $\xi G(x, f(x), X)$ is compact. Thus $\min \xi G(x, f(x), X) \in \xi G(x, f(x), X)$ and hence $F_n(x)$ is nonempty for any $x \in X$. Also, by Lemma 1.2, the multifunction $u \mapsto \xi G(x, y, u)$ is quasi $\mathbb{R}^+$-convex and hence by Lemma 1.1, $F_n(x)$ is convex for any $x \in X$.

Furthermore, for any $z \in X$, we have

$$
\begin{aligned}
F_n^{-1}(z) &= \{x \in X : z \in F_n(x)\} \\
&= \left\{x \in X : [\xi G(x, f(x), z) - \min \xi G(x, f(x), X)] \cap \left(\frac{1}{n} - int\mathbb{R}^+\right) \neq \phi\right\}.
\end{aligned}
$$

By Lemma 1.4, the multifunction $(x, z) \mapsto \xi G(x, f(x), z)$ is u.s.c. and the multifunction $x \mapsto \xi G(x, f(x), z)$ is l.s.c. and hence by Lemma 1.5, the multifunction $x \mapsto \xi G(x, f(x), X)$ is continuous and compact-valued. Hence we can easily check that the function $x \mapsto \min \xi G(x, f(x), X)$ is continuous. Since the multifunction $x \mapsto \min \xi G(x, f(x), z)$ is l.s.c., the multifunction $x \mapsto \xi G(x, f(x), z) - \min \xi G(x, f(x), X)$ is also l.s.c. and hence $F_n^{-1}(z)$ is open for $z \in X$.

Consequently, for each $n = 1, 2, \cdots$, $F_n : X \to 2^X$ is a nonempty convex-valued multifunction such that $F_n^{-1}(z)$ is open for any $z \in X$. Hence by Fan-Browder fixed point theorem, there exists $x_n \in X$ such that

$$
x_n \in F_n(x_n), \quad n = 1, 2, \cdots. \tag{2.1}
$$

Since X is compact, we may assume that $x_n \to \bar{x} \in X$ and so $f(x_n) \to f(\bar{x}) \in T(\bar{x})$. Moreover, it follows from the definition of F_n and (2.1) that for each $n = 1, 2, \cdots$, there exists $s_n \in \xi G(x_n, f(x_n), x_n)$ such that

$$
s_n < \min \xi G(x_n, f(x_n), X) + \frac{1}{n}. \tag{2.2}
$$

Since ξG is u.s.c. and compact-valued, by Lemma 1.3, $\xi G(X\times f(X)\times X)$ is compact and hence we may assume that $s_n \to \bar{s} \in \xi G(X\times f(X)\times X)$. Since the multifunction $(x,z)\mapsto \xi G(x,f(x),z)$ is u.s.c. and compact-valued, by Lemma 1.3, its graph is closed and hence $\bar{s}\in \xi G(\bar{x},f(\bar{x}),\bar{x})$. Again since the function $x\mapsto \min \xi G(x,f(x),X)$ is continuous, from (2.2), $\bar{s}\leq \min \xi G(\bar{x},f(\bar{x}),X)$ and hence $\bar{s}= min\xi G(\bar{x},f(\bar{x}),X)$. Moreover we have $\min \xi G(\bar{x},f(\bar{x}),X)\in \xi G(\bar{x},f(\bar{x}),\bar{x})$. Thus there exists $\bar{z}\in G(\bar{x},f(\bar{x}),\bar{x})$ such that $\xi(\bar{z})=\min \xi G(\bar{x},f(\bar{x}),X)$. Since ξ is strictly monotonically increasing, for any $x\in X$ and any $z\in G(\bar{x},f(\bar{x}),x)$,

$$z-\bar{z}\notin -intC. \tag{2.3}$$

Finally we prove that $z\notin -intC$. Indeed, suppose the contrary, $z\in -intC$. Since $\bar{z}\in G(\bar{x},f(\bar{x}),\bar{x})$, by assumption (2)(i) $\bar{z}\in C$. Hence $z-\bar{z}\in (-intC)+(-C)=-intC$, which contradicts (2.3).

Let $\overline{y}=f(\overline{x})$. Consequently, there exist $\overline{x}\in X$ and $\overline{y}\in T(\overline{x})$ such that for any $x\in X$ and any $z\in G(\overline{x},\overline{y},x)$, $z\notin -intC$.

This completes the proof of Theorem2.1.

3. Existence Theorem of Solutions for (GVQVI). Now we give an existence theorem for the generalized vector quasivariational inequality (GVQVI) for multifunctions, using the Fan-Glicksberg-Kakutani fixed point theorem.

Theorem 3.1. *Let X be a compact convex subset of a locally convex Hausdorff topological vector space, Y a locally convex Hausdorff topological vector space and Z a Hausdorff topological vector space with a convex cone C such that $intC\neq \phi$. Let $S: X\to 2^X$ be a continuous multifunction with convex and compact values, $T: X\to 2^Y$ a closed multifunction with convex values, and $G: X\times N\times X\to 2^Z$ a continuous multifunction with compact values, where N is a compact convex subset of Y containing $T(X)$. Suppose further that*

(i) $G(x,y,x)\subset C$ for any $(x,y)\in X\times N$; and

(ii) the multifunction $u\mapsto G(x,y,u)$ is quasi $\bar{C}$-convex.

Then (GVQVI) is solvable.

Proof. By Lemma 1.2, there exists a continuous and strictly monotonically increasing function ξ from Z to $\mathbb{R}$. By Lemma 1.4, ξG is a continuous compact-valued multifunction. Also, by Lemma 1.3, $T: X\to 2^N$ is u.s.c.. Define a function $M: X\times N\to \mathbb{R}$ by

$$M(x,y)=\min_{s\in S(x)} \xi G(x,y,s),\ (x,y)\in X\times N.$$

Now we show that M is continuous. Indeed, suppose that M is not continuous at (x_0, y_0). There exist a neighborhood U of $M(x_0, y_0)$ and a net $(x_j, y_j)_{j\in J}$ in $X \times N$ such that $(x_j, y_j) \to (x_0, y_0)$ and $M(x_j, y_j) \notin U$, for all $j \in J$. Since $M(x_j, y_j) = \min_{s\in S(x_j)} \xi G(x_j, y_j, s)$, there exists $s_j \in S(x_j)$ such that $M(x_j, y_j) \in \xi G(x_j, y_j, s_j)$. Since $S(X)$ and $\xi G(X \times N \times X)$ is compact, we may assume that $s_j \to s_0 \in X$ and $M(x_j, y_j) \to z_0 \in \mathbb{R}$. Since S and ξG are closed multifunctions, $s_0 \in S(x_0)$ and $z_0 \in \xi G(x_0, y_0, s_0)$.

Suppose that $z_0 \neq M(x_0, y_0)$. Then there exist $s^* \in S(x_0)$ and $z^* \in \xi G(x_0, y_0, s^*)$ such that $z^* < z_0$, i.e.,

$$(3.1) \qquad z_0 - z^* \in int\mathbb{R}^+.$$

Since the multifunction $(x, y) \mapsto \xi G(x, y, S(x))$ is l.s.c., there exists a net $\{z_j^*\}$ in $\mathbb{R}$ such that $z_j^* \in \xi G(x_j, y_j, S(x_j))$ and $z_j^* \to z^*$. By (3.1), for j large enough $M(x_j, y_j) - z_j^* \in int\mathbb{R}^+$, i.e., $M(x_j, y_j) > z_j^*$, which contradicts the definition of $M(x_j, y_j)$. Hence $z_0 = M(x_0, y_0)$. Therefore $M(x_j, y_j) \to M(x_0, y_0)$, which contradicts the fact that $M(x_j, y_j) \notin U$, for all $j \in J$. Thus M is continuous.

Next we define a multifunction $V : X \times N \to 2^X$ by

$$V(x, y) = \{u \in S(x) : \min_{s\in S(x)} \xi G(x, y, s) \in \xi G(x, y, u)\}, \quad (x, y) \in X \times N.$$

Since $S(x)$ is compact, and ξG is u.s.c. and compact-valued, for any $x \in X$, $\xi G(x, y, S(x))$ is compact in $\mathbb{R}$ and hence $V(x, y)$ is nonempty for any $(x, y) \in X \times N$. Now we show that $graph(V)$ is a closed subset of $X \times N \times X$. Indeed, let $\{(x_j, y_j, u_j)\}_{j\in J}$ be a net in $graph(V)$ such that $(x_j, y_j, u_j) \to (x_0, y_0, u_0) \in X \times N \times X$. Then we have $u_j \in S(x_j)$ and $\min_{s\in S(x_j)} \xi G(x_j, y_j, s) = M(x_j, y_j) \in \xi G(x_j, y_j, u_j)$. Since S is a closed multifunction, $u_0 \in S(x_0)$. Since M is a continuous function, $M(x_j, y_j) \to M(x_0, y_0)$. Again since ξG is a closed multifunction, we have $M(x_0, y_0) = \min_{s\in S(x_0)} \xi G(x_0, y_0, s) \in \xi G(x_0, y_0, u_0)$. Thus $u_0 \in V(x_0, y_0)$ and hence $graph(V)$ is a closed subset of $X \times N \times X$.

Therefore, by Lemma 1.3, V is u.s.c.. It can be easily checked that V is closed-valued.

Now we show that $V(x, y)$ is a convex subset of X. For any $u_1, u_2 \in V(x, y)$ and $\lambda \in [0, 1]$, let $t_0 = \min_{s\in S(x)} \xi G(x, y, s)$, then $t_0 \in \xi G(x, y, u_i)$, $i = 1, 2$. Since S is convex- valued, $\lambda u_1 + (1 - \lambda)u_2 \in S(x)$. By Lemma 1.2, for each fixed $(x, y) \in X \times N$, the multifunction $u \mapsto \xi G(x, y, u)$ is quasi $\mathbb{R}^+$-convex and hence by Lemma 1.1 the set

$$A := \{u \in X : \text{ there is a } t \in \xi G(x, y, u) \text{ such that } t \leq t_0\}$$

is convex. Since $u_1, u_2 \in A$, $\lambda u_1 + (1-\lambda)u_2 \in A$ and hence there exists $t \in \xi G(x, y, \lambda u_1 + (1-\lambda)u_2)$ such that $t \leq t_0$. By the definition of t_0, $t = t_0$. This implies that $\min_{s \in S(x)} \xi G(x, y, s) \in \xi G(x, y, \lambda u_1 + (1-\lambda)u_2)$, and hence $\lambda u_1 + (1-\lambda)u_2 \in V(x, y)$. Thus $V(x, y)$ is a convex set.

Therefore V is a nonempty, convex-valued, closed-valued and u.s.c. multifunction.

Next we define a multifunction $W : X \times N \to 2^{X \times N}$ by

$$W(x, y) = V(x, y) \times T(x), \quad (x, y) \in X \times N.$$

Then W is a nonempty, convex-valued, closed-valued and u.s.c. multifunction. By Fan-Glicksberg-Kakutani fixed point theorem, there exists $(\bar{x}, \bar{y}) \in W(\bar{x}, \bar{y})$. Hence we have $\bar{x} \in S(\bar{x})$, $\min_{s \in S(\bar{x})} \xi G(\bar{x}, \bar{y}, s) \in \xi G(\bar{x}, \bar{y}, \bar{x})$ and $\bar{y} \in T(\bar{x})$. It follows from $\min_{s \in S(\bar{x})} \xi G(\bar{x}, \bar{y}, s) \in \xi G(\bar{x}, \bar{y}, \bar{x})$ that there exists $\bar{z} \in G(\bar{x}, \bar{y}, \bar{x})$ such that $\xi(\bar{z}) = \min_{s \in S(\bar{x})} \xi G(\bar{x}, \bar{y}, s)$. Since ξ is strictly monotonically increasing, we can check that for any $x \in S(\bar{x})$ and any $z \in G(\bar{x}, \bar{y}, x)$, $\quad z \notin -intC$. The desired assertion is proved.

Consequently, there exist $\bar{x} \in S(\bar{x})$ and $\bar{y} \in T(\bar{x})$ such that for any $x \in S(\bar{x})$ and any $z \in G(\bar{x}, \bar{y}, x)$, $\quad z \notin -intC$. This completes the proof of Theorem 3.1.

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received July 14, 1997

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