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# Measures of Dependence for ARMA Models with Stable Innovations

Abstract. A relation between different measures of dependence for stable time series is explained in this paper. We point out some errors in Kokoszka and Taqqu (1994) leading to a wrong asymptotic behavior of the dependence structure for stable ARMA processes and demonstrate this in the simplest case of AR(1) model.

**1. Introduction.** Let $\{X_n\}$ $(n = 0, \pm 1, \pm 2, \ldots)$ be a symmetric $\alpha$-stable $(S\alpha S)$ stationary time series with $0 < \alpha \leq 2$. When $\alpha = 2$, the covariance function $\mathrm{Cov}(n) = E(X_n X_0)$ describes the dependence structure of the process $\{X_n\}$. As for $\alpha < 2$ the covariance function is not defined, let us consider three most popular measures of dependence which are extensions of the covariance and are defined for stable time series.

**Definition 1.1.** Measures of dependence.

- *Covariation* $CV(n)$ of $X_n$ on $X_0$, defined for $1 < \alpha \leq 2$ (cf. [2])

$$CV(n) = CV(X_n, X_0) = \int_{S_2} s_1 s_2^{<\alpha-1>} \Gamma(ds)\,, \tag{1.1}$$

where $\Gamma$ is the spectral measure of $X_0$ and $X_n$, $z^{<p>} = |z|^{p-1}\overline{z}$,

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- *Codifference* $CD(n)$ (cf. [9])

$$\begin{aligned} CD(n) = CD(X_n, X_0) &= \ln E\exp\{i(X_n - X_0)\} \\ &- \ln E\exp(iX_n) - \ln E\exp(-iX_0)\,, \end{aligned} \tag{1.2}$$

- *Dynamical functional* $DF(n)$ (cf. [3])

$$DF(n) = DF(X_n, X_0) = E\exp\{i(X_n - X_0)\}\,. \tag{1.3}$$

**Remark 1.1.** Interpretation of the dynamical functional.
Let us consider two stationary symmetric $\alpha$-stable sequences $\{X_n\}$ and $\{\widetilde{X}_n\}$ with scaling parameters equal to 1 and suppose that for some $n$

$$DF_X(n) \le DF_{\widetilde{X}}(n)\,. \tag{1.4}$$

Formula (1.4) is, by (1.3), equivalent to

$$E\exp\{i(X_n - X_0)\} \le E\exp\{i(\widetilde{X}_n - \widetilde{X}_0)\}\,.$$

It follows from the form of the characteristic function for $S\alpha S$ variables that

$$\exp(-\sigma_X^\alpha) \le \exp(-\sigma_{\widetilde{X}}^\alpha)\,,$$

so $\sigma_X^{-1}\sigma_{\widetilde{X}} \le 1$ ($\sigma_X$ and $\sigma_{\widetilde{X}}$ are scale parameters of $X_n - X_0$ and $\widetilde{X}_n - \widetilde{X}_0$, respectively). Since $\sigma_X^{-1}(X_n - X_0)$ and $\sigma_{\widetilde{X}}^{-1}(\widetilde{X}_n - \widetilde{X}_0)$ have the same distribution, we get for any $a > 0$

$$\begin{aligned} P(|X_n - X_0| > a) &= P(\sigma_X^{-1}|X_n - X_0| > \sigma_X^{-1}a) \\ &= P(\sigma_{\widetilde{X}}^{-1}|\widetilde{X}_n - \widetilde{X}_0| > \sigma_X^{-1}a) = P(|\widetilde{X}_n - \widetilde{X}_0| > \sigma_X^{-1}\sigma_{\widetilde{X}}a) \\ &\ge P(|\widetilde{X}_n - \widetilde{X}_0| > a)\,. \end{aligned}$$

The above inequality means that $\widetilde{X}_n$ and $\widetilde{X}_0$ are less likely to differ than $X_n$ and $X_0$, and so are "more dependent".

**Remark 1.2.** One can easily define the following more general measures of dependence for any $\theta_1, \theta_2 \in \mathbb{R}$ :

$$DF(n; \theta_1, \theta_2) = \exp\{i(\theta_1 X_n + \theta_2 X_0)\}\,, \tag{1.5}$$

(1.6) $$CD(n;\theta_1,\theta_2) = -I_n(\theta_1,\theta_2)$$
$$= \ln E\exp\{i(\theta_1 X_n + \theta_2 X_0)\} - \ln E\exp(i\theta_1 X_n) - \ln E\exp(i\theta_2 X_0).$$

**Remark 1.3.** For $\alpha = 2$ the following identities hold

$$CV(n) = \frac{1}{2}\operatorname{Cov}(n),$$

$$CD(n) = \operatorname{Cov}(n),$$

$$DF(n) = \exp\{\operatorname{Cov}(n) - \operatorname{Var} X_0\}.$$

For $0 < \alpha \le 2$ the following relation between $DF(n)$ and $CD(n)$ holds for every $n$

(1.7) $$DF(n) = \exp\{CD(n) - CD(0)\}.$$

**2. ARMA processes with stable innovations.** Let ARMA$(p,q)$ process be defined by the equations

(2.1) $$X_n - b_1 X_{n-1} - \ldots - b_p X_{n-p} = \varepsilon_n + a_1\varepsilon_{n-1} + \ldots + a_q\varepsilon_{n-q},$$

where the innovations $\{\varepsilon_n\}$ are independent, symmetric $\alpha$–stable with the scale parameter 1, i.e. with the characteristic function given by

(2.2) $$E\exp(i\theta\varepsilon_n) = \exp(-|\theta|^\alpha), \quad 0 < \alpha \le 2.$$

The polynomials $B(z) = 1 - b_1 z - \ldots - b_p z^p$ and $A(z) = 1 + a_1 z + \ldots + a_q z^q$ are assumed to have no common roots and $B(z)$ is assumed to have no roots in the closed unit disk $\{z : |z| \le 1\}$. By Proposition 2.1 in [4], the system (2.1) has a unique solution of the form

(2.3) $$X_n = \sum_{j=0}^{\infty} c_j \varepsilon_{n-j} \quad \text{a.s.}$$

with real $c_j$s satisfying $|c_j| < Q^{-j}$ eventually, for some $Q > 1$. ("$a_j < b_j$ eventually" means that there is a $j_0$ such that $a_j < b_j$ for all $j > j_0$.) The $c_j$s are the coefficients in the series expansion of $A(z)/B(z), |z| < 1$.

**Remark 2.1.** Note that the coefficients $a_1, \ldots, a_q$ and $b_1, \ldots, b_p$ are time-invariant and if $1 < \alpha \le 2$, then the innovations $\{\varepsilon_n\}$ are independent, symmetric $\alpha$–stable with finite first order moments. Therefore, for index of stability $1 < \alpha \leq 2$, Proposition 2. 1 in [4] can be considered as a special case of a more general result obtained in [7]. Indeed, the $L^1$–ARMA models with time–dependent coefficients and nonstationary noise processes are described in [7] and regularity conditions under which there exists a unique, purely nondeterministic solution of such an ARMA equation are given.

**Proposition 2.1.** *For* ARMA *models defined in this section*

$$CV(n) = \sum_{j=0}^{\infty} c_{j+n} c_j^{<\alpha-1>}, \quad \alpha > 1, \tag{2.4}$$

$$CD(n) = \sum_{j=0}^{\infty} (|c_j|^\alpha + |c_{j+n}|^\alpha - |c_{j+n} - c_j|^\alpha), \tag{2.5}$$

$$DF(n) = \exp\left( -\sum_{j=0}^{n-1} |c_j|^\alpha - \sum_{j=0}^{\infty} |c_{j+n} - c_j|^\alpha \right). \tag{2.6}$$

**Proof.** Using (1.3), we have

$$\begin{aligned} DF(n) &= E\exp\left\{ i\left( \sum_{j=0}^{\infty} c_j \varepsilon_{n-j} - \sum_{j=0}^{\infty} c_j \varepsilon_{-j} \right) \right\} \\ &= E\exp\left\{ i\left( \sum_{j=0}^{n-1} c_j \varepsilon_{n-j} + \sum_{j=0}^{\infty} (c_{j+n} - c_j)\varepsilon_{-j} \right) \right\} \\ &= \exp\left( -\sum_{j=0}^{n-1} |c_j|^\alpha - \sum_{j=0}^{\infty} |c_{j+n} - c_j|^\alpha \right). \end{aligned}$$

For (2.4) and (2.5) see [5]. □

Kokoszka and Taqqu (cf. [4]) investigate the function $I_n$ (given by (1.6)) for ARMA processes in two cases. When the "smallest" root of $B(z)$ (i.e. the root with the smallest modulus) is real and positive, they determine the asymptotic behavior of $I_n$. In the case when the "smallest" root is complex, they give an exact formula for the function by which $|I_n|$ is asymptotically bounded from above. Unfortunately, there are some errors in theorems describing the asymptotic behavior of $I_n$ for $\alpha < 1$ and $\alpha = 1$. Namely, in

Theorem 3.1 in [4] there should be the sign "−" in front of the expression of the limit, formula (3.11), page 212, and in Theorem 3.3 there should be "−" between two terms of sum instead of "+", formula (3.23), page 215. These incorrect formulas are consequences of a misprint in formula (3.13) (page 212) where the sign "−" in front of the last term is missing. Moreover, errors in theorems in [4] cause incorrect results in the analysis of their AR(2) example.

The analysis carried out in [4] is extended in [8]. Three classes of roots, i.e. real positive, real negative and complex, are considered there and the correct asymptotic behavior of the covariation and the codifference in these cases is determined. In AR(1) example in the next section we do not use, however, general results, since we are able to give explicit proofs in this simple case.

**3. AR(1) model.** Let us consider the autoregressive process of order 1 defined by

$$X_n - b_1 X_{n-1} = \varepsilon_n \tag{3.1}$$

as an example of ARMA$(p, q)$ process defined in Section 2.

The covariance and the codifference are given by (2.4) and (2.5), respectively, in terms of the coefficients $c_j$. Therefore, we have to determine them in order to study the behavior of the measures of dependence. For AR(1) process the coefficients $c_j$ depend on a root $z_1$ of the polynomial $B(z) = 1 - b_1 z$. The assumption $|z_1| > 1$ is made in order to get a unique moving average (2.3) representation of the process $\{X_n\}$. The explicit formulas for $c_j$s in the series expansion of $1/B(z)$ in two possible cases (positive and negative) are presented in Proposition 3.1 and the asymptotic behavior of the covariation and the codifference is stated in Theorems 3.1 and 3.2, respectively.

**Proposition 3.1.** The coefficients $c_j$.

a) *If $B(z)$ has one positive root $z_1 = e^{\kappa_1}$, $\kappa_1 > 0$, then*

$$c_j = e^{-\kappa_1 j}. \tag{3.2}$$

b) *If $B(z)$ has one negative root $z_1 = -e^{\nu_1}$, $\nu_1 > 0$, then*

$$c_j = (-1)^j e^{-\nu_1 j}. \tag{3.3}$$

**Proof.** If $z_1 = e^{\kappa_1}$, $\kappa_1 > 0$, then $b_1 = e^{-\kappa_1}$. Thus, the coefficients $c_j$ in the series expansion of $1/B(z) = 1/(1 - e^{-\kappa_1} z)$ are given by (3.2).

Similarly, if $z_1 = -e^{\nu_1}$, $\nu_1 > 0$, then $b_1 = -e^{-\nu_1}$. Since $1/B(z) = 1/(1+e^{-\nu_1}z)$, it is not difficult to see that $c_j = (-1)^j e^{-\nu_1 j}$.

**Theorem 3.1.** The asymptotic behavior of $CV$.

*Suppose $1 < \alpha \leq 2$.*

a) *If $B(z)$ has one positive root $z_1 = e^{\kappa_1}, \kappa_1 > 0$, then*

$$\lim_{n\to\infty} e^{\kappa_1 n} CV(n) = \frac{1}{1-e^{-\kappa_1 \alpha}}. \tag{3.4}$$

b) *If $B(z)$ has one negative root $z_1 = -e^{\nu_1}, \nu_1 > 0$, then*

$$\lim_{n\to\infty} e^{\nu_1 n} CV(n) = \begin{cases} -1/(1-e^{-\nu_1\alpha}) & \text{for odd } ns \\ 1/(1-e^{-\nu_1\alpha}) & \text{for even } ns. \end{cases} \tag{3.5}$$

**Proof.** a) If $B(z)$ has one positive root $z_1 = e^{\kappa_1}$, $\kappa_1 > 0$, then, by the formula (3.2) defining the coefficients $c_j$ and by (2.4), we have

$$CV(n) = \sum_{j=0}^{\infty} e^{-\kappa_1(j+n)} e^{-\kappa_1 j(\alpha-1)} = e^{-\kappa_1 n} \frac{1}{1-e^{-\kappa_1\alpha}}. \tag{3.6}$$

Thus,

$$e^{\kappa_1 n} CV(n) = \frac{1}{1-e^{-\kappa_1\alpha}} \tag{3.7}$$

and the formula (3.4) holds.

b) If $B(z)$ has one negative root $z_1 = -e^{\nu_1}, \nu_1 > 0$, then, by (3.3), $\operatorname{sign}(c_j) = (-1)^j$. Thus

$$CV(n) = \sum_{j=0}^{\infty} (-1)^{j+n} e^{-\nu_1(j+n)} e^{-\nu_1 j(\alpha-1)} \operatorname{sign}(c_j) = e^{-\nu_1 n} \frac{(-1)^n}{1-e^{-\nu_1\alpha}} \tag{3.8}$$

and so

$$e^{\nu_1 n} CV(n) = \frac{(-1)^n}{1-e^{-\nu_1\alpha}} = \begin{cases} -1/(1-e^{-\nu_1\alpha}) & \text{for odd } ns \\ 1/(1-e^{-\nu_1\alpha}) & \text{for even } ns, \end{cases} \tag{3.9}$$

implying (3.5). □

If the root of $B(z)$ is real and positive then $CV(n)$ is proportional to $e^{-\kappa_1 n}$. If the root of $B(z)$ is real and negative then $CV(n)$ is oscillating and the best idea is to investigate its behavior taking even and odd $ns$ separately. $CV(n)$ is proportional to $e^{-\nu_1 n}$ for each of these two subsequences of $ns$.

**Theorem 3.2.** The asymptotic behavior of $CD$.

a) *Suppose $B(z)$ has one positive root $z = e^{\kappa_1}, \kappa_1 > 0$.*

(i) *If $1 < \alpha \leq 2$, then*

$$\lim_{n\to\infty} e^{\kappa_1 n} CD(n) = \frac{\alpha}{1 - e^{-\kappa_1 \alpha}}. \tag{3.10}$$

(ii) *If $\alpha = 1$, then*

$$\lim_{n\to\infty} e^{\kappa_1 n} CD(n) = \frac{2}{1 - e^{-\kappa_1}}. \tag{3.11}$$

(iii) *If $0 < \alpha < 1$, then*

$$\lim_{n\to\infty} e^{\kappa_1 \alpha n} CD(n) = \frac{1}{1 - e^{-\kappa_1 \alpha}}. \tag{3.12}$$

b) *Suppose $B(z)$ has one negative root $z = -e^{\nu_1}, \nu_1 > 0$.*

(i) *If $1 < \alpha \leq 2$, then*

$$\lim_{n\to\infty} e^{\nu_1 n} CD(n) = \begin{cases} -\alpha/(1 - e^{-\nu_1 \alpha}) & \text{for odd } ns \\ \alpha/(1 - e^{-\nu_1 \alpha}) & \text{for even } ns. \end{cases} \tag{3.13}$$

(ii) *If $\alpha = 1$, then*

$$\lim_{n\to\infty} e^{\nu_1 n} CD(n) = \begin{cases} 0 & \text{for odd } ns \\ 2/(1 - e^{-\nu_1}) & \text{for even } ns. \end{cases} \tag{3.14}$$

(iii) *If $0 < \alpha < 1$, then*

$$\lim_{n\to\infty} e^{\nu_1 \alpha n} CD(n) = \frac{1}{1 - e^{-\nu_1 \alpha}}. \tag{3.15}$$

**Proof.** a) If $B(z)$ has one positive root $z = e^{\kappa_1}, \kappa_1 > 0$, then $CD(n)$ is given by the following formula

$$CD(n) = \frac{e^{-\kappa_1 \alpha n} + [1 - (1 - e^{-\kappa_1 n})^{\alpha}]}{1 - e^{-\kappa_1 \alpha}}. \tag{3.16}$$

Indeed, in this case the coefficients $c_j$ are positive and $c_j > c_{j+n}$. Therefore, using (3.2) in (2.4), we get

$$CD(n) = \sum_{j=0}^{\infty} \left(e^{-\kappa_1 j\alpha} + e^{-\kappa_1 \alpha(j+n)} - (e^{-\kappa_1 j} - e^{-\kappa_1 (j+n)})^\alpha\right)$$

$$= \left(1 + e^{-\kappa_1 \alpha n} - (1 - e^{-\kappa_1 n})^\alpha\right) \sum_{j=0}^{\infty} e^{-\kappa_1 j\alpha} = \frac{1 + e^{-\kappa_1 \alpha n} - (1 - e^{-\kappa_1 n})^\alpha}{1 - e^{-\kappa_1 \alpha}}.$$

If $1 < \alpha \leq 2$, then the following facts

$$\lim_{n\to\infty} e^{-\kappa_1(\alpha-1)n} = 0 \tag{3.17}$$

and

$$\lim_{n\to\infty} e^{\kappa_1 n}[1 - (1 - e^{-\kappa_1 n})^\alpha] = \alpha \tag{3.18}$$

imply that (3.10) holds.
Setting $\alpha = 1$ in (3.16) we obtain the simple formula describing $CD(n)$

$$CD(n) = \frac{2e^{-\kappa_1 n}}{1 - e^{-\kappa_1}}, \tag{3.19}$$

which, in turn, yields (3.11).

If $0 < \alpha < 1$, then, by (3.16),

$$\lim_{n\to\infty} e^{\kappa_1 \alpha n} CD(n) = \lim_{n\to\infty} e^{\kappa_1 \alpha n} \frac{e^{-\kappa_1 \alpha n} + [1 - (1 - e^{-\kappa_1 n})^\alpha]}{1 - e^{-\kappa_1 \alpha}}$$

$$= \frac{1}{1 - e^{-\kappa_1 \alpha}} \lim_{n\to\infty} \left(1 + e^{\kappa_1 \alpha n}[1 - (1 - e^{-\kappa_1 n})^\alpha]\right)$$

and in order to get (3.12) it is enough to notice that

$$\lim_{n\to\infty} e^{\kappa_1 \alpha n} \left[1 - (1 - e^{-\kappa_1 n})^\alpha\right] = 0\,. \tag{3.20}$$

b) Suppose $B(z)$ has one negative root $z = -e^{\nu_1}, \nu_1 > 0$. In this case (2.5) and (3.3) yield

$$CD(n) = \frac{e^{-\nu_1 \alpha n} + 1 - [(-1)^n e^{-\nu_1 n} - 1]^\alpha}{1 - e^{-\nu_1 \alpha}}. \tag{3.21}$$

Notice that considering even $ns$ we get the same formula for $CD(n)$ as in the case of one positive root. Thus, using arguments as in part a) of this proof one can obtain (3.13), (3.14) and (3.15) for the sequence of even $ns$.

We now focus on the case of odd $ns$.

If $1 < \alpha \leq 2$, then applying (3.17) and

$$\lim_{n\to\infty} e^{\nu_1 n}\left[1-(1+e^{-\nu_1 n})^{\alpha}\right] = -\alpha \tag{3.22}$$

in

$$\lim_{n\to\infty} e^{\nu_1 n} CD(n) = \lim_{n\to\infty} e^{\nu_1 n}\frac{e^{-\nu_1 \alpha n}+[1-(1+e^{-\nu_1 n})^{\alpha}]}{1-e^{-\nu_1\alpha}}, \tag{3.23}$$

we get (3.13) for the sequence of odd $ns$.

If $\alpha = 1$, then (3.14) results immediately from the following explicit formula for $CD(n)$

$$CD(n) = \frac{e^{-\nu_1 n}+[1-(1+e^{-\nu_1 n})]}{1-e^{-\nu_1}} = 0. \tag{3.24}$$

If $0 < \alpha < 1$, then, using the simple fact that

$$\lim_{n\to\infty} e^{\nu_1 \alpha n}[1-(1+e^{-\nu_1 n})^{\alpha}] = 0, \tag{3.25}$$

we get (3.15) in the case of odd $ns$. Indeed,

$$\lim_{n\to\infty} e^{\nu_1 \alpha n} CD(n) = \lim_{n\to\infty} e^{\nu_1 \alpha n}\frac{e^{-\nu_1 \alpha n}+[1-(1+e^{-\nu_1 n})^{\alpha}]}{1-e^{-\nu_1\alpha}}$$

$$= \frac{1}{1-e^{-\nu_1\alpha}}\lim_{n\to\infty} e^{\nu_1\alpha n}[1-(1+e^{-\nu_1 n})^{\alpha}] = \frac{1}{1-e^{-\nu_1\alpha}}.$$

□

**Theorem 3.3.** *If $1 < \alpha \leq 2$, then for* AR(1) *processes the following formula holds*

$$\lim_{n\to\infty}\frac{CD(n)}{CV(n)} = \alpha. \tag{3.26}$$

**Proof.** If $1 < \alpha \leq 2$, then both the codifference and the covariation are defined. If $B(z)$ has one positive root, then one can get (3.26) dividing

(3.10) by (3.4). In the case of one negative root, formula (3.26) results from (3.13) and (3.5).

This fact can be considered as a natural extension of the behavior of $CD(n)$ and $CV(n)$ for $\alpha = 2$.

□

The method of computer approximation of the covariation and the codifference for AR(1) time series provides us with the visualization of the theoretical results obtained in Theorem 3.3 and can be used as a tool to show errors in [4].

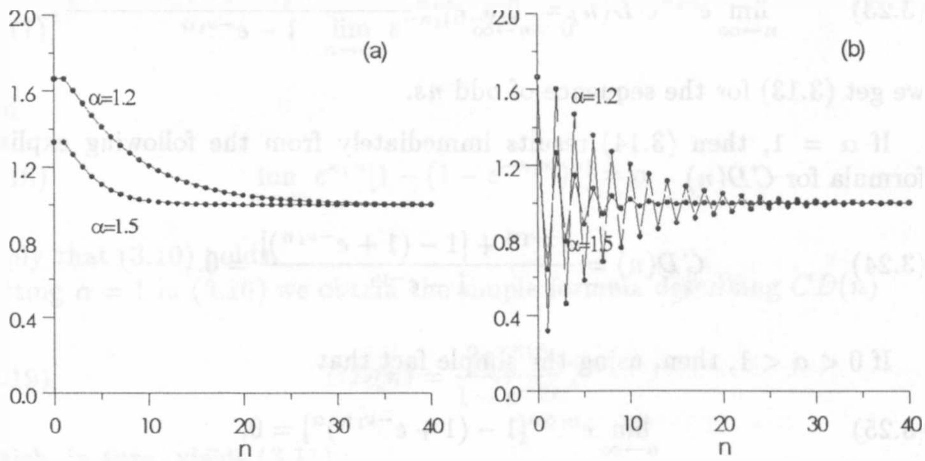


Figure 3.1: The plot of the $\dfrac{CD(n)}{\alpha CV(n)}$ vs $n = 0, 1, \ldots, 40$ for $\alpha = 1.2$ and $\alpha = 1.5$ in the case of (a) one positive root $z_1 = 2$, (b) one negative root $z_1 = -2$.

Fig. 3.1 contains the plot of the numerical approximation of the function $CD(n)/\alpha CV(n)$ for $n = 0, 1, \ldots, 40$ and for two selected values of $\alpha$, i.e. for $\alpha = 1.2$ and for $\alpha = 1.5$. Fig. 3.1(a) presents the case of one real positive root $z_1 = 2$ and Fig. 3.1(b) shows the case of one real negative root $z_1 = -2$. It can be noticed that for small $n$ values of $CV(n)$ and $CD(n)$ are different, but the numerical evaluation of the fraction $CD(n)/\alpha CV(n)$ gives 1 for sufficiently large $ns$. This observation is not surprising because, as stated in Theorem 3.3, formula (3.26) holds for AR(1) processes. Moreover, if $B(z)$ has a positive root, presented function is decreasing. In case of negative root, in turn, there are apparent oscillations. More precisely, taking odd $ns$ and even $ns$ one can distinguish two sequences of values (the first one is increasing while the second one is decreasing) that reach the same limit. This is the consequence of the behavior of the covariation and the codifference - there are two different formulas for two subsequences of $ns$ in Theorem 3.1 b) and Theorem 3.2 b) (i).

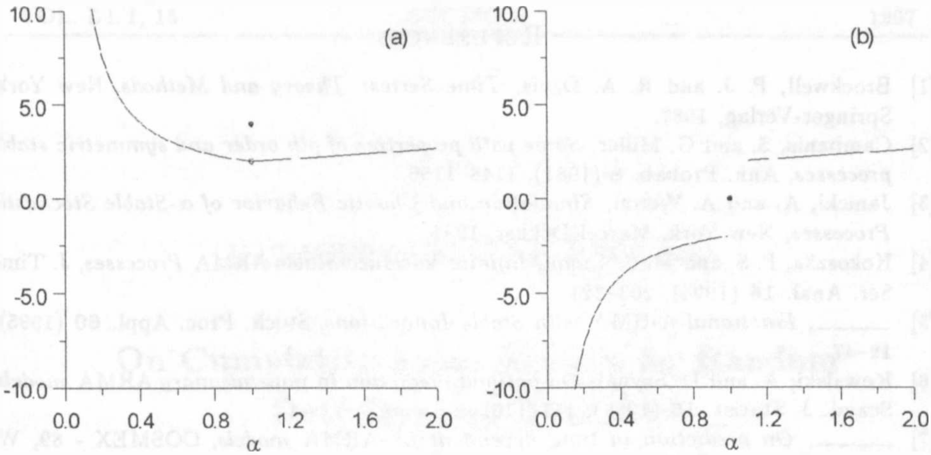


Figure 3.2: (a) Theoretical limit values (cf. (3.10), (3.11) and (3.12)) for $CD(n)$ vs $\alpha$.
(b) Incorrect theoretical limit values for $-I_n(1,-1)$ vs $\alpha$ obtained in Kokoszka and Taqqu (1994).

Suppose now that $B(z)$ has the positive root $z_1 = 2$. Since for every $n$

$$CD(n) = -I_n(1,-1),$$

general theorems for the asymptotic behavior of $I_n$ should, in particular, give the same results for AR(1) process as we state in this paper in Theorem 3.2. There are, however, some errors in [4], which are discussed in detail in Section 2.

Fig. 3.2 allows us to compare theoretical limit values and incorrect results obtained in [4]. For $0 < \alpha < 1$ Fig. 3.2(a) gives the plot of

$$\lim_{n\to\infty} e^{\kappa_1 \alpha n} CD(n) = \lim_{n\to\infty} 2^{\alpha n} CD(n)$$

(see RHS. of (3.12)) against the index of stability $\alpha$. For $\alpha = 1$ and $1 < \alpha \leq 2$, in turn, this figure presents

$$\lim_{n\to\infty} e^{\kappa_1 n} CD(n) = \lim_{n\to\infty} 2^{n} CD(n)$$

(see RHS. of (3.10) and (3.11), respectively). Fig. 3.2(b) illustrates the incorrect asymptotic behavior of $-I_n(1,-1)$ as presented in [4].

**Remark 3.1.** Since the relation (1.7) holds for every $n$, an information about the asymptotic behavior of the codifference $CD(n)$ gives us the asymptotic behavior of the dynamical functional $DF(n)$.

## References

[1] Brockwell, P. J. and R. A. Davis, *Time Series: Theory and Methods*, New York, Springer-Verlag, 1987.

[2] Cambanis, S. and G. Miller, *Some path properties of pth order and symmetric stable processes*, Ann. Probab. **8** (1981), 1148–1156.

[3] Janicki, A. and A. Weron, *Simulation and Chaotic Behavior of $\alpha$-Stable Stochastic Processes*, New York, Marcel Dekker, 1994.

[4] Kokoszka, P. S. and M. S. Taqqu, *Infinite Variance Stable* ARMA *Processes*, J. Time Ser. Anal. **15** (1994), 203–221.

[5] ———, *Fractional* ARIMA *with Stable Innovations*, Stoch. Proc. Appl. **60** (1995), 19–47.

[6] Kowalski, A. and D. Szynal, *On optimal prediction in nonstationary* ARMA *models*, Scand. J. Statist. **15** (1988), 111–116.

[7] ———, *On prediction in time-dependent $L^1$*-ARMA *models*, COSMEX - 89, W. Kasprzak & A. Weron (Eds), World Scientific, Singapore, 1990.

[8] Nowicka, J., *Asymptotic behavior of the covariation and the codifference for* ARMA *models with stable innovations*, Stochastic Models **13** (1997), 79–82.

[9] Samorodnitsky, G. and M. S. Taqqu, *Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance*, Nwe York, Chapman & Hall, 1994.

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