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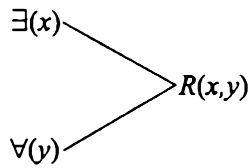
Logical Entailment on Multidimensional Branching Quantifier Representations

Wynikanie logiczne w reprezentacja opartych na wielowymiarowych kwantyfikatorach rozgałęzionych

Consider a branching-quantifier sentence, such as

(1)



which, as Hintikka observes, translates as

(2) $$(\exists x)(\forall y)\, R(x,y)$$

the latter being understood as a linear (non-branching) first-order sentence.[1]

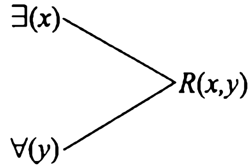
The same translatability applies of course to instances of (1) and (2) in the reverse order. Thus,

[1] J. Hintikka, *Game-Theoretical Semantics as a Synthesis of Verificationist and Truth-Conditional Meaning Theories*, [in:] E. LePore, [ed.], *New Directions in Semantics*, Academic Press, London, 1987, p. 254.

(2) $$(\exists x)(\forall y)\, R(x,y)$$

translates, respectively, as

(1)

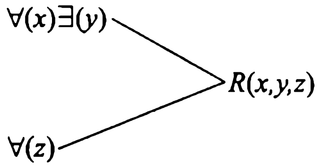


while

(3) $$(\forall x)(\exists y)(\forall z)\, R(x,y,z)$$

owing to its being itself a translation from (4), translates as

(4)



Anyone versed in the theory of quantifier scope would tell us that in

(5) $$(\forall y)(\exists x)\, R(x,y)$$

the existential quantifier depends on the universal quantifier since it falls within the scope of the universal quantifier. When the order of the quantifiers is reversed, so is the directionality of quantifier dependence, as it is assumed to be happening in the opposite case of

(2) $$(\exists x)(\forall y)\, R(x,y).$$

However, quite contrary to what the scope theory would aspire to predict, the universal quantifier in (2) does *not* fall within the scope of the existential quantifier, since otherwise it would be informationally dependent and (2) would not be equivalent with its informationally independent translation in (1). The essence of the contradistinction between examples in (5) and (2) can thus be shown to be lying somewhere else, and in something entirely different than the order of quantifiers *per se*; namely, in the distinction between a case of branching quantifiers in (2), represented more adequately by a vertical configuration of these quantifiers in (1), and a case of linear quantifiers in (5), where their configuration happens to be horizontal. The issue, however, that immediately begs the question is whether we have two quantifier configurations here, one in (2) and another in (5), of which (2) *defies* laws of quantifier scope, while (5) *complies* to them, or whether we in fact have two quantifier configurations such that to *both* of them laws of quantifier scope do not and, for some reasons, cannot apply. Then, if only Frege was right in contending that, since (in his view) thought is timeless and spaceless, its real structure defeats any linear representation (which in his case was supported by his necessarily two-dimensional representation of the conditional), quantifier scope distinctions would follow not from the real exigencies of logical form as such but, rather, would appear as a mere artifact only serving to compensate for the inadequacies of logical form in its necessarily linear rendering in the received convention of logical syntax. In the latter case, quantifier scope distinctions would appear only as mere vagaries of the linguistic expression of canonical form, or what Quine calls, more to the point, canonical *idiom*, but not as facts pertaining to the description of logical form *per se*.

This latter point can be best illustrated by the consideration of the form-content distinctions in Frege's attempt to provide an analytic explanation to the synthetic truth of the expressions of identity which he undertook in his theory of *Sinn und Bedeutung*. This theory is in fact a theory of form-content relationships in expressions of the form "$A = B$", which explains that A and B are different expressions because they are expressions of different thoughts which, as different thoughts, are identical in that they have, or refer to, the same truth-value. The form-content distinctions are thus rendered as logical structure being the form and its truth-value, the content, of A and B. This, however, goes contrary to what Frege says elsewhere; namely, that

> "[...] we must not fail to recognize that the same sense, the same thought, may be variously expressed; [...] If all transformation of the expression were forbidden on the plea that this would alter the content as well, logic would simply be crippled; for the task of logic can hardly be performed without trying to recognize the thought in its manifold guises".[2]

However, according to this latter claim, A and B would be expressions not of different, but of the same thought, and the form-content distinctions would be thus rendered not as logical structure being the form and its truth-value, the content but, rather, as the various linguistic guises of the same thought being the form, and the thought itself, the (necessarily identical) content of both A and B.

What criterion should we choose so as to find the way out of this apparent contradiction? Bearing in mind, however, that the main task of Frege's considerations on the issue was to find the way to explain how the expressions of identity can be informative, the best way to explain how the sign of logical identity can be informative in the case of quantificational structures (hence, intrinsically analytic expressions) is to view the sign of logical identity not as an expression

[2] G. Frege, *Über Begriff und Gegenstand*, „Vierteljahrschrift für wissenschaftliche Philosophie" 1892, 16, p. 196. English translation [in:] P. Geach and M. Black, [eds.], *Translations from the Philosophical Writings of Gottlob Frege*, Basil Blackwell, Oxford 1952, p. 46.

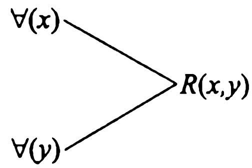
of the identity of their truth-values, but, instead, as an expression of the identity of the thought – as well as the identity of its respective underlying structure – which appears in its various 'canonical' manifestations to the left and to the right of the sign of logical identity.

To see the real benefit of adopting this position, consider the logical equivalence in

(6) $$(\forall x)(\forall y)\, R(x,y) \equiv (\forall y)(\forall x)\, R(x,y)$$

which, in fact, is nothing else but a demonstration of the obvious fact that the two different quantifier structures with a varying, but logically equivalent, order of the quantifiers in (6) are nothing but two linear translations of one and the same branching quantifier structure in

(7)



this structure (but not, as is generally thought, its truth-value) presenting itself as the same *Bedeutung* which turns up to be identical for both quantifier expressions flanking the sign of logical identity in (6), in much the same way as the object called "Venus" appears in the role of the same *Bedeutung* for both 'the morning star' and 'the evening star' in 'The morning star is identical with the evening star'.

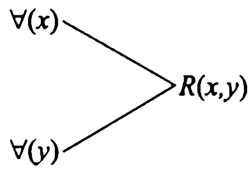
What the existence of the two linear translations of the branching structure in (7) shows is that, contrary to what the theory of quantifier dependence endorses, the two quantifiers appearing in the same quantifier prefix, if they are both universal, are never dependent on one another, not even if one of them appears, linearly, within the scope of another.

Returning to examples (2) and (5), it will be seen that, accordingly, the universal quantifier is never dependent on the existential quantifier, since it is either that the universal quantifier precedes, and thus governs, the 'dependent' existential one, as in (5), or, otherwise, if it appears, linearly, within the scope of the latter, as is the case in (2), then it is independent due to the logical equivalence of (2) with the corresponding branching structure in (1).

What we may observe, now, is that it is not the order of the quantifiers that matters: for what really matters is that we have either the case where the existential quantifier is dependent on the universal quantifier and, thus, the representation is linear, or the case is that the former is independent of the latter, so the respective representation is necessarily branching. Notice, that since the shift in the linear ordering of quantifiers really amounts to shifting from the linear representation to the branching one, the only relevant contradistinction boils down to the directionality of their appearance in the quantifier prefix, their order in either of the cases being quite irrelevant. But since it is this directionality, but not their order, which is really relevant, then the notion of the quantifier scope appears to be quite useless, and thus should be entirely dispensed with.

This claim gets further support by the gain in simplicity that we get in the respective apparatus of proof theory. As a matter of fact, we do no longer need any proof, since now we are in a position to define entailment directly on logical structure, as a function of its purely structural properties. As will be seen in the case of examples (6) and (7), for example, we do no longer need any rules of inference or any proof procedure whatsoever so as to demonstrate that $(\forall x)(\forall y)\,R(x,y)$ entails $(\forall y)(\forall x)\,R(x,y)$ and, respectively, $(\forall y)(\forall x)\,R(x,y)$ entails $(\forall x)(\forall y)\,R(x,y)$, since what we have here is a trivial case of $p \supset p$, where the logical structure of p *is* the branching structure in

(7)

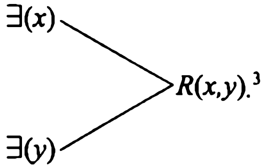


Accordingly, the logical equivalence of both parts of the biconditional in

(6')
$$(\exists x)(\exists y)R(x,y) \equiv (\exists y)(\exists x)R(x,y)$$

would then follow logically from the fact of their being necessarily various linear representations of the same non-linear logical form in

(7')



On the other hand, the only rule that we will need for the demonstration of the fact that

(2)
$$(\exists x)(\forall y)\,R(x,y)$$

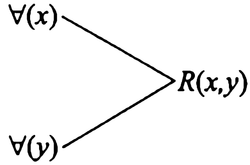
entails

[3] Cf., in this connection, a certain parallel with Quine's interpretation of identity in terms of the identity of objects that are not *relatively* discernible (*Word and Object*, Cambridge, Mass., 1960, p. 230).

(8) $$(\forall y)(\exists x)\, R(x,y)\,,$$

but not vice versa, will amount to stating that it is a branching quantifier structure in (1) which entails a linear one in (8), in much the same way as

7)



can be shown to entail either of the respective linear quantifier representations in

(6) $$(\forall x)(\forall y)\, R(x,y) \equiv (\forall y)(\forall x)\, R(x,y)\,.$$

In order to show the substantial content of this rule, consider Fauconnier's examples in

(*H*)	$\exists y$	$\forall x$	(DANCE (*x*,*y*))
	girl	boy	

and

(*H'*)	$\forall x$	$\exists y$	(DANCE (*x*,*y*))
	boy	girl	

which he renders as logical representations of the two readings, between which

(D1) *All the boys danced with a girl*

happens to be ambiguous.[4] (*H*) implies that there was one particular girl all the boys danced with, (*H'*) does not.

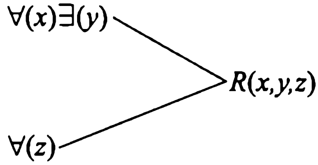
Since, as Fauconnier emphasises, it is (*H*) that logically entails (*H'*), and not vice versa, it is clear that, the difference in the quantifier scope between (*H*) and (*H'*) notwithstanding, the interpretation of (*D1*) designated as (*H*) represents, necessarily, a particular case of all interpretations of (*D1*) designated by (*H'*), but not vice versa. In other words, if there was one particular girl all the boys danced with, then the reading, (*H*), which implies this situation also *contains*, necessarily, the identification that a person that all the boys danced with was a girl, i.e., exactly the reading on the interpretation whose logical representation is formally contrasted with (*H*). What we may also observe is that the contrasting reading, (*H'*), *does not* necessarily *contain* the respective identification, provided by (*H*), namely that a girl with which each of the boys danced was one and the same person. It appears, then, that a more reasonable and logically consistent way of accounting for the

[4] G. Fauconnier, *Do Quantifiers Branch?*, „Linguistic Inquiry" 1975, 6, pp. 561–562.

meaning differences between the two interpretations of (*D1*) is to represent them as a difference between two logical structures, one containing both the vertical and the horizontal arrangement of quantifiers (*C*), and another, the horizontal arrangement alone (*c*). Then, the fact of one of them entailing another would be read off from the respective configurations considered in terms of their part:whole relationship: the larger configuration, *C*, which would have the smaller configuration, *c*, as its part, would also logically entail it, whereas *c* would not entail *C* as a consequence of the fact that *C* would not be a part of *c*.

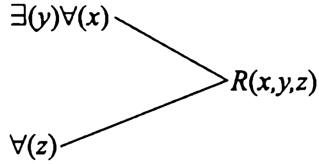
For further demonstration that the notion of quantifier scope and the apparatus of logical proof, attendant to this notion, are unsustainable and in fact logically transcendental, consider, once again, example (4):

(4)



Here, the order of $(\forall x)$ and $(\exists y)$ is linear, and the order of $(\forall x)(\exists y)$, on the one hand, and $(\forall z)$, on the other, is branching. But if we invert, in (4), the order of $(\forall x)(\exists y)$, as it appears in

(9)



then the part in the quantifier prefix in $(\exists y)(\forall x)$ would itself require a branching translation so as to get rid of the contradiction that arises between the predictions as to the dependence of the quantifiers necessarily following from their linear ordering and the logical reality of $(\exists y)(\forall x)$. Since the arrangement of the quantifiers in the quantifier prefix of (4) clearly consists of both the horizontal arrangement of $(\forall x)$ and $(\exists y)$ as well as the vertical arrangement of both $(\forall x)$ and $(\exists y)$ with respect to $(\forall z)$, it is obvious that the respective branching representation of $(\exists y)(\forall x)$ would require an additional dimension, in which the inevitable branching of $(\exists y)(\forall x)$ would appear to be standing in branching contrast to the already two-dimensional branching structure of the quantifier prefix in (4).

The notion of the multidimensionality of branching, thus obtained, appears to be extremely helpful in disclosing the logical substance of quantifier entailment. Since, heretofore, we have only been dealing with branching, as opposed to linear, quantifier structures, the revealed multidimensional nature of quantifier branching urges us to drop all mention of linearity considered as something that we can place in contradistinction to branching as such. As our consideration of examples (9) and (4) above shows, the horizontal and the vertical arrangement of the quantifiers in the quantifier prefix which we categorised in terms of the contradistinction between linear and branching quantification as such turns up, in fact, to be none but the same branching quantification, the only difference being in what could be best described as the dimensionality of branching. The notion of quantifier entailment will then be conceptualised as the containment, by any n-dimensional quantifier structure, of all the respective quantifier structures whose dimensionality is equal to, or less than, n. In the fist case, it would be a trivial case of p which entails itself, as in $p \supset p$, while in the second case it would be a no less trivial case of reasoning from $p\&q$ to q. As a corollary, the invalidity of the linear representation in $(\forall y)(\exists x)Fxy \supset (\exists x)(\forall y)Fxy$, for instance, will then be read off directly from the respective multidimensional formula,[5] thus wholly dispensing with the much more elaborate and much more time-consuming apparatus of invalidity proofs[6] that has so far been employed to essentially the same effect.

The novelty of the vantage point outlined above appears to be in a stark contrast with Quine's extremely conservative views on the logic of linear quantification, which he opposes as the only true logic to what is claimed by him to be the 'deviant' logic of branching quantification.[7] However, such cursory remarks as these on the philosophy of logical identity can aspire at most to sort out issues and sketch a position; not to persuade.

STRESZCZENIE

W artykule proponuje się całkowicie nową koncepcję logicznej syntaktyki dla kwantyfikacji. W teorii kwantyfikacji złożoność relacji zależności między zmiennymi związanymi przerasta w oczywisty sposób możliwość jej liniowego wyrażenia, co dodatkowo zmusza do uzupełnienia w pewnych przypadkach syntaktyki aparaturą pojęciową kwantyfikacji skończenie częściowo uporządkowanej (FPO kwantyfikacji). Pokazuje się też, że teoria FTO posiada swoje własne ograniczenia, których jednakże można uniknąć odwołując się do modeli topologicznych. Zaletą proponowanych modeli jest to, że prawdziwość logiczna może być przedstawiona bezpośrednio jako funkcja od własności rozważanych modeli topologicznych.

[5] As a case of invalid reasoning from q to $p\&q$.

[6] Like the one to be found in J. A. Faris, *Quantification Theory*, London, 1964, pp. 138–139.

[7] Cf. Chapter 6 on *Deviant Logics*, [in:] his *Philosophy of Logic*, Englewood Cliffs, 1970, pp. 89–93; also his essay on *Existence and Quantification*, [in:] *Ontological Relativity and Other Essays*, New York, 1969, specifically pp.108-113.