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A nonlinear Abelian ergodic theorem for asymptotically nonexpansive mappings in a Hilbert space

Abstract. Let C be a closed convex subset of a real Hilbert space and let T be an asymptotically nonexpansive nonlinear self–mapping of C. We prove a nonlinear Abelian ergodic theorem which deals with the weak convergence of the Abelian averages $A_r[T]x,\ 0 < r < 1$, of the iterates $\{T^n x\}$ for each x in C.

1. Introduction. Throughout this paper H will denote a Hilbert space over the real number field. Let C be a nonempty closed convex subset of H and let T be a mapping of C into itself. If the inequality $\|Tx-Ty\| \le \|x-y\|$ holds for all x, y in C, the mapping T is called nonexpansive on C. More generally, the mapping T is said to be $\{\alpha_n\}$-asymptotically nonexpansive on C if the inequality $\|T^n x - T^n y\| \le (1+\alpha_n)\|x-y\|$ holds for all x, y in C, where $\{\alpha_n\}$ is a sequence of real numbers such that $\lim_{n\to\infty} \alpha_n = 0$. The

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latter notion was introduced by Goebel and Kirk [4]. The object of this investigation is the so–called Abelian average $A_r[T]x$ of the iterates $\{T^n x\}$ for each x in C, which is defined by

$$A_r[T]x = (1-r)\sum_{n=0}^{\infty} r^n T^n x \;=\; (1-r)(I-rT)^{-1}x \;,\; 0<r<1$$

whenever $(I-rT)^{-1}x$ exists. If a norm–bounded sequence $\{w_n : n \in \mathbb{N}\}$ is given in C, we define

$$\begin{aligned} \Gamma_m(x) &= \sup\{\|x-w_k\| : k \ge m\} \;,\; x \in C \;, \\ \Gamma(x) &= \inf\{\Gamma_m(x) : m \in \mathbb{N}\} \;,\; x \in C \;, \\ \Gamma &= \inf\{\Gamma(x) : x \in C\} \;. \end{aligned}$$

For the numbers $\Gamma(x)$ and Γ so defined, the set $A_C(\{w_n\}) = \{x \in C : \Gamma(x) = \Gamma\}$ (the number Γ) is called the asymptotic center (asymptotic radius) of $\{w_n : n \in \mathbb{N}\}$ in C. This definition is due to Lim [7]. It is well known (cf. [2], [3], [4], [7]) that a unique point x exists in C such that the asymptotic center $A_C(\{w_n\})$ is a single–element set $\{x\}$ which satisfies the equality

$$\limsup_{n\to\infty}\|x-w_n\| = \inf\{\limsup_{n\to\infty}\|y-w_n\| : y \in C\} \;.$$

The study of the Abel limit seems to be particularly appropriate and interesting. If we set $\lambda = 1/r$ then $A_r[T]x = (\lambda-1)R(\lambda;T)x$, where $R(\lambda;T)x = \sum_{n=0}^{\infty}\lambda^{-n-1}T^n x$. In general, $R(\lambda;T)x$ does not satisfy the resolvent equation unless T is linear. Nevertheless, an interesting relation exists between the Cesàro $(C,1)$-limit and the Abel limit which reminds us of the equivalence relation concerning Abelian ergodic theorem for asymptotically nonexpansive nonlinear mappings. And then we shall clarify the relation between the Cesàro $(C,1)$ and the Abel limits just mentioned in the nonlinear case. Some related topics are also discussed.

2. The main results.

Theorem 1. *Let C be a nonempty closed convex subset of H and let T be an asymptotically nonexpansive self–mapping of C. Suppose that for each x in C, $\{A_r[T]x\}$ is norm–bounded and there exists an integer $m_0 \ge 1$ such that* $\mathrm{s-}\lim_{r\to 1-0}(I-T^m)\,A_r[T]x = 0$ *for each $m \ge m_0$. Then for each x in C, $A_r[T]x$ converges weakly to a fixed point of T as $r \to 1-0$.*

The method of proof is based upon the Opial property of Hilbert space and the role of the asymptotic center defined by Lim [7].

Lemma 1 [9, Lemma 1]. *If $\{x_n\}$ is a sequence in H which converges weakly to a point x_0 in H, then for any x in H with $x \neq x_0$*

$$\liminf_{n\to\infty} \|x_n - x_0\| < \liminf_{n\to\infty} \|x_n - x\|.$$

Lemma 2. *If a norm–bounded sequence $\{w_n\}$ in C converges strongly to a point x_0 in C, then*

$$A_C\left(\{w_n\}\right) \cap \left(\bigcap_n \overline{\text{co}}\{w_k : k \geq n\}\right) = \{x_0\}.$$

Proof. It is clear that x_0 belongs to $A_C(\{w_n\}) \cap (\bigcap_n \overline{\text{co}}\{w_k : k \geq n\})$. On the contrary, suppose that $A_C(\{w_n\}) \cap (\bigcap_n \overline{\text{co}}\{w_k : k \geq n\})$ contains a point u different from the point x_0. Then, by Lemma 1

$$\begin{aligned}\limsup_{n\to\infty} \|w_n - x_0\| &= \liminf_{n\to\infty} \|w_n - x_0\| \\ &< \liminf_{n\to\infty} \|w_n - u\| \leq \limsup_{n\to\infty} \|w_n - u\|\,.\end{aligned}$$

Here, if we define $E = \{z \in H : \|z - x_0\| \leq \|z - u\|\}$ then E is a closed convex subset of H. Hence there is an integer $k_0 \geq 1$ such that $\{w_k : k \geq k_0\} \subset E$, so that $\overline{\text{co}}\{w_k : k \geq k_0\} \subset E$. Since u is obviously not in E, u does not belong to $\overline{\text{co}}\{w_k : k \geq k_0\}$. This is, however, impossible and the lemma follows. $\square$

Lemma 3 [6, Lemma 3]. *Let C be a nonempty closed convex subset of H and let T be an asymptotically nonexpansive self–mapping of C. Suppose that $\{T^n x\}$ is norm–bounded for each x in C. Then for each x in C the asymptotic center of $\{T^n x\}$ is a fixed point of T.*

Proof of Theorem 1. We may assume that T is α_n-asymptotically nonexpansive. Let x be arbitrarily fixed in C. Since $\{A_r[T]x\}$ is weakly sequentially compact, there exists a subsequence $\{A_{r_i}[T]x\}$ $(\lim_{i\to\infty} r_i = 1)$ of $\{A_r[T]x\}$ which converges weakly to a point x_0 in C.

We wish to show that x_0 is a fixed point of T. To show this, it suffices to prove that $\{T^n x_0\}$ converges strongly to x_0. On the contrary, suppose that $\{T^n x_0\}$ does not converge strongly to x_0. Then there exists a number $\varepsilon_0 = \varepsilon_0(x_0) > 0$ and a subsequence$\{T^{k_i} x_0\}$ of $\{T^n x_0\}$ such that

$$\|T^{k_i} x_0 - x_0\| > \varepsilon_0 \text{ for all } i \geq 1\,.$$

Now as in [6] put $p(x_0) = \liminf_{i\to\infty} \|A_{r_i}[T]x - x_0\|$ and choose a number $\delta = \delta(x, x_0, \varepsilon_0) > 0$ such that

$$\{p(x_0) + \delta\}^2 - \{p(x_0)\}^2 < \frac{\varepsilon_0^2}{4}\,.$$

We can find a subsequence $\{s_i\}$ of $\{r_i\}$ for which $p(x_0) = \lim_{i\to\infty} \|A_{s_i}[T]x - x_0\|$, so that there exists an integer $i_0 = i_0(x, x_0, \delta)$ such that

$$\|A_{s_i}[T]x - x_0\| < p(x_0) + \frac{\delta}{3} \quad \text{for all } \ i \geq i_0\,.$$

Furthermore, it follows from Lemma 1 that for any ξ in H with $\xi \neq x_0$

$$\liminf_{i\to\infty} \|A_{s_i}[T]x - x_0\| < \liminf_{i\to\infty} \|A_{s_i}[T]x - \xi\|\,.$$

Noting that $\lim_{n\to\infty} \alpha_n = 0$, we choose an integer $n_0 = n_0(x, x_0, \delta)$ such that

$$\alpha_n \left\{ p(x_0) + \frac{\delta}{3} \right\} \leq \frac{\delta}{3} \quad \text{for all } \ n \geq n_0\,.$$

Let m_0 be the integer given in the assumption of the theorem and take m to be an integer such that $m \geq \max(n_0, m_0)$ and $\|t^m x_0 - x_0\| > \varepsilon_0$. Then there exists by assumption a number $r_0 = r_0(x, m, \delta)$, $0 < r_0 < 1$, such that

$$\|A_r[T]x - T^m A_r[T]x\| < \frac{\delta}{3} \quad \text{for } \ r_0 < r < 1\,.$$

Fixing such an integer m and choosing an integer $i_1 = i_1(x, m, \delta)$ so that $r_0 < s_i < 1$ for all $i \geq i_1$, we have for all $i \geq \max(i_0, i_1)$

$$\begin{aligned}
\|A_{s_i}[T]x - T^m x_0\| &\leq \|A_{s_i}[T]x - T^m A_{s_i}[T]x\| + \|T^m A_{s_i}[T]x - T^m x_0\| \\
&\leq \|A_{s_i}[T]x - T^m A_{s_i}[T]x\| + (1 + \alpha_m)\|A_{s_i}[T]x - x_0\| \\
&\leq \frac{\delta}{3} + (1 + \alpha_m) \left\{ p(x_0) + \frac{\delta}{3} \right\} < p(x_0) + \delta
\end{aligned}$$

and hence

$$\begin{aligned}
\left\| A_{s_i}[T]x - \frac{1}{2}(T^m x_0 + x_0) \right\|^2 &= 2\left\| \frac{1}{2}(A_{s_i}[T]x - T^m x_0) \right\|^2 \\
&\quad + 2\left\| \frac{1}{2}(A_{s_i}[T]x - x_0) \right\|^2 - \left\| \frac{1}{2}(T^m x_0 - x_0) \right\|^2 \\
&< \frac{1}{2}\{p(x_0) + \delta\}^2 + \frac{1}{2}\{p(x_0) + \frac{\delta}{3}\}^2 - \frac{\varepsilon_0^2}{4} \\
&< \{p(x_0)\}^2\,.
\end{aligned}$$

Consequently, taking $\xi = (T^m x_0 + x_0)/2$ yields

$$\liminf_{i\to\infty} \|A_{s_i}[T]x - \xi\| < \liminf_{i\to\infty} \|A_{s_i}[T]x - x_0\| \ .$$

This is a contradiction which asserts that $Tx_0 = x_0$. We conclude that the orbit $\{T^n x\}$ is norm–bounded for each x in C. Therefore, taking into account that every closed bounded convex subset of H is weakly compact, we deduce from Lemma 2 that

$$A_C(\{T^n x_0\}) \cap \left(\bigcap_n \overline{\text{co}}\{T^k x : k \geq n\} \right) = \{x_0\} \ .$$

In addition, we see by means of Lemma 3 that

$$A_C(\{T^n x_0\}) = A_C(\{T^n x\}) = \{x_0\} \ .$$

This shows that $A_r[T]x$ converges weakly to the point x_0 as $r \to 1-0$ and the proof of Theorem 1 is complete. $\square$

Now it seems to be somewhat interesting to ask whether the weak convergence of the Abelian averages $A_r[T]x$ implies the norm–boundedness of the orbit $\{T^n x\}$. In particular, when T is nonexpansive on C being a nonempty closed convex subset of H, the weak convergence of the Cesàro $(C,1)$ averages $C_n[T]x$ $(= [x + Tx + \cdots + T^{n-1}x]/n,\ n \geq 1)$ for each x in C is known to imply the norm–boundedness of the orbit $\{T^n x\}$ ([6], Theorem 2). In connection with this question we have the following theorem which is characteristic of asymptotically nonexpansive mappings. In what follows, $F(T)$ stands for the set of fixed points of T.

Theorem 2. *Let C be a nonempty closed convex subset of H and let T be a $\{\alpha_n\}$-asymptotically nonexpansive self mapping of C. Let x be arbitrarily fixed in C. Then the following conditions are equivalent:*

(1) *The set $F(T)$ is not empty.*

(2) *The orbit $\{T^n x\}$ is norm–bounded.*

(3) *The set $\{A_r[T]x\}$ is norm–bounded and for any $\varepsilon > 0$ there exists an integer $m_0 = m_0(\varepsilon) \geq 1$ such that for each $m \geq m_0$ there is a number $r_0 = r_0(\varepsilon, m)$, $0 < r_0 < 1$, satisfying*

$$\|A_r[T]x - T^m A_r[T]x\|^2 < \varepsilon M(x) \ \ \textit{for } r_0 < r < 1 \ ,$$

where $M(x)$ is a positive constant depending only on x.

Proof. Implication (1)⇒(2) is obvious. Implication (3)⇒(1) is a direct consequence of Theorem 1. We now prove implication (2)⇒(3). In general, for any ξ in H

$$\|A_r[T]x-\xi\|^2 = (1-r)^2 \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} r^{n+k} < T^n x-\xi, T^k x-\xi >$$

and

$$2 < T^n x-\xi, T^k x-\xi >= \|T^n x-\xi\|^2 + \left\|T^k x-\xi\right\|^2 - \left\|T^n x - T^k x\right\|^2 .$$

After replacing ξ with $A_r[T]x$, one gets

$$(1-r)^2 \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} r^{n+k}\left\|T^n x - T^k x\right\|^2 = 2(1-r)\sum_{n=0}^{\infty} r^n \|T^n x - A_r[T]x\|^2 ,$$

so that

$$\|A_r[T]x-\xi\|^2 = (1-r)\sum_{n=0}^{\infty} r^n \|T^n x-\xi\|^2 - (1-r)\sum_{n=0}^{\infty} r^n \|T^n x - A_r[T]x\|^2 .$$

Again, on taking $\xi = T^m A_r[T]x$ for $m \geq 1$, we have

$$\begin{aligned}
\|A_r[T]x - T^m A_r[T]x\|^2 &= (1-r)\sum_{n=0}^{\infty} r^n \|T^n x - T^m A_r[T]x\|^2 \\
&\quad - (1-r)\sum_{n=0}^{\infty} r^n \|T^n x - A_r[T]x\|^2 \\
&\leq (1-r)\sum_{n=0}^{m-1} r^n \|T^n x - T^m A_r[T]x\|^2 \\
&\quad + (1-r)(1+\alpha_m)^2 \sum_{n=0}^{\infty} r^{n+m} \|T^n x - A_r[T]x\|^2 \\
&\quad - (1-r)\sum_{n=0}^{\infty} r^n \|T^n x - A_r[T]x\|^2 .
\end{aligned}$$

Let $\varepsilon > 0$ be arbitrarily small. Since $\lim_{n\to\infty} \alpha_n = 0$, one can choose an integer $m_0 = m_0(\varepsilon) \geq 1$ such that

$$(1+\alpha_m)^2 - 1 < \varepsilon \quad \text{for all } m \geq m_0$$

and such that for each $m \geq m_0$ there is a number $r_0 = r_0(\varepsilon, m)$, $0 < r_0 < 1$, satisfying

$$\max\{|r^m - 1|, m(1-r)(1+\alpha_m)^2\} < \varepsilon \quad \text{for } r_0 < r < 1 \ .$$

Hence, observing that

$$\sup_n \sup_r \|T^n x - A_r[T]x\|^2 \leq 4\{\sup_n \|T^n x\|\}^2$$

and

$$\sup_n \sup_r \|T^n x - T^m A_r[T]x\|^2 \leq 16(1+\alpha_m)^2 \{\sup_n \|T^n x\|\}^2 \ ,$$

we have for each $m \geq m_0$ and each r with $r_0 < r < 1$

$$\begin{aligned}
\|A_r[T]x - T^m A_r[T]x\|^2 &\leq 4\{4m(1-r)(1+\alpha_m)^2 \\
&\quad + |r^m(1+\alpha_m)^2 - 1|\}\{\sup_n \|T^n x\|\}^2 \\
&\leq 4\{4m(1-r)(1+\alpha_m)^2 \\
&\quad + (1+\alpha_m)^2 - 1 + |r^m - 1|\}\{\sup_n \|T^n x\|\}^2 \\
&< \varepsilon M(x) \ ,
\end{aligned}$$

where $M(x) = 24\{\sup_n \|T^n x\|\}^2 + 1$, and the theorem is proved. □

We next consider the case of the mapping T nonexpansive on C. If we take $\alpha_n = 0$, $n = 1, 2, \ldots$, this is just the case. Then Theorem 1 becomes

Theorem 3 (cf. [11], Theorem 2.6.1)**.** *Let C be a nonempty closed convex subset of H and let T be a nonexpansive self–mapping of C. Suppose that for each x in C, $\{A_r[T]x\}$ is norm–bounded and* $\mathrm{s-}\lim_{r\to 1-0}(I-T)A_r[T]x = 0$. *Then for each x in C, $A_r[T]x$ converges weakly to a fixed point of T as $r \to 1-0$.*

Theorem 2 becomes

Theorem 4. *Let C be a nonempty closed convex subset of H and let T be a nonexpansive self–mapping of C. Let x be an arbitrary element in C. Then the following conditions are equivalent:*

(1) *The set $F(T)$ is not empty.*

(2) *The orbit $\{T^n x\}$ is norm–bounded.*

(3) *The set $\{A_r[T]x\}$ is norm–bounded and* $\mathrm{s-}\lim_{r\to 1-0}(I-T)A_r[T]x = 0$.

Theorem 5. *Let C be a nonempty closed convex subset of H and let T be a nonexpansive self–mapping of C. Then the following conditions are equivalent:*

(1) $\overline{\mathrm{Range}\,(I-T)}$ *contains* 0.

(2) *For each x in C, $A_r[T]x$–$A_r[T]Tx$ converges weakly to* 0 *as* $r \to 1-0$.

(3) *For each x in C, $A_r[T]x - A_r[T]Tx$ converges strongly to* 0 *as* $r \to 1-0$.

Proof. We first prove implication (1)⇒(3). Let x be in C and let $\varepsilon > 0$ be arbitrarily small. Put $S_n[x] = x + Tx + \cdots + T^{n-1}x$ for $n \geq 1$, and so $C_n[T]x = S_n[x]/n$. Since $0 \in \overline{\mathrm{Range}\,(I-T)}$, there exists by Lemma 4 of [6] an integer $n_0 = n_0(x, \varepsilon) \geq 1$ such that

$$\|C_n[T]x - C_n[T]Tx\| < \varepsilon \quad \text{for all} \quad n \geq n_0 \,.$$

For arbitrary integers p, q with $p \geq n_0\,,\ q \geq 1$, one has

$$\begin{aligned}\sum_{n=p}^{p+q} r^n T^n x &= \sum_{n=p}^{p+q} r^n (x + S_n[Tx] - S_n[x]) \\ &= \sum_{n=p}^{p+q} r^n x + \sum_{n=p}^{p+q} n r^n \{C_n[T]Tx - C_n[T]x\}\end{aligned}$$

and thus

$$\|\sum_{n=p}^{p+q} r^n T^n x\| \leq \sum_{n=p}^{p+q} r^n \|x\| + \varepsilon \sum_{n=p}^{p+q} (n+1) r^n \,.$$

Hence

$$\lim_{\substack{p\to\infty \\ q\to\infty}} \|\sum_{n=p}^{p+q} r^n T^n x\| = 0 \,,$$

and $A_r[T]x$ is well defined for $0 < r < 1$. Now we have

$$A_r[T]x - A_r[T]Tx = (1-r)^2 \sum_{n=0}^{\infty} (n+1) r^n \{C_{n+1}[T]x - C_{n+1}[T]Tx\} \,.$$

Therefore

$$\begin{aligned}\|A_r[T]x - A_r[T]Tx\| \leq &(1-r)^2 \sum_{n=0}^{n_0-1} (n+1) r^n \|C_{n+1}[T]x - C_{n+1}[T]Tx\| \\ &+ \varepsilon (1-r)^2 \sum_{n=n_0}^{\infty} (n+1) r^n \,,\end{aligned}$$

which implies

$$\limsup_{r\to 1-0} \|A_r[T]x - A_r[T]Tx\| \le \varepsilon\,.$$

Since ε is arbitrary, condition (3) holds. We next prove implication (3)$\Rightarrow$(1). When using the equality

$$A_r[T]x - A_r[T]Tx = (1-r)\sum_{n=0}^{\infty} r^n (I-T)T^n x\,,$$

condition (3) asserts that 0 is contained in $\overline{\mathrm{co}}$ Range $(I-T)$. However, according to Lemmas 4 and 5 of [10], $\overline{\mathrm{Range}\,(I-T)}$ is convex and Range $(I-T)$ has the minimum property. Therefore 0 is contained in $\overline{\mathrm{Range}\,(I-T)}$. Implications (3)$\Rightarrow$(2) and (2)$\Rightarrow$(1) are obvious. This completes the proof of Theorem 5. $\square$

In [6], Hirano and Takahashi obtained a generalization of Baillon's theorem to more general asymptotically nonexpansive mappings. They in fact proved that if C is a nonempty closed convex subset of H and if T is an asymptotically nonexpansive self–mapping of C such that for each x in C the orbit $\{T^n x\}$ is norm–bounded, then for each x in C, $C_n[T]x$ converges weakly to a fixed point of T. We remark that the weak convergence of $\{C_n[T]x\}$ remains true even if the norm–boundedness of $\{T^n x\}$ is replaced by the norm–boundedness of $\{C_n[T]x\}$ and the existence of an integer $m_0 \ge 1$ such that $\mathrm{s}-\lim_{n\to\infty}(I-T^m)C_n[T]x = 0$ for each $m \ge m_0$. Taking this fact into account, we have the following theorem pertaining to the relation between the Cesàro $(C,1)$ limit and the Abel limit.

Theorem 6. *Let C be a nonempty closed convex subset of H and let T be a asymptotically nonexpansive self–mapping of C. Suppose that for each x in C there exists an integer $m_0 \ge 1$ such that $\mathrm{s}-\lim_{n\to\infty}(I-T^m)C_n[T]x = 0$ and $\mathrm{s}-\lim_{r\to 1-0}(I-T^m)A_r[T]x = 0$ for each $m \ge m_0$. Then for each x in C, $C_n[T]x$ converges weakly to a point x_0 in C as $n \to \infty$ if and only if $A_r[T]x$ converges weakly to x_0 as $r \to 1-0$.*

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