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The limits of weighing

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Mass determination in the gravitational field covers a wide measuring range and, as a comparative method, facilitates very sensitive measurements. However, gravity is a very weak force. First by application of acceleration fields and with mass spectrometric methods, the measuring range could be extended towards atomic and sub-atomic masses. There are only practical limits of weighing heavy masses. The mass of celestial objects can be determined by means of Kepler's Laws.

1. INTRODUCTION

Weighing means using a balance for mass determination (Fig. 1). In science, we are usually interested in mass as a characteristic parameter of matter, and not in weight, which is a force that depends on the local gravitational field. However, mass is an abstract concept and can be determined only indirectly, by weighing. Thus, different methods are necessary to cover the wide range of mass of articles to be determined (Tab. 1):

1. Observation of the movement.
2. Weighing in the gravitational field.
3. Measurement of the impulse of an accelerated object.
4. Measurement of the frequency shift of an oscillating or rotating object.

According to Einstein's equivalence principle, rest mass and moving mass cannot be distinguished [1-2] and thus, all these methods are equivalent and give the same result.

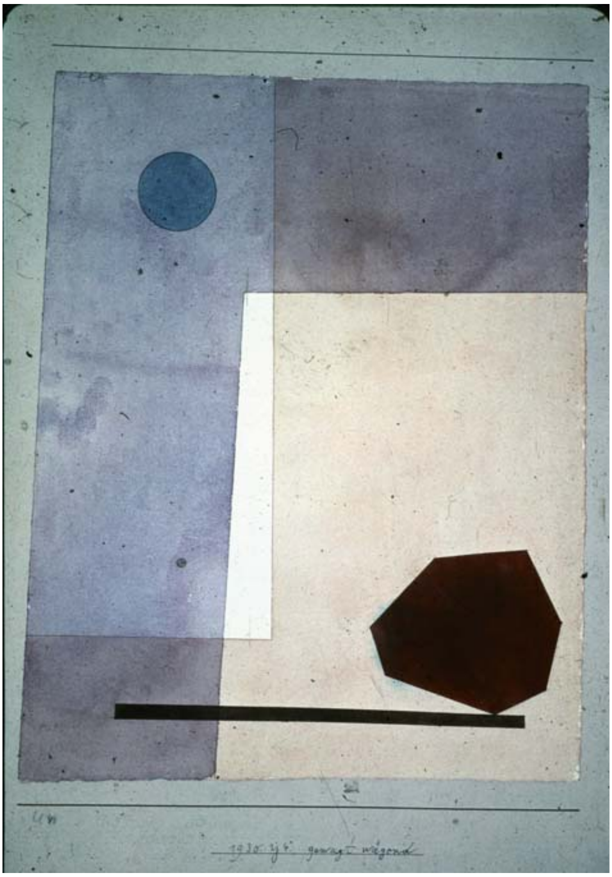


Fig. 1. Paul Klee: Gewagt wägend (Risky weighing). The original colored aquarelle is exhibited in the Zentrum Paul Klee, Bern, © Zentrum Paul Klee.

Table 1. Mass of some objects in rest.

Object	Mass	Object	Mass [kg]
Photon	0	Sun	$1.989 \cdot 10^{30}$
Higgs particle	114.4 > m_{Higgs} >170 GeV	Moon	$7.349 \cdot 10^{22}$
Quark Up	1.5-4.0 MeV c^{-2}	Mercury	$3.302 \cdot 10^{23}$
Quark Down	4-8 MeV c^{-2}	Venus	$4.869 \cdot 10^{24}$
Quark Strange	80-130 MeV c^{-2}	Earth	$5.9736 \cdot 10^{24}$
Quark Charm	1150-1350 MeV c^{-2}	Mars	$6.419 \cdot 10^{23}$
Quark Bottom	4100-4400 MeV c^{-2}	Jupiter	$1.899 \cdot 10^{27}$
Quark Top	170900 ± 1800 MeV c^{-2}	Saturn	$5.685 \cdot 10^{26}$
Gibbs Boson	126 GeV c^{-2}	Uranus	$8.683 \cdot 10^{25}$
Electron	510 998.9 eV = $5.485\,799\,110(12) \cdot 10^{-4}$ u $9.109\,381\,88(72) \cdot 10^{-31}$ kg	Neptun	$1.0243 \cdot 10^{26}$
Proton	1.007 276 466 88(13) u $1.672\,621\,58(13) \cdot 10^{-27}$ kg	Neutron star	$2.7\text{-}4.2 \cdot 10^{30}$
Mist particle	~ 0.01-0.3 g	Black holes	$\sim 5 \cdot 10^{30} – 10^{40}$
Rain droplet	~ 0.05 g	Galaxy Milky Way	$3.6 \cdot 10^{41}$
1 l = 1 dm Water	1 kg		
Water on Earth	$1.384 \cdot 10^{21}$ kg	Universe (visible matter)	$10^{52} – 10^{54}$

In many calculations the gravitational constant $G = 6.67428(67) \cdot 10^{-11}$ m^3 kg^{-1} s^{-2} plays a role. The value of the gravitational constant is less accurate than that of all the other fundamental constants; its standard uncertainty is $1.4 \cdot 10^{-3}$. That means that the accuracy of all formulae including G is limited to 1 in 10000. Many geological, meteorological, astronomical calculations, as well as those of space operations are burdened with that basic uncertainty.

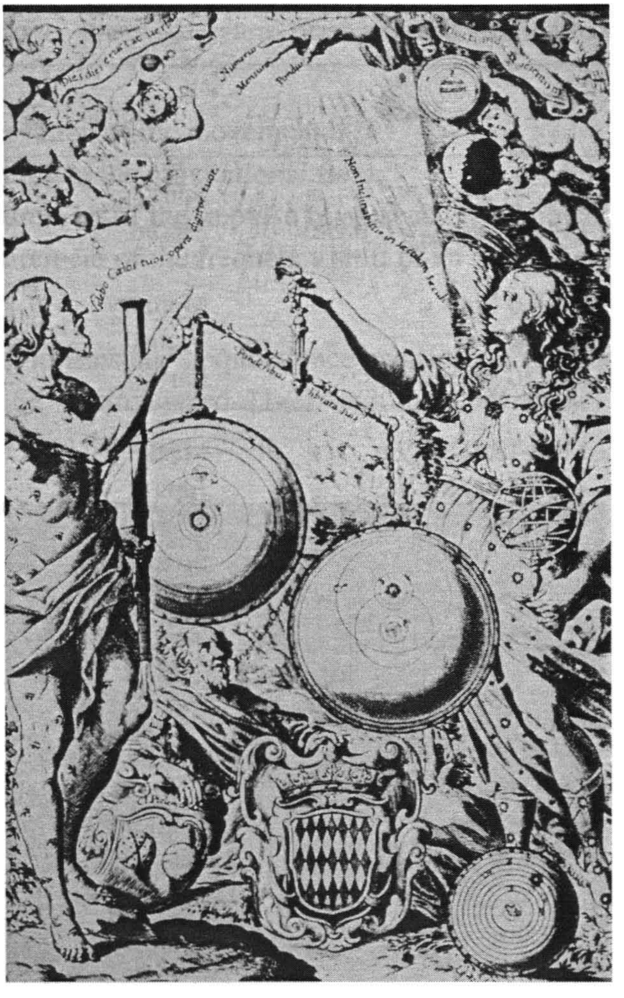


Fig. 2. Comparison of the geocentric (Ptolemaic) and the heliocentric (Copernican) model by weighing of the solar system.

However, in weighing, the mass of an article is compared with the mass of a prototype. This relative mass can be measured with high accuracy, restricted only by the error of the balance and influences of the environment. Finally, the resolution of weighing using a classical balance is limited downwards to the nanogram range by the Brownian motion of the gas molecules which collide with the movable parts [3-4].

Astronomy

Let us start with the upper limits. Archimedes of Syracuse (~287 BC-~212 BC) supposedly claimed that he could lift the Earth off its foundation if he were given a place to stand, and a long enough lever [5].

This way he could also weigh the Earth. Unfortunately the so-called Archimedean point outside an experimental set-up is not available. So we cannot weigh astronomical objects with a balance (Fig. 2).

In astronomy the orbits of planets, stars, and nebulae are observed. In general, the rotation of a binary star system, the motions of a planet around the sun or of a moon around a planet can be regarded as a two-body system, and irregularities of the movement on account of various influences can be neglected (Fig. 3). The mass of such objects can be calculated by means of Kepler's 3rd Law [6]. Even star clusters and galaxies at sufficiently large distance may be regarded as punctiform and then Kepler's laws can be applied approximately. Other methods include measurements of the acceleration of a celestial body or aberration of rays of light due to gravity.

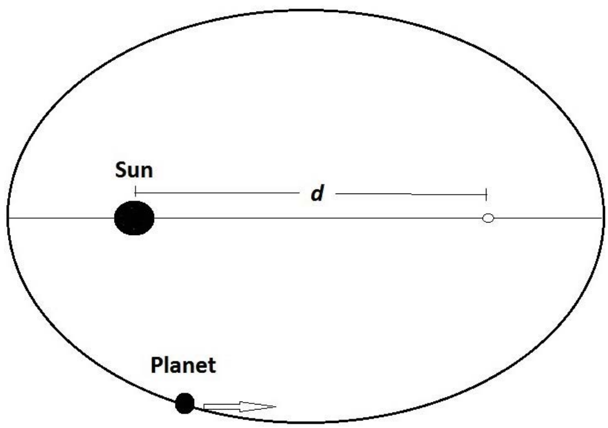


Fig. 3. Kepler's 3rd Law: $t_p^2 \approx \dfrac{4\,\pi^2\,d^3}{G\,m_1} \quad if \quad m_1 >> m_2$.

From these results and by counting all the visible objects on heaven we can calculate the mass of the visible universe. The visible universe with a radius of 10^{28} m contains at least 10^{11} galaxies. Our galaxy contains at least 10^{11} stars and $5 \cdot 10^{10}$ planets. The mass of the sun is about a $1.99 \cdot 10^{30}$ kg. Calculation of the mass of our galaxy results to about 10^{42} kg. That means the visible mass of the universe is at least 10^{53} kg. This very rough estimate is based on an approximate model, because:

1. We cannot see the boundaries of our Universe.
2. On account of the finite value of light velocity, we look into the past and extrapolate the present.

3. Only about 4 % is visible. About 23 % is considered to be composed of dark matter [7] and the remaining 73 % is thought as dark energy, distributed diffusely in space, and 0.3 % of matter may be neutrinos.

Nevertheless, we can determine approximately the mass of all such items which had been identified so far.

Classical weighing

When weighing in the gravitational field of the earth the test sample is always in a local constant field and the force is measured against Earth mass. Upper limits of weighing of earthly objects are governed by practical considerations. Up to the present time, all things of interest could be weighed. The Eiffel Tower in Paris has a mass of about 10,000 tons and deriving its weight could be based on hydraulic presses in the foundations which had been planned for the correction of the upright position of the tower [8]. A need for this correction never occurred. For practical reasons large quantities of material are usually weighed by proportioning.

The usual weighing range is between 1μg and 1000 t (Fig. 4) [9]. The lower limits of weighing and the sensitivity of balances have gradually improved according to the development of techniques and of the demands of users.

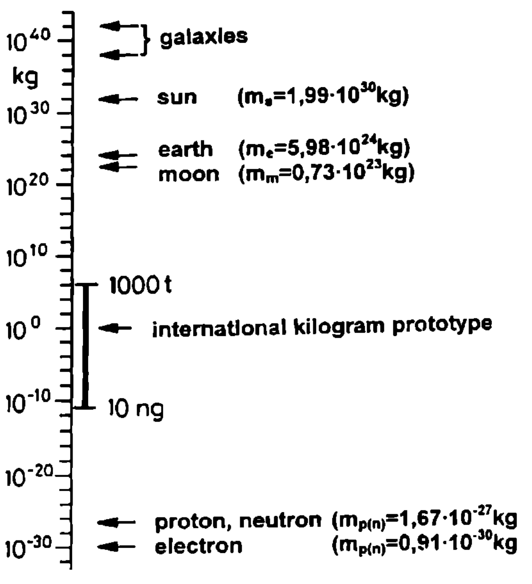


Fig. 4. Survey of masses and usual of the weighing range. © Kochsiek, Schwarz.

We may characterize the ability of balance by the relative resolution, i.e. resolution divided by the maximum load. The oldest relict of a balance is a balance beam found in Upper Egypt and about 5500 years old (Fig. 5) [10].

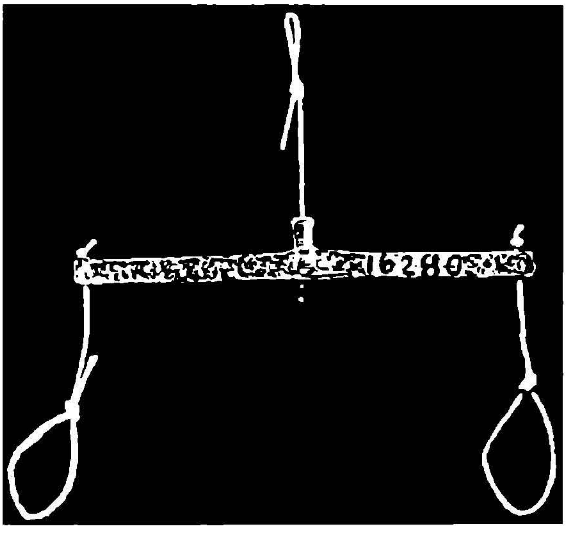


Fig. 5. Oldest balance beam, reddish limestone 85 mm long. The ropes are added for demonstration. Pre-dynastic Naqada period ~ 3500 B.C., Naqada, Upper Egypt. © Petrie Collection, London.

Balances of that time had a relative resolution in the per mille range. 3000 years later, in Egyptian's New Kingdom that value had been improved to 10^{-4} (Fig. 6) [11]. The "Balance of Wisdom", a hydrological balance built by Al Chazini 1120 AD achieved a relative resolution of $2 \cdot 10^{-5}$ (Fig. 7) [12].

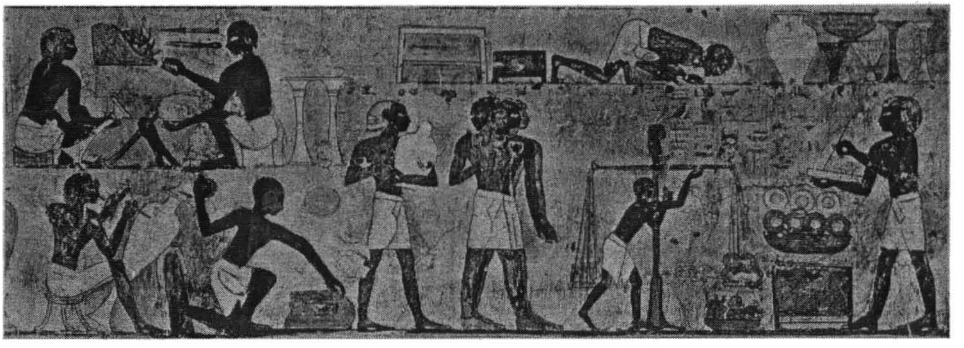
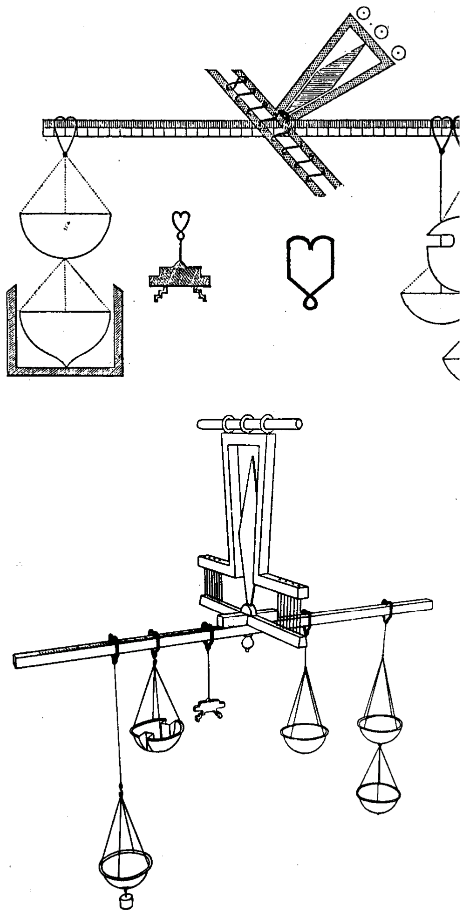


Fig. 6. Weighing of tributes. Papyrus, Egypt, 18. Dynasty.



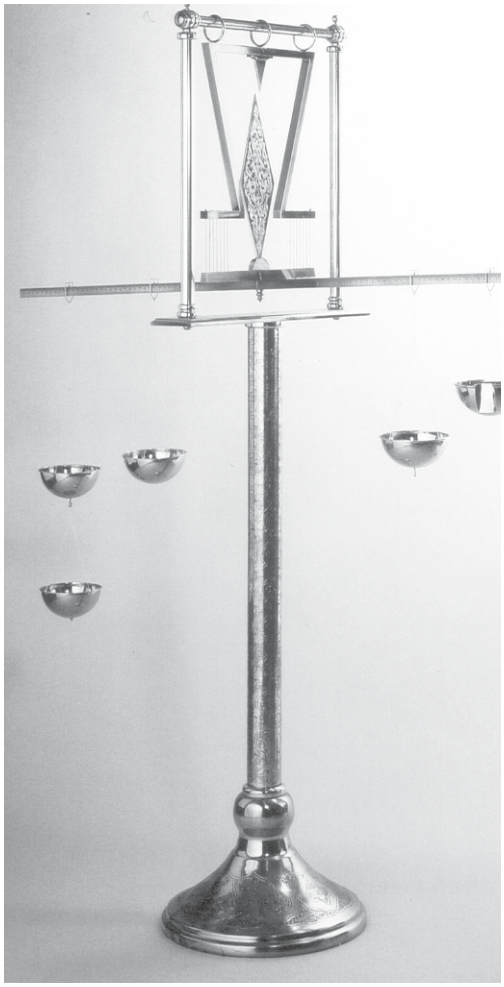


Fig. 7. Balance of wisdom of Al-Chazini 1120. Above: original drawing, below: perspective view.

Reconstruction, Islamisches Institut, Johann-Wolfgang-Goethe Universität, Frankfurt am Main. Germany.

Real progress took place when scientists like Lavoisier (1743–1794) founded modern chemistry and established the metric system [13]. To weigh the mass of prototypes (*in vacuo)* highly sensitive balances were needed (Fig. 8) [14].

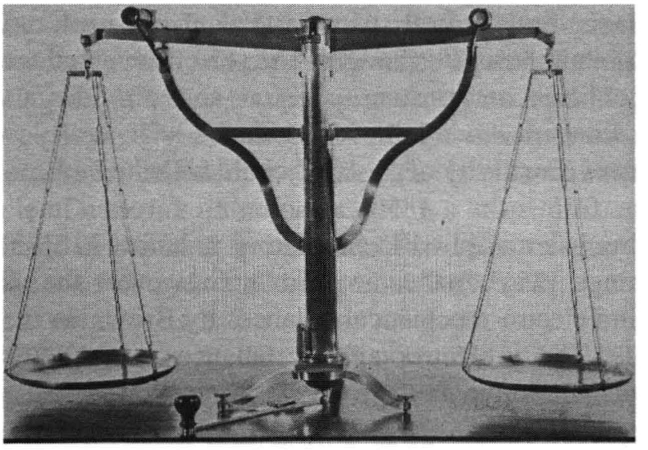


Fig. 8. Balance of Fortin for Lavoisier. © Musée des Arts et Métiers, Paris, France.

At the end of the 19th century mechanical metrological comparator balances for kilogram prototypes achieved a relative sensitivity of $2.5 \cdot 10^{-9}$. At present, the most accurate balance is an equal-arm beam balance for loads up to 2 kg at the Bureau International des Poids et Mésures at Sèvres, France (Fig. 9) [15].

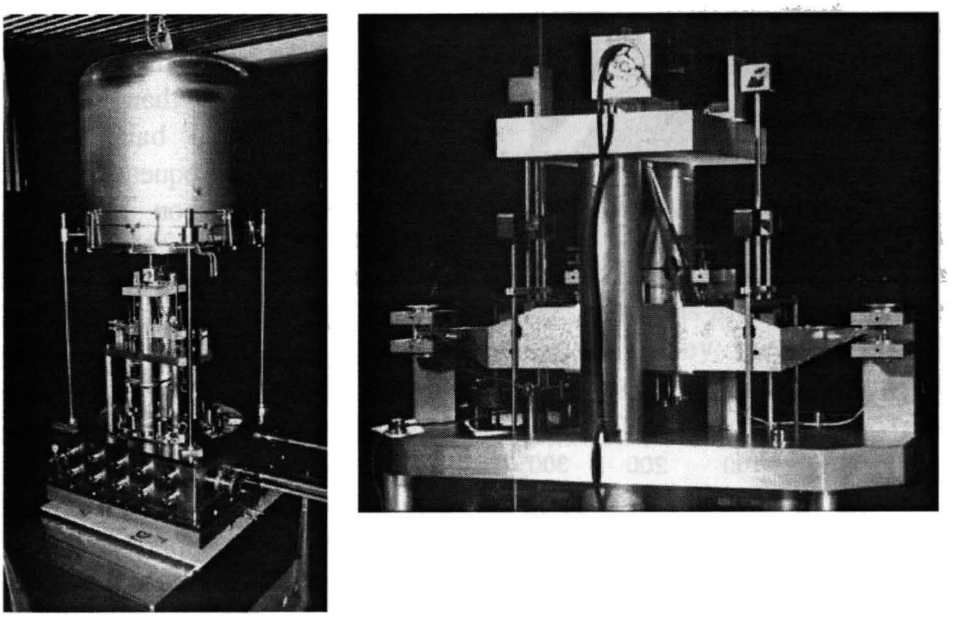


Fig. 9. Flexure-strip balance FB-2, © Bureau International des Poids et Mesures (BIPM), Sèvres, France.

This balance has flexural strips instead of bearings and is equipped with an automatic sample changer. An electromagnetic servo-control maintains the beam at constant position and provides the imbalance reading. The standard deviation of the balance is generally within 0.1 µg. Indeed a relative sensitivity of $3 \cdot 10^{-12}$ could be achieved.

Starting from the 1850s, Lorentz forces in, addition to counterweights, were applied in laboratory balances [16]. This way, the nanogram range was made accessible; however the limitation of mechanical and electro-mechanical balances by Brownian motion become clearly visible. Today electromagnetic balances with relative resolution down to about 10^{-7} dominate the market of precision scales.

Oscillators

The gravitational force is by far the weakest of the four basic forces, and we can apply it for weighing only on account of the big mass of the Earth. The weakness, however, limits the ability of balances. In 1957 Sauerbrey invented the oscillating quartz crystal balance [17-18]. Oscillators produce an acceleration field which can be a million times stronger than the gravitational field of the Earth [19]. Thus, measuring range and resolution of such a balance can be far better than that of a conventional balance. By observing the frequency shift of oscillating carbon nanotubes [20], silica nanorods or graphene foils recently, mass changes in the attogram (10^{-18} g) and zeptogram (10^{-21} g) range have been observed. It was even possible to measure forces in sub-atomic regions.

Using micromechanical cantilevers as applied in micromechatronic systems MEMS), it is possible to investtigate samples with nanogram mass and mass changes in the picogram range [21] (Fig. 10).

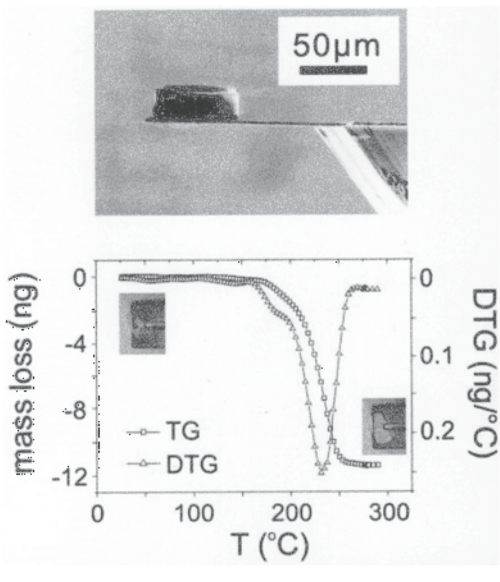


Fig. 10. Scanning electron micrograph of a zeolite single crystal on a micromechanical cantilever. Heating of the sample (TG) resulted in a mass loss of 12 ng. Furthermore the differential mass loss is (DTG) is plotted. © Rüdiger Berger.

Observing the resonance frequency of the elastic lever the curve showing dependence with temperature can be recorded. With the atomic force microscope (AFM) it is possible to catch and displace single molecules or clusters. The manipulation allows simultaneous determining of binding forces. So-called optical tweezers is a laser beam which can catch a particle of nano- or micrometer size in a 'trap'. If such a particle is moved by means of the laser beam to another object, it may be edge out of that trap and the forces exerted can be measured [22].

Mass spectrometer

In mass spectrometers, by means of electric and magnetic fields, molecules and atoms from a molecular beam can be separated and their mass relation determined. Recently, a device called Penning trap mass spectrometer SHIPTRAP at the für Schwerionenforschung GSI Darmstadt (Germany) has been used to separate three nobelium isotopes $^{252\text{-}254}No$ and to catch each single ion in an ion cage. The cyclotron frequency of the rotating ion is determined, which is directly related to its mass. These are the first direct mass measurements of transuranium elements, which provide new anchor points in this region [23-24]. The exact statement of an atomic mass in the kilogram unit is possible still only by indirect methods because the mass of an atom is about 10^{-26} kg. The uncertainties for the differences between two atomic masses correspond to 10^{-36} kg.

2. CONCLUSION

Mass determination in the gravitational field covers the range of nanograms to about thousand tons. As a comparative method a relative resolution of 10^{-12} was achieved. However, gravity is a very weak force. First, by application of acceleration fields or by means of mass spectrometric methods, the measuring range has been extended towards atomic and sub-atomic masses. For mass determination no limits are noticeable [25].

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