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Ishikawa iterative processes with errors for approximations of zeros of strongly accretive operator equations

Abstract. In this paper we consider the strong convergence of the sequence of the Ishikawa iterative process with errors to fixed points and solutions of quasi-strongly accretive and quasi-strongly pseudo-contractive operator equations in Banach spaces. Considered error terms are not necessarily summable. Our main results improve and extend the corresponding results recently obtained by Chidume [1], [2], Deng [4], [5], Deng and Ding [6], Liu [8], Xu [11] and Zhou and Jia [12].

1. Introduction. Suppose X is an arbitrary Banach space, X^* is the dual space of X and $\langle\cdot,\cdot\rangle$ is the pairing between X and X^*. The mapping $\dot{J}: X \to 2^{X^*}$ defined by

$$\dot{J}(x) = \{f \in X^* : \langle x, f\rangle = \|f\|\cdot\|x\|,\ \|f\| = \|x\|\}$$

is called the normalized duality mapping. It is known that if X is uniformly smooth (or equivalently, X^* is uniformly convex), then $\dot{J}$ is single-valued and uniformly continuous on any bounded subset of X.

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An operator $T : X \to X$ is called *strongly accretive* if for all $x, y \in D(T)$ ($D(T)$ - the domain of T) there exists $j(x-y) \in \dot{J}(x-y)$ and a constant $k > 0$ such that

$$\mathrm{Re}\,\langle Tx - Ty, j(x-y)\rangle \geq k\|x-y\|^2. \tag{1}$$

Without loss of generality we may assume $k \in (0,1)$. T is called *accretive* if T satisfies (1) with $k = 0$.

We will say that an operator T is quasi-strongly-accretive if there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that for any $x, y \in D(T)$:

$$\mathrm{Re}\,\langle Tx - Ty, j(x-y)\rangle \geq \phi(\|x-y\|). \tag{2}$$

An operator T is said to be *strongly* (*quasi-strongly*) *pseudo-contractive* if $I - T$ (where I denotes the identity mapping) is strongly (quasi-strongly) accretive mapping.

Every strongly accretive operator is quasi-strongly accretive with $\phi : [0,\infty) \to [0,\infty)$ defined by $\phi(t) = kt^2$. The following example shows that the class of strongly accretive operators is a proper subclass of the class of quasi-strongly accretive operators.

Example. Let $X = \mathbb{R}$ (the reals with the usual norm) and let $K = [0,\infty)$. Define

$$Tx = x^3.$$

Since $x^2 + xy + y^2 \geq |x-y|^2$ on K, T is quasi-strongly accretive on K with $\phi : [0,\infty) \to [0,\infty)$ defined by

$$\phi(t) = t^4.$$

However, for any fixed $x \in (0,1)$ and $y = 0$ we have

$$\langle Tx - Ty, j(x-y)\rangle < k|x-y|^2$$

for all $x \in K$ with $0 < x < \sqrt{k}$. Therefore, T is not strongly accretive operator.

The study of accretive operators has an important role in the existence theory for nonlinear evolution equation in Banach spaces (see for example [3]).

If $T : X \to X$ is strongly accretive and the equation $Tx = f$ has a solution, methods for approximating the solution have been studied extensively by several researchers. Many authors have applied the Mann iteration method and the Ishikawa iteration method to approximate solutions of $Tx = f$.

Recently Liu [8] introduced the following iteration method which he called *Ishikawa (Mann) iteration method with errors.*

For a nonempty subset K of X and a mapping $T : K \to X$, the sequence $\{x_n\}$ defined for arbitrary x_0 in X by

$$y_n = (1 - \beta_n)x_n + \beta_n T x_n + v_n,$$
$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T y_n + u_n$$

for all $n = 0, 1, 2, \ldots$, where $\{u_n\}$ and $\{v_n\}$ are two summable sequences in X (i.e., $\sum_{n=0}^{\infty} \|u_n\| < \infty$ and $\sum_{n=0}^{\infty} \|v_n\| < \infty$), $\{\alpha_n\}$ and $\{\beta_n\}$ are two real sequences in $[0, 1]$ satisfying suitable conditions, is called the *Ishikawa iterates with errors.* If $\beta_n = 0$ and $v_n = 0$ for all n, then the sequence $\{x_n\}$ is called the *Mann iterates with errors.*

If $u_n \equiv 0$, $v_n \equiv 0$ then the Ishikawa and Mann iteration methods with errors reduce to the original Ishikawa and Mann iteration methods.

Unfortunately, the definitions of Liu, which depend on the convergence of the error terms, is against the randomness of errors.

The purpose of this paper is to define the Ishikawa iterates with errors where the error terms are not necessarily summable. We refine and improve the method of proof which have been used for strongly accretive operators, as this method is not directly applicable for quasi-strongly accretive operators. We prove the strong convergence of the Ishikawa iterates with errors to fixed points and solutions of quasi-strongly accretive and quasi-strongly pseudo-contractive operator equations. Our main results improve and extend the corresponding results recently obtained by Chidume [1], [2], Deng [4], [5], Deng and Ding [6], Liu [8], Xu [11] and Zhou and Jia [12].

2. Main results. First we state the following lemma, which we shall use in the proof of our main theorem.

Lemma 1 ([11])**.** *Let X be a Banach space. Then for all $x, y \in X$ and $j(x + y) \in \dot{J}(x + y)$,*

$$\|x + y\|^2 \le \|x\|^2 + 2\mathrm{Re}\,\langle y, j(x + y)\rangle.$$

Our main theorem is the following.

Theorem 1. *Let X be a uniformly smooth Banach space and let $T : X \to X$ be a quasi-strongly accretive mapping, i.e.*

$$\mathrm{Re}\,\langle Tx - Ty, j(x - y)\rangle \ge \phi(\|x - y\|), \tag{3}$$

where $\phi : [0, +\infty) \to [0, +\infty)$ is a strictly increasing function with $\phi(0) = 0$. Suppose that there exists a solution of the equation $Tx = f$ for some $f \in X$. For $f \in X$ define $S : X \to X$ by $Sx = f + x - Tx$ for all $x \in X$, and suppose *that the range of S is bounded. Let for arbitrary $x_0 \in X$ the Ishikawa iteration sequence $\{x_n\}$ with errors be defined by*

$$y_n = (1-\beta_n)x_n + \beta_n S x_n + b_n v_n, \tag{4}$$

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n S y_n + a_n u_n, \tag{5}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, $\{a_n\}$ and $\{b_n\}$ are sequences in $[0,1]$ satisfying

$$\lim_{n\to\infty} \alpha_n = 0, \quad \lim_{n\to\infty} \beta_n = 0, \tag{6}$$

$$\sum_{n=0}^{\infty} \alpha_n = \infty, \tag{7}$$

$$a_n \le \alpha_n^{1+c} \quad (c>0), \quad b_n \le \beta_n, \tag{8}$$

and $\{u_n\}$ and $\{v_n\}$ are two bounded sequences in X. Then $\{x_n\}$ converges strongly to the unique solution of the equation $Tx = f$.

Proof. Let $Tq = f$, so that q is a fixed point of S. Since T is quasi-strongly accretive, it follows from definition of S that

$$\operatorname{Re}\langle Sx - Sy, j(x-y)\rangle \le ||x-y||^2 - \phi(||x-y||). \tag{9}$$

Setting $y = q$ we have

$$\langle Sx - q, j(x-q)\rangle \le ||x-q||^2 - \phi(||x-q||). \tag{10}$$

If p is a fixed point of S, then (10) with $x = p$ implies $p = q$.

We prove that $\{x_n\}$ and $\{y_n\}$ are bounded. Let

$$\begin{aligned} A &= \sup\{\|Sx_n - q\| + \|Sy_n - q\| : n \ge 0\} + \|x_0 - q\|, \\ B &= \sup\{\|u_n\| + \|v_n\| : n \ge 0\}, \\ M &= A + B. \end{aligned}$$

From (5) and (8) we get

$$\begin{aligned} \|x_{n+1} - q\| &\le (1-\alpha_n)\|x_n - q\| + \alpha_n\|Sx_n - q\| + a_n\|u_n\| \\ &\le (1-\alpha_n)\|x_n - q\| + \alpha_n A + \alpha_n B \end{aligned}$$

and hence

$$\|x_{n+1} - q\| \le (1-\alpha_n)\|x_n - q\| + \alpha_n M. \tag{11}$$

From (4) and (8) we have

$$\begin{aligned} \|y_n - q\| &\le (1-\beta_n)\|x_n - q\| + \beta_n\|Sy_n - q\| + b_n\|v_n\| \\ &\le (1-\beta_n)\|x_n - q\| + \beta_n A + \beta_n B \end{aligned}$$

and hence

$$\|y_n - q\| \le (1-\beta_n)\|x_n - q\| + \beta_n M. \tag{12}$$

Now we show by induction that

$$\|x_n - q\| \le M \tag{13}$$

for all $n \ge 0$. For $n = 0$ we have $\|x_0 - q\| \le A \le M$, by definition of A and M. Assume now that $\|x_n - q\| \le M$ for some $n \ge 0$. Then by (11) we have

$$\|x_{n+1} - q\| \le (1-\alpha_n)M + \alpha_n M = M.$$

Therefore, by induction we conclude that (13) holds.
Substituting (13) into (12) we get

$$\|y_n - q\| \le M. \tag{14}$$

From (12) we have

$$\|y_n - q\|^2 \le (1-\beta_n)^2\|x_n - q\|^2 + 2\beta_n(1-\beta_n)M\|x_n - q\| + \beta_n^2 M^2.$$

Since $1-\beta_n \le 1$ and $\|x_n - q\| \le M$, we get

$$\|y_n - q\|^2 \le \|x_n - q\|^2 + 2\beta_n M^2. \tag{15}$$

Using Lemma 1 we get

$$\begin{aligned}\|x_{n+1} - q\|^2 &= \|(1-\alpha_n)(x_n - q) + a_n u_n + \alpha_n(Sx_n - q)\|^2\\ &\le \|(1-\alpha_n)(x_n - q) + a_n u_n\|^2 + 2\alpha_n \mathrm{Re}\,\langle Sx_n - q, j(x_{n+1} - q)\rangle\\ &\le (1-\alpha_n)^2\|x_n - q\|^2 + 2(1-\alpha_n)a_n\|x_n - q\|\cdot\|u_n\| + a_n^2\|u_n\|^2\\ &+ 2\alpha_n \mathrm{Re}\,\langle Sx_n - q, j(y_n - q)\rangle\\ &+ 2\alpha_n \mathrm{Re}\,\langle Sx_n - q, j(x_{n+1} - q) - j(y_n - q)\rangle.\end{aligned}$$

Hence, using (3) and definition of M, we get

$$\begin{aligned}\|x_{n+1} - q\|^2 \le\ & \|x_n - q\|^2 - 2\alpha_n\|x_n - q\|^2 + \alpha_n^2\|x_n - q\|^2\\ &+ 2(1-\alpha_n)a_n M^2 + a_n^2 M^2 + 2\alpha_n\|y_n - q\|^2\\ &- 2\alpha_n\phi(\|y_n - q\|) + 2\alpha_n c_n,\end{aligned}$$

where

$$c_n = \mathrm{Re}\,\langle Sx_n - q, j(x_{n+1} - q) - j(y_n - q)\rangle. \tag{16}$$

By (13) and (15), and using that $a_n \leq \alpha_n \alpha_n^c$ and $-2\alpha_n a_n + a_n^2 \leq 0$, we obtain

$$\begin{aligned} \|x_{n+1} - q\|^2 \leq \|x_n - q\|^2 - 2\alpha_n \|x_n - q\|^2 + \alpha_n^2 M^2 + 2\alpha_n \alpha_n^c M^2 \\ + 2\alpha_n \|x_n - q\|^2 + 4\alpha_n \beta_n M^2 - 2\alpha_n \phi(\|y_n - q\|) + 2\alpha_n c_n \end{aligned}$$

and hence

$$\|x_{n+1} - q\|^2 \leq \|x_n - q\|^2 - 2\alpha_n \phi(\|y_n - q\|) + \alpha_n \lambda_n, \tag{17}$$

where

$$\lambda_n = (\alpha_n + 2\alpha_n^c + 4\beta_n) M^2 + 2c_n.$$

First we show that $c_n \to 0$ as $n \to \infty$. Observe that from (4) and (5) we have

$$\begin{aligned} &\|x_{n+1} - y_n\| \\ &\quad = \|(\beta_n - \alpha_n)(x_n - q) + \alpha_n(Sx_n - q) - \beta_n(Sy_n - q) + a_n u_n - b_n v_n\| \\ &\quad \leq (\beta_n + \alpha_n)\|x_n - q\| + \alpha_n \|Sx_n - q\| + \beta_n \|Sy_n - q\| \\ &\quad + \alpha_n \|u_n\| + \beta_n \|v_n\| \end{aligned}$$

and hence, by (13) and definition of M,

$$\|x_{n+1} - y_n\| \leq 2(\alpha_n + \beta_n) M. \tag{18}$$

Therefore,

$$\|x_{n+1} - q - (y_n - q)\| \to 0 \text{ as } n \to \infty.$$

Since $\{x_{n+1} - q\}$, $\{y_n - q\}$ and $\{Sx_n - q\}$ are bounded and j is uniformly continuous on any bounded subsets of X, we have

$$\begin{aligned} j(x_{n+1} - q) - j(y_n - q) \to 0 \text{ as } n \to \infty, \\ c_n = \langle Sx_n - q, j(x_{n+1} - q) - j(y_n - q) \rangle \to 0 \text{ as } n \to \infty. \end{aligned}$$

Thus

$$\lim_{n \to \infty} \lambda_n = 0.$$

Set

$$\inf \{\|y_n - q\| : n \geq 0\} = \delta \geq 0.$$

We prove that $\delta = 0$. Assume the contrary, i.e. $\delta > 0$. Then $\|y_n - q\| \geq \delta > 0$ for all $n \geq 0$. Since ϕ strictly increases and $\phi(0) = 0$,

$$\phi(\|y_n - q\|) \geq \phi(\delta) > 0.$$

Thus from (17)

$$\|x_{n+1} - q\|^2 \leq \|x_n - q\|^2 - \alpha_n \phi(\delta) - \alpha_n [\phi(\delta) - \lambda_n] \tag{19}$$

for all $n \geq 0$. Since $\lim \lambda_n = 0$, there exists a positive integer n_0 such that $\lambda_n \leq \phi(\delta)$ for all $n \geq n_0$. Therefore, from (19) we have

$$\|x_{n+1} - q\|^2 \leq \|x_n - q\|^2 - \alpha_n \phi(\delta),$$

or rewritten,

$$\alpha_n \phi(\delta) \leq \|x_n - q\|^2 - \|x_{n+1} - q\|^2$$

for all $n \geq n_0$. Hence

$$\phi(\delta) \sum_{j=n_0}^{n} \alpha_j = \|x_{n_0} - q\|^2 - \|x_{n+1} - q\|^2 \leq \|x_{n_0} - q\|^2,$$

which implies $\sum_{n=0}^{\infty} \alpha_n < \infty$, contradicting (7). Therefore, $\delta = 0$. From definition of δ, there exists a subsequence of $\{\|y_n - q\|\}$, which we will denote by $\{\|y_j - q\|\}$, such that

$$\lim_{j \to \infty} \|y_j - q\| = 0. \tag{20}$$

Observe that from (4) for all $n \geq 0$ we have

$$\begin{aligned} \|x_n - q\| &= \|y_n - q + \beta_n(x_n - q) - \beta_n(Sy_n - q) - b_n v_n\| \\ &\leq \|y_n - q\| + \beta_n \|x_n - q\| + \beta_n \|Sy_n - q\| + b_n \|v_n\|. \end{aligned}$$

Since $b_n \leq \beta_n$, by definition of A, B and M we get

$$\|x_n - q\| \leq \|y_n - q\| + 2\beta_n M \tag{21}$$

for all $n \geq 0$. Thus by (6), (21) and (20) we have

$$\lim_{j \to \infty} \|x_j - q\| = 0. \tag{22}$$

Let $\varepsilon > 0$ be arbitrary. Since $\lim \alpha_n = 0$, $\lim \beta_n = 0$ and $\lim \lambda_n = 0$, there exists a positive integer N_0 such that

$$\alpha_n \leq \frac{\varepsilon}{8M}, \quad \beta_n \leq \frac{\varepsilon}{8M}, \quad \lambda_n \leq \phi\left(\frac{\varepsilon}{2}\right)$$

for all $n \geq N_0$. From (22), there exists $k \geq N_0$ such that

$$\|x_k - q\| < \varepsilon. \tag{23}$$

We prove by induction that

$$\|x_{k+n} - q\| < \varepsilon \tag{24}$$

for all $n \geq 0$. For $n = 0$ we see that (24) holds by (23). Suppose that (24) holds for some $n \geq 0$ and that $\|x_{k+n+1} - q\| \geq \varepsilon$. Then by (18) we get

$$\begin{aligned}\varepsilon \leq \|x_{k+n+1} - q\| &= \|y_{k+n} - q + x_{k+n+1} - y_{k+n}\| \\ &\leq \|y_{k+n} - q\| + \|x_{k+n+1} - y_{k+n}\| \\ &\leq \|y_{k+n} - q\| + 2(\alpha_{k+n} + \beta_{k+n})M \leq \|y_{k+n} - q\| + \frac{\varepsilon}{2}.\end{aligned}$$

Hence

$$\|y_{k+n} - q\| \geq \frac{\varepsilon}{2}.$$

Since ϕ is increasing, from (17) we get

$$\begin{aligned}\varepsilon^2 \leq \|x_{k+n+1} - q\|^2 &\leq \|x_{k+n} - q\|^2 - 2\alpha_{k+n}\phi\left(\frac{\varepsilon}{2}\right) + \alpha_{k+n}\phi\left(\frac{\varepsilon}{2}\right) \\ &\leq \|x_{k+n} - q\|^2 < \varepsilon^2,\end{aligned}$$

which is a contradiction. Thus we proved (24). Since ε is arbitrary, from (24) we have

$$\lim_{n\to\infty} \|x_n - q\| = 0,$$

which completes the proof of the Theorem. □

If T is strongly accretive (i.e. $\phi(t) = kt^2$) and demicontinuous (i.e. $x_n \xrightarrow{s} x \Rightarrow Tx_n \xrightarrow{w} Tx$), then the existence of a solution of the equation $Tx = f$ for each $f \in X$ follows from Deimling [3].

Remark 1. If in Theorem 1, $\beta_n = 0$, $b_n = 0$, then we obtain a result that deals with the Mann iterative process with errors.

Now we state the Ishikawa and Mann iterative process with errors for the quasi-strongly pseudo-contractive operators.

Theorem 2. *Let X be a uniformly smooth Banach space, let K be a non-empty bounded closed convex subset of X and $T : K \to K$ be a quasi-strongly pseudo-contractive mapping, i.e.*

$$\operatorname{Re}\langle Tx - Ty, j(x-y)\rangle \leq \|x - y\|^2 - \phi(\|x - y\|), \tag{25}$$

where $\phi : [0, \infty) \to [0, +\infty)$ is a strictly increasing function with $\phi(0) = 0$. Let q be a fixed point of T and let for $x_0 \in K$ the Ishikawa iteration sequence $\{x_n\}$ be defined by

$$\begin{aligned} y_n &= \overline{\beta}_n x_n + \beta_n T x_n + b_n v_n, \\ x_{n+1} &= \overline{\alpha}_n x_n + \alpha_n T y_n + a_n u_n, \quad n \geq 0,\end{aligned}$$

where $\{u_n\}$, $\{v_n\} \subset K$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{a_n\}$, $\{b_n\}$ *are sequences as in Theorem 1 and*

$$\overline{\alpha}_n = 1 - \alpha_n - a_n,$$
$$\overline{\beta}_n = 1 - \beta_n - b_n.$$

Then $\{x_n\}$ *converges strongly to the unique fixed point of* T.

Proof. Obviously $\{x_n\}$ and $\{y_n\}$ are both contained in K and, therefore, bounded. Since T is quasi-strongly pseudo-contractive, $I - T$ is quasi-strongly accretive. Further, since (25) with $y = q$ and $T = S$ becomes (10), the proof of Theorem 1 follows. □

Remark 2. If in Theorem 2, $\beta_n = 0$, $b_n = 0$, then we obtain the corresponding result for the Mann iteration process with errors.

Remark 3. Theorem 2 extends and improves the main result of Liu [8, Theorem 1] in the following ways:

(1) the assumption that $\{u_n\}$ and $\{v_n\}$ are two summable sequences is replaced by the assumption that $\{u_n\}$ and $\{v_n\}$ are two bounded sequences;
(2) T need not be Lipschitz;
(3) the assumption that T is a strongly accretive mapping is replaced by the assumption that T is quasi-strongly accretive.

If T is a strongly pseudo-contractive (i.e. $\phi(t) = kt^2$) and continuous mapping in Theorem 2, then T has a fixed point by Proposition 3 of Martin [9]. So, Theorem 2 gives and affirmative answer to the open problem (Chidume [2]) in the more general setting and generalizes Theorem 2 in [2] in several aspects.

Also, Theorem 2 improves and extends the results of Chidume [2, Theorem 4], Deng [4,5], Deng and Ding [6], Liu [8, Theorem 1], Tan and Hu [10, Theorem 4.2], Xu [11, Theorem 3.3] and Zhou and Jia [12, Theorem 2.1].

Remark 4. A mapping T with domain $D(T)$ and the range $R(T)$ in X we will be called a quasi-hemicontraction if $F(T) = \{x \in D(T) : x = Tx\} \neq \emptyset$ and if for all $x \in D(T)$ and $q \in F(T)$ there exist $j(x - q) \in \dot{J}(x - q)$ and a strictly increasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ such that

$$\text{Re}\,\langle Sx - q, j(x - q)\rangle \leq \|x - q\|^2 - \phi(\|x - q\|). \tag{26}$$

Since (26) is identical with (10) in the proof of Theorem 1, we have the following.

Corollary 1. *Let* X *be as in Theorem 1 and let* $T : X \to X$ *be a quasi-hemicontraction. Then* $F(T)$ *is singleton. Suppose that* $R(T)$ *is bounded* *and let* $\{\alpha_n\}$, $\{\beta_n\}$, $\{a_n\}$, $\{b_n\}$ *in* $[0,1]$ *and* $\{u_n\}$, $\{v_n\}$ *in* X *be sequences as in Theorem 1. Let* $\{x_n\}$ *satisfy*

$$y_n = (1-\beta_n)x_n + \beta_n T x_n + b_n v_n,$$
$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n + a_n u_n.$$

Then the sequence $\{x_n\}$ *converges strongly to the fixed point of* T.

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